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PAULUSVERLAG FREIBURG SCHWEIZ

Uwe Meixner

Aquinas on the essential composition of objects

I. In this paper a formal language will be constructed in which an essential part of Aquinatic ontology can be precisely formulated. In the formal language an axiomatization of this part of Aquinatic ontology will be given, and its exegetic adequacy shown by deducing a long series of theorems that are all in accordance with the ontological teachings of Aquinas. It will be made plausible that no theorem contradicting Aquinatic ontology can be derived. Finally the consistency of the axiom-system will be demonstrated by providing a model for it. The texts referred to are the *Summa theologiae, Summa contra gentiles* and *De ente et essentia*.

II. Before beginning a remark concerning method is in order. This paper has been written in the conviction that the logical reconstruction of philosophical doctrines can be of value for our understanding of them (if they can be at all subjected to such treatment). This conviction is not uncontroversial. It is in the nature of a logical reconstruction that it contains certain deviations from the original. In a logical reconstruction inconsistencies are avoided, that is, inessential inconsistencies due to carelessness; for essentially inconsistent theories are not amenable to logical reconstruction. (Sometimes, however, an attempt at logical reconstruction is necessary in order to show that a philosophical theory is essentially inconsistent.) In a logical reconstruction instances of ambiguity and vagueness are unravelled into alternative non-ambiguous and non-vague logical sub-reconstructions. The theoretical horizon of a logical reconstruction is normally wider than that of the original; it normally points out conclusions that the author of the original did not think of or at least did not mention; these 'new insights', however, must not be contrary to the spirit of the original; else, the logical reconstruction is inadequate. A logical reconstruction employs logical resources of which the author had no or only an inadequate idea. A logical reconstruction demonstrates conclusions that the author of the original merely stated on the strength of his intuitions or arrived at by entirely inconclusive arguments. A logical reconstruction is more, sometimes much more systematic than the original, connecting results that are not connected in the latter; but it may also disconnect results that are connected in the original, if no justifiable logical bond can be found between them.

If the original is open to logical reconstruction, then the mentioned deviations of its logical reconstruction from it, if they remain within limits, will not contribute to its distortion but rather to its clarification, revealing, as it were, what the author would have said if he had had the modern logical techniques at his disposal.

III. The present logical reconstruction refers to the ontological doctrines of Aquinas concerning the composition *(compositio)* of (existent) objects *(res per se subsistentes, substantiae primae)* by their essential aspects. Aquinas knows five essential aspects of an object: its *matter,* its *pure substantial form,* its *being,* its *essence,* and its *actuating substantial form.* Accordingly five functional terms are introduced: m(0), f(0), s(0), w(0), a(0), where 0 can be replaced by any object-variable or object-name. They are to be read as 'the matter of 0', 'the pure form of 0' (short for 'the pure substantial form of 0'), 'the being of 0', 'the essence of 0', 'the actuating form of 0' (short for 'the actuating substantial form of 0').

Normally object-aspects are not objects (there are, however, exceptions); thus it is not generally meaningful to speak, for example, of the essence of the being of an object, or of the being of the being of an object. The formal language will consequently be constructed in such a manner that iterated functional terms like w(f(0)), s(w(f(0))), f(s(m(0))) etc. are not well-formed. To allow such functional-terms to be well-formed is warranted by nothing in the writings of Aquinas.

Certain objects according to Aquinas have no matter; for such objects the function *the matter of* is initially not defined. However, a complete definition (that is, a definition for all objects) is secured for this function by assuming an empty aspect of every object, and by stipulating that if an object has no matter its matter is its empty aspect. Correspondingly, the functional term c(0) is introduced, which is to be read as 'the empty aspect of 0'.

Aspects of the same object, according to Aquinas, combine to form an aspect of the object or the object itself, and Aquinas, as has been said, knows five aspects of an object. In correspondence to this we have a dyadic functional term + such that only functional expressions with + of the forms $(\beta(0)+\beta'(0))$, $((\beta(0)+\beta'(0))+\beta''(0))$, $(\beta''(0)+(\beta(0)+\beta'(0)))$ are well-formed, where β, β', β'' may each be replaced by m, f, s, and c. Why not also by w and a? Because we are here referring to the language ***without*** defined expressions, and with the help of the compositional functor w and a can be defined in keeping with the writings of Aquinas. The first definition is:
(i) $w(0) := (f(0)+m(0))$ – the essence of an object is its pure form combined with its matter.

