Rashba Precession in Quantum Wires

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The length over which electron spins reverse direction due to the Rashba effect when injected with an initial polarization along the axes of a quantum wire is investigated theoretically. A soft wall confinement of the wire renormalizes the spin–orbit parameter (and the effective mass) stronger than hard walls. Electron–electron interactions enhance the Rashba precession while evidence is found that the coupling between transport channels may suppress it.

Spin transport has regained considerable interest in recent years, partly because of the prospective for qualitatively novel electronic devices and partly since fundamentally new mesoscopic quantum coherence phenomena could be investigated in the magnetic degree of electronic freedom. A prime example is the spin transistor proposed by Datta and Das [1].

Here we investigate how Rashba spin precession [2] is affected by many body effects [3] as expected to be important in semiconducting devices. We consider electrons moving along a one-dimensional (1D) quantum "wave guide." Also the cases of two interacting channels and different transversal confinement potentials is addressed. Of prime interest will be the length λ_R over which spins initially polarized along the channel axes reverse spin-direction while moving along the channel.

In a 2D layer spin splitting is a consequence of spin-orbit coupling

$$H^{\rm so} = \alpha(\sigma_x p_z - \sigma_z p_x) \qquad (1)$$

proportional to the Rashba parameter α and thus to the intrinsic or by means of gates externally applied electric field in y-direction, perpendicular to the layer [2]. $H^{\rm so}$ is also proportional to the momentum of spins (p_x,p_z) in the "active" $x-z$-plane, $\sigma_{x,z}$ are Pauli matrices. Precession then occurs on the length scale $|k_+ - k_-|^{-1}$ of the spin split wavenumbers $k_\pm$ at the Fermi energy. If the effective mass approximation $\epsilon(k) = k^2/2m$ is applicable, as for many semiconductors, the Rashba length

$$\frac{2\pi}{|k_+ - k_-|} = \frac{\pi}{m\alpha} \qquad (2)$$

does *not* depend on carrier density or Fermi energy.

With electrons being confined in z-direction, to zero'th order in $H^{\rm so}$ this picture carries over to 1D. Only to higher order, $H^{\rm so}$ will weakly mix different transport channels of the wave guide, which up to $O(\alpha^5)$ can be accounted for by renormalizing $\alpha \to \alpha^*$ in $\epsilon_\pm(k)$ and, within the effective mass approximation, by $m \to m^*$. In the latter case, the Rashba length modifies according to $(m\alpha)^{-1} \to (m^*\alpha^*)^{-1}$. For a quantitative estimate of both renormalizations the intra-subband eigenfunctions

$$\psi_{kns}(x, z) = e^{ikx}\phi_n(z)(\cos(m\alpha z)|s\rangle + {\rm i}\,\sin(m\alpha z)|-s\rangle) \qquad (3)$$

are needed, which are plane waves of momentum k along the wave guide and, without inter-subband scattering, slightly modified subband states ϕ_n (subband index n) in z-direction, spin-polarized $s = \pm$ on the axes at $z = 0$. For a harmonic confinement (subband energy ω_0), as relevant for example for samples defined by gating [4], the perturbative estimate yields $\alpha^* = \alpha(1-\eta)$ and $m^* = m(1+8\eta^2)$ in the ground subband. The dimensionless parameter $\eta = (wm\alpha/2)^2$ measures the bare value of the spin–orbit strength α in (1) by comparing the wave guide

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width $w = 2/\sqrt{mw_0}$ with the length $(m\alpha)^{-1}$. For a hard wall confinement on the other hand (again of width w), as possibly more relevant for wires fabricated by the cleaved edge technique [5], the renormalizations become $\alpha^* = \alpha(1 - (1/6 - 1/\pi^2)\eta)$ and $m^* = m(1 + 3(4/3\pi)^6\eta^2)$ i.e. they are significantly reduced compared to the soft wall case since $1/6 - 1/\pi^2 \approx 0.065$ and $3(4/3\pi)^6/8 \approx 0.002$.

The main idea of the transistor operation [1] is to vary the strength of the electric field and thereby $\pi/m\alpha$. This is achieved by charging a gate parallel to the layer. Unless carefully compensated by a second back gate [6], however, the carrier density in the structure will change at the same time. Ignoring interactions, this would be unimportant when $\epsilon(k) = k^2/2m^*$ but the interaction strength depends, even quite sensitively, on carrier density through the r_s-parameter. In order to investigate possible consequences for the Rashba precession, we employ the Tomonaga–Luttinger (TL) model [7] as the most precise low energy description of 1D metals. Here we shall not focus on various characteristic power laws predicted by this model, also regarding spin properties [8] coming from the charge sector, but rather focus on the question how interactions influence the length λ_R.