According to Aquinas this definition is not adequate for all objects; it is only adequate for material objects. But the introduction of c(0) makes it possible for us to regard it as the general definition of essence. Let 'g' designate some immaterial object; then $w(g)=(f(g)+m(g))$ is equivalent to $w(g)=f(g)$, which corresponds to the Aquinatic definition of essence for immaterial objects: the essence of an immaterial object is its pure form. Since g is immaterial we have $m(g)=c(g)$, hence $(f(g)+m(g))=(f(g)+c(g))$, hence $(f(g)+m(g))=f(g)$ – the empty aspect of g adds nothing to the pure form of g. Aquinas says:

1. In hoc ergo differt essentia substantiae compositae ‹sive materialis› et substantiae simplicis ‹sive immaterialis›, quod essentia substantiae compositae non est tantum forma sed complectitur formam et materiam, essentia autem substantiae simplicis est forma tantum.
   (De ente et essentia, 4, 25) ‹Divi Thomae Aquinatis *Opuscula Philosophica*, cura et studio P. Fr. Raymundi, M. Spiazzi O. P., Marietti, Roma 1954›.

While Aquinas speaking of composition always means ***proper composition,*** that is, the composition of different, non-empty aspects (of the same object), we also have ***improper composition,*** that is, the composition of an aspect with itself or with the empty aspect (of the same object). (By being different, object-aspects – at least the essential object-aspects considered by Aquinas – are distinct, since they cannot be proper parts of each other or overlap).

The second definition is:
(ii) $a(0) := (f(0) + s(0))$ – the actuating form of an object is its pure form combined with its being; or: the actuating form of an object is the composition of its pure form and its being.

Aquinas does not verbally distinguish between the actuating form of an object and its pure form, and on the whole he seems to be unaware of their being distinct (in most objects). However, his doctrines can only be consistently interpreted by considering the pure form of an object to be normally distinct from its actuating form. In the following quotations Aquinas is referring to the actuating form of an object:

2. ex forma et materia relinquitur esse substantiale quando componuntur (De ente et essentia, 6, 34).

3. per formam enim, quae est actus materiae, materia efficitur ens actu et hoc aliquid
(De ente et essentia, 2, 6).

And Aquinas adds:

4. unde illud quod superadvenit non dat esse actu simpliciter materiae, sed esse actu tale ... Unde, quando talis forma acquiritur, non dicitur generari simpliciter, sed secundum quid.
(De ente et essentia, 2, 6).

In the quotation below, however, Aquinas is referring to the pure form of an object:

5. esse substantiae compositae non est tantum formae neque tantum materiae, sed ipsius compositi; essentia autem est secundum quam res esse dicitur. Unde oportet ut essentia, qua res denominatur ens, non tantum sit forma, nec tantum materia, sed utrumque, quamvis huiusmodi esse suo modo sola forma sit causa.
(De ente et essentia, 2, 6bis).

Here Aquinas names a third ultimate distinct component in the composition of a material substance beside form and matter: its being *(esse)*; while in the previous quotations he only mentions form and matter, obviously intending that they by themselves suffice to constitute the object. This apparent discrepancy can be resolved by supposing that in the last quotation Aquinas means by 'forma' the pure form of the object which together with the matter of the object composes its essence, which in its turn enters into composition with the being of the object to constitute the object itself; while in the previous quotations he means by 'forma' the pure form of the object *in composition with its being,* that is, the actuating form of the object, which together with the matter of the object composes the object itself. It amounts to the same, whether pure form and matter are first composed to constitute essence, and then essence and being to constitute the object; or whether pure form and being are first composed to constitute actuating form, and then actuating form and matter to constitute the object.

In the next quotation the first instance of the word 'forma' means the actuating form of the object, the second instance, however, its pure form:

6. In substantiis autem compositis ex materia et forma est duplex compositio actus et potentiae: prima quidem ipsius substantiae, quae componitur ex materia et forma; secunda vero, ex ipsa substantia iam composita et esse; quae etiam potest dici ex *quod est* et *esse,* vel ex *quod est* et *quo est.* (Summa contra gentiles, 2, 54).

In this passage we also have an example of the equivocal use of the word 'substantia' in the writings of Aquinas; the first instance of this word signifies the same as 'res' ('object', 'first substance'), the second and third, however, the same as 'essentia seu natura' ('essence', 'second substance'). Aquinas is aware of this equivocation; in *Summa theologiae,* I,29,2 – following Aristotle – he explicitly distinguishes the two meanings of 'substantia':

7. substantia dicitur dupliciter. Uno modo dicitur substantia *quidditas rei,* quam significat definitio, secundum quod dicimus quod *definitio significat substantiam rei:* quam quidem substantiam Graeci *usiam* vocant, quod nos *essentiam* dicere possumus. – Alio modo dicitur substantia *subiectum vel suppositum quod subsistit in genere substantiae.*

In contrast, Aquinas seems not to be aware of the equivocation in his use of the word 'forma'; he apparently does not differentiate between what we have here been calling 'pure form' and what we have here been calling 'actuating form'. The identification of what on the strength of his own theory is non-identical is bound to lead to some confusion, as we shall see. (The use of 'forma' to refer to the actuating form of an object is, it seems, predominant over the use of 'forma' to refer to the pure form of an object.)

The matter of a material object cannot enter into composition with the being of that object (while the pure form of any object enters into composition with its being to constitute its actuating form); there is no 'actuating matter' of a material object; matter is actuated by the actuating form (compare quotations 2 and 3); the complementary view of pure form being actuated by the actuating matter is absurd for Aquinas; not matter but pure form is the 'vehicle' of being:

8. forma tamen potest dici quo est, secundum quod est essendi principium (Summa contra gentiles, 2,54).
9. quamvis huiusmodi esse suo modo sola forma sit causa (the last phrase of quotation 6).
10. materia vero non habet esse nisi per formam (De ente et essentia, 6,36).

In consequence the composition-function is initially not defined if the arguments are the matter of a material object and the being of that object. We can, however, stipulate that for any material object the composition of its matter with its being is its empty aspect.

IV. It has become apparent that the possibilities of expression by means of the compositional-functor + are drastically limited in the intended formal language. To sum up:

(a) A well-formed compositional expression contains at most two instances of +.
(b) Only expressions having the form f(0), m(0), s(0), c(0) may occur in a well-formed compositional expression (without defined expressions) as argument expressions that are not themselves compositional expressions.
(c) Exactly one object-variable or exactly one object-name occurs in a well-formed compositional expression.

These restrictions can be justified as follows:

(a') Aquinas does not consider more complex compositions than can be expressed by the compositional expressions allowed to be well-formed.
(b') Aquinas does not in general consider the composition of an object with an object or with an object-aspect; he only considers the composition of an object-aspect with an object-aspect (occasionally, however, an object-aspect is identical with an object).
(c') Aquinas does not in general consider the composition of aspects of different objects; he only considers the composition of aspects of the same object (occasionally, however, an aspect of one object is identical with an aspect of another object).

In spite of the restrictions there still can be generated infinitely many well-formed compositional expressions; but for each object-designator (object-variable or object-name) the number of well-formed compositional expressions 'around' it is finite.

The axiom-system will be constructed in such a manner that all well-formed compositional expressions around the object-designator 0 are reducible to 0, c(0), m(0), f(0), s(0), w(0) (= (f(0) + m(0))) and a(0) (= (f(0) + s(0)). As has been said, Aquinas knows only five aspects of an object; in addition to these we have for reasons of formal simplification the empty aspect of an object; and by composition of the aspects of an object there issues an aspect of this object or the object itself. In special cases the reduction of compositional expressions can be carried further than this. For example, if 0 designates an immaterial object, we have m(0) = c(0) and w(0) = f(0).

Which predicates should belong to our formal language? As basic predicate only the identity-predicate =. With respect to the sentences and open sentences generable with the help of = no further restrictions are made; such – as, for example, requiring that one and the same object designator has to occur left and right of = – would not be justified by the writings of Aquinas. As will become apparent a great many other predicates for Aquinatic ontological distinctions can be defined with the help of the identity-predicate, the aspect-expressions and the logical expressions. The rendering of 'est' by 'is identical with' in the present context of a treatment of the composition of objects by their aspects is, of course, a matter of interpretation; this rendering can be said to be overwhelmingly suggested by the relevant passages in the writings of Aquinas.

V. The reflections in sections III. and IV. are summed up and made precise by the following definition of the formal language T:

1. Object-variables (OVs) of T
(a) 'x' is an OV of T;
(b) if 0 is an OV of T, then 0' is an OV of T;
(c) OVs of T are only expressions satisfying (a) and (b).