As a second striking property the TL-model predicts charge-spin separation which, interestingly and contrary to statements in the literature [8,9], is *not* spoiled in semiconducting quantum wires unless spin-orbit coupling is not exceedingly strong $\eta \gtrsim 1$ to devastate the effective mass description. On the other hand, for nonquadratic dispersion relations, charge-spin separation is in general destroyed with H^{so}. An example of this are carbon nanotubes where $v_\pm = v_F \pm \alpha$ with α originating in this case from the curved surface [10] so that $v_+ \neq v_-$ spoils charge-spin separation.

A realistic interaction in 1D

$$V(x) = \frac{e^2}{\kappa}\left(\frac{1}{\sqrt{x^2+w^2}} - \frac{1}{\sqrt{x^2+w^2+4R^2}}\right) \quad (4)$$

is specified by the wire width w and a cut-off at large distance R because of screening by the nearest metals, such as the gates that help defining the quantum channel. Since spin properties involve finite momenta q of the Fourier-transformed interaction $\hat{V}(q)$ (and contrary to charge properties not the dominant long wavelength limit $\hat{V}(q \to 0)$) the interaction range R plays only a minor rôle if $(R/w, k_F R) \gg 1$. In carbon nanotubes, for example, interactions will alter the spin velocity only weakly, in marked contrast to the charge velocity [11].

How to include the Rashba term in the TL-model? In previous work [8,9] the Fermi velocities v_+ and v_- have simply been put to different values, which for $\varepsilon(k) = k^2/2m^*$ does not describe the leading effect of H^{so}. Rather *both* velocities change slightly but obey $v_+ = v_-$ and charge-spin separation. In effective mass systems, H^{so} acts solely in the spin sector of the corresponding TL low energy model (of length L), where the topological term

$$\frac{\pi}{4L}\left(v_N N_\sigma^2 + v_J J_\sigma^2\right) - m^*\alpha^* v_F J_\sigma \quad (5)$$

is most relevant for the following. N_σ and J_σ denote the usual currents of velocities $v_{N/J}$ where with Coulomb repulsion $(v_N, v_J) \neq v_F$ [7]. In strictly spin isotropic systems $v_N = v_J$. Since we expect this isotropy to be broken only weakly, both of these velocities should be similar in magnitude and also similar to the velocity v_σ of spin wave propagation. This latter quantity has been determined recently by extensive quantum Monte-Carlo simulations [12]. With increasing interaction strength, equivalent to a decreasing carrier density, v_σ/v_F was found to decrease, even below 0.5 when accounting for parameters of existing quantum wires [5].

Many quantities of interest can be calculated exactly using (5). In particular it can be shown [13] that spins polarized in x-direction along the wire precess now over a length that acquires a factor v_J/v_F compared to the length of Eq. (2). With $v_J = v_\sigma$ we conclude from [12] that this length *decreases* with increasing interaction strength. A similar conclusion has been drawn for two-dimensional electrons after treating the interaction perturbatively [3]. This trend is opposite to what is expected for almost linear single-electron dispersions (such as narrow gap semiconductors) but agrees with experimental observations [14].

For very large and very small $k_F w$, asymptotic expressions can be given for v_σ which result in the following dependencies for the Rashba length:

$$\lambda_R \sim \frac{\pi}{m^*\alpha^*}[1 - f(2k_F w)] \quad \text{if } f(2k_F w) < \frac{1}{2\pi}$$

at large k_F, $f(x) := \sqrt{\frac{2}{\pi}\frac{w}{\alpha_B}}x^{-3/2}e^{-x}$, and

$$\lambda_R \sim \frac{\pi^2}{3m^*\alpha^*}\frac{a_B}{w}\frac{2k_F w}{\ln 2R/w} \quad \text{if } k_F \ll 1/R,$$

a_B is the Bohr radius. This latter estimate follows from the conjecture [15] that the system falls into the universality class of the Hubbard model at small fillings in which case the interaction range R becomes relevant.

Now we turn to the case of two occupied channels in the quantum wire. Then the renormalizations $\alpha \to \alpha_j$ and $m \to m_j$ due to H^{so} acquire a channel index j; for example, in a harmonic confining potential α_2 of the upper channel renormalizes by a factor of 3 more than α_1.