2. Object-names (ONs) of T
(a) 'g' is an ON of T;
(b) if 0 is an ON of T, then 0' is an ON of T;
(c) ONs of T are only expressions satisfying (a) and (b).

3. Object-designators (ODs) of T
0 is an OD of T $=_{Df}$ 0 is an OV of T, or 0 is an ON of T

4. Primary aspect-expressions (PAEs) of T
(a) If 0 is an OD of T, then m(0), f(0), s(0) and c(0) are PAEs of T;
(b) PAEs of T are only expressions satisfying (a).

5. Secondary aspect-expressions (SAEs) of T
(a) If 0 is an OD of T and β(0) and β'(0) are PAEs of T, then (β(0) + β'(0)) is a SAE of T;
(b) SAEs of T are only expressions satisfying (a).

6. Aspect-expressions (AEs) of T
(a) PAEs and SAEs of T, are AEs of T;
(b) if 0 is an OD of T and β(0) is a PAE of T and (β'(0) + β''(0)) is a SAE of T, then (β(0)+ (β'(0) + β''(0))) and ((β'(0) + β''(0)) + β(0)) are AEs of T;
(c) AEs of T are only expressions satisfying (a) and (b).

7. Compositional expressions (CEs) of T
0 is a CE of T $=_{Df}$ 0 is an AE of T, but 0 is not a PAE of T

8. Tertiary aspect-expressions (TAEs) of T
0 is a TAE of T $=_{Df}$ 0 is an AE of T, but 0 is neither a PAE nor a SAE of T

9. Entity-designators (EDs) of T
0 is an ED of T $=_{Df}$ 0 is an OD of T, or 0 is an AE of T

10. Primary sententials (PSLs) of T
(a) If β and β' are EDs of T, then (β = β') is a PSL of T;
(b) PSLs of T are only expressions satisfying (a).

11. Sententials (SLs) of T
(a) PSLs of T are SLs of T;
(b) if β and β' are SLs of T, then ¬β, (β∧β'), (β∨β'), (β⊃β'), (β ≡ β') are SLs of T;
(c) if β is a SL of T in which in certain places X there occurs a certain ON of T, namely, 0, and is v a OV of T that does not occur in β, then – if v replaces 0 in all places X (β⟨v⟩ resulting from β) – ⋀vβ⟨v⟩ and ⋁vβ⟨v⟩ are SLs of T;
(d) SLs of T are only expressions satisfying (a), (b) and (c).

12. Primary sentences (PSs) of T
β is a PS of T $=_{Df}$ β is a PSL of T in which no OV of T occurs

13. Sentences (Ss) of T
β is a S of $T =_{Df} \beta$ is a SL of T in which no OV of T occurs free.

1. – 13 determines the syntax of T. The intended interpretation of T has been outlined, but of course there remains much to be said about it. The logical operators $\neg, \wedge, \vee, \supset, \equiv, \bigwedge, \bigvee$ are to be read as 'not', 'and', 'or' (in the sense of' not neither_, nor_'), 'if_, then_', '_if and only if_', 'for all objects', 'for some object'. In view of the intended interpretation relative to Aquinatic doctrine we define:
D1 $w(0) := (f(0) + m(0))$ (for all ODs 0 of T)
D2 $a(0) := (f(0) + s(0))$ (for all ODs 0 of T)
Finally, parentheses may be omitted in accordance with the following rules:
(a) External parentheses can be omitted.
(b) In the sequence $+, =, \neg, \wedge, \vee, \supset, \equiv$ binding-power is decreasing from left to right.
(c) In a conjunction (disjunction) the parentheses around an immediate member can be omitted if it is itself a conjunction (disjunction).

VI. Before we go on to formulate in T an axiom-system adequate in the intended interpretation of T for part of Aquinatic ontology, the logic has to be described by the use of which we intend to deduce theorems from the axioms. It is classical first-order predicate-logic with the identity-predicate and functional terms. However, there is one deductive restriction: Only ODs of T are quantifiable, which means that the deduction rules $\bigwedge v\beta\langle v\rangle \rightarrow \beta\langle 0'\rangle$ and $\beta\langle 0\rangle \rightarrow \bigvee v\beta\langle v\rangle$ ($\rightarrow$: 'logically implies') may only be applied if 0 is an OD of T. This restriction is in keeping with the intended interpretation of $\bigwedge$ and $\bigvee$: 'for all objects', 'for some object'; under the intended interpretation an AE of T, for example f(g'), will normally not refer to an object but only to an aspect of it. Aspects of objects which are not objects are thus not quantified over, and it is impossible to directly refer to them; that is, it is impossible to refer to them without referring at the same time to some object, which is a consequence of there being no simple names in T for aspects of objects that are not objects. Under the intended interpretation these semantic features mirror the ontologically dependent status of object-aspects which are not objects in contrast to the ontologically independent status of objects. Aquinas would have said that object-aspects which are not objects are less real than objects; the former have their being only in the latter. (The second-order deduction rules *If* $\beta' \rightarrow \beta\langle 0\rangle$, *and 0 does not occur* *free in* $\beta'\rightarrow\bigwedge v\beta\langle v\rangle$, *then* $\beta'\rightarrow\bigwedge v\beta\langle v\rangle$ and *If* $\beta\langle 0\rangle\rightarrow\beta'$, *and 0 does not occur free in* $\bigvee v\beta\langle v\rangle\rightarrow\beta'$, *then* $\bigvee v\beta\langle v\rangle\rightarrow\beta'$ may in any case only be applied *if 0 is an OD of T:* $f(g') = g'\rightarrow\bigvee x(f(x) = x)$, $f(g')$ does not occur in $\bigvee x'(x' = g')\rightarrow\bigvee x(f(x) = x)$; but clearly $\bigvee x'(x' = g')$ does not logically imply $\bigvee x(f(x) = x)$.)

Since only ODs of T are to be quantifiable and we nevertheless want to make unrestricted use of the deduction-rules referring to identity, these cannot be codified in the following manner: $\rightarrow\bigwedge x(x = x)$, $\rightarrow\bigwedge x\bigwedge x'(x = x' \supset (\beta\langle x\rangle \supset \beta\langle x'\rangle))$, but must rather be formulated thus: $\rightarrow 0 = 0$, $\rightarrow 0 = 0' \supset (\beta\langle 0\rangle \supset \beta\langle 0'\rangle)$.

VII. The axiom-system TO ('Thomasic ontology') consists of the following axioms:

($\beta(x)$ and $\beta'(x)$ is a PAE or a SAE of T having x as its OV)

A1 Every S of T having the form
$\bigwedge x(\beta(x) + \beta'(x) = \beta'(x) + \beta(x))$
is an axiom of TO.

A2 (a) Every S of T having the form
$\bigwedge x(\beta(x) + \beta(x) = \beta(x))$
is an axiom of TO.
(b) Every S of T having the form
$\bigwedge x(\beta(x) + c(x) = \beta(x))$
is an axiom of TO.

A3 Every S of T having the form
$\bigwedge x(\beta(x) + \beta'(x) = \beta(x) \supset \beta'(x) = \beta(x) \vee \beta'(x) = c(x))$
is an axiom of TO

B1 $\bigwedge x(x = (f(x) + m(x)) + s(x))$

B2 $\bigwedge x((f(x) + m(x)) + s(x) = (f(x) + s(x)) + m(x))$

B3 (a) $\bigwedge x\neg f(x) = m(x)$
(b) $\bigwedge x\neg s(x) = m(x)$
(c) $\bigwedge x\neg f(x) + s(x) = m(x)$

B4 (a) $\bigwedge x\neg x = c(x)$
(b) $\bigwedge x\neg f(x) = c(x)$
(c) $\bigwedge x\neg s(x) = c(x)$
(d) $\bigwedge x\neg f(x) + s(x) = c(x)$
(e) $\bigwedge x\neg f(x) + m(x) = c(x)$

B5 $\bigwedge x(x = f(x) \supset m(x) = c(x))$

B6 $\bigwedge x(\neg m(x) = c(x) \supset m(x) + s(x) = c(x))$

B7 $\bigwedge x(\neg m(x) = c(x) \supset (f(x) + m(x)) + f(x) = c(x) \wedge (f(x) + m(x)) + m(x) = c(x))$

B8 $\bigwedge x(\neg f(x) = s(x) \supset (f(x) + s(x)) + f(x) = c(x) \wedge (f(x) + s(x)) + s(x) = c(x))$

Ad A1: The composition-function is commutative. The composition of aspect b of an object and aspect c of the same object is identical with the composition of aspect c of that object with aspect b of that object. Aquinas