Between the channels the same microscopic (screened Coulomb type) interaction takes place (4) as within each channel. Up to now no theory is available to determine spin velocities in multichannel situations and we would like to sketch a possible solution (details will be published elsewhere). We assume 1) sufficiently different carrier densities in both channels to conserve the TL-phase [16] (otherwise spin gaps could spoil the Rashba precession), and 2) charge-spin separation. The Fermi momentum in the upper channel may be taken as $k_2 = \sqrt{\frac{m_2}{m_1}k_1^2 - 2m_2\omega_0}$ where ω_0 is the subband spacing and $m_{2/1}$ are the effective masses of the upper/lower channel after renormalization through H^{so2}.

We expect two spin velocities $v_{\sigma 1}$ and $v_{\sigma 2}$, both of which will be relevant for the Rashba precession, similar to the single channel case. They can be obtained from the microscopic interaction through generalized susceptibilities

$$(v_N)_{ij} = \frac{L}{\pi}\frac{\partial^2 E_0}{\partial N_i \partial N_j} \quad (6)$$

and

$$(v_J)_{ij} = \frac{L}{\pi}\frac{\partial^2 E_0}{\partial J_i \partial J_j} \quad (7)$$

when suitably generalizing the relation $v_\sigma = \sqrt{v_N v_J}$ observed in one channel. Here, N_j and J_j are the currents (5) in channel j.

How can we obtain v_N and v_J for a given microscopic interaction? In principle we already know from the single channel case how notoriously difficult spin velocities are to evaluate [12]. Leading corrections at not-too-small carrier densities to the ground state energy E_0/L per length can be obtained perturbatively. Fock-exchange terms $\sim \hat{V}(k_i \pm k_j)$ have to be included adequately for spin properties, which goes beyond the RPA approximation mostly used for estimating charge velocities [17] since the $\hat{V}(q=0)$ contribution drops out here.

The result is

$$v_N = \begin{pmatrix} v_1 - \frac{\hat{V}(2k_1)}{\pi} + \hat{V}_- & -\hat{V}_+ \\ -\hat{V}_+ & v_2 - \frac{\hat{V}(2k_2)}{\pi} + \hat{V}_- \end{pmatrix} \quad (8)$$

and

$$v_J = \begin{pmatrix} v_1 + \hat{V}_- & -\hat{V}_- \\ -\hat{V}_- & v_2 + \hat{V}_- \end{pmatrix}. \quad (9)$$

Here, v_j are the bare Fermi velocities and $\hat{V}_\pm = (\hat{V}(k_1 - k_2) \pm \hat{V}(k_1 + k_2))/2\pi$. Both limits, $k_2 \to 0$ (almost empty upper channel) and $k_2 \to k_1$ (limit of equivalent subbands), indicate instabilities in v_N and v_J when $\hat{V}(q=0) > \pi v_j$ (long range interactions, note that $V(q)$ decreases with increasing q for any realistic electron–electron interaction). These are precursors of the Cooper or charge density wave instabilities [16], that occur in repulsively interacting two-channel systems near the threshold for opening the second channel or near equal carrier densities in both channels, respectively. We also see in (9) that v_J leaves the Galilei mode (1,1) independent of the interaction. For $v_1 = v_2$ this mode is an eigenmode so that perturbation theory pretends Galilei invariance of the spin sector, similar as in the single channel case [15]. Then perturbative renormalization of the spin velocities is solely due to v_N.

Resulting spin velocities are shown in Fig. 1. One of the velocities, $v_{\sigma 1}$, is decreasing with decreasing carrier density $2k_1/\pi$, similar as in the single channel case [12]. More strikingly, $v_{\sigma 2}$ first increases and exceeds the larger of the two Fermi velocities v_1. Then according to Eq. (2) we have evidence for a *suppression* of

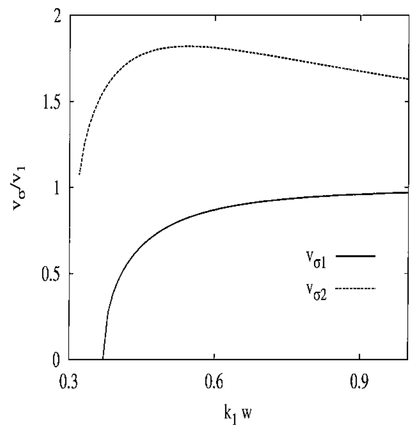

Fig. 1. Spin velocities in a two channel quantum wire in units of the larger of the Fermi velocities v_1, versus $k_1 w$ in perturbation theory.

[2] Here $2k_j/\pi$ denote the densities in the j'th channel with the effect density adjustment due to the inter-channel repulsion already included.

the Rashba precession as a result of interchannel coupling by electron–electron interaction.

In conclusion, we have discussed how a hard or soft confining potential, and the intra-and interchannel interaction affect the Rashba precession along a quantum wire.

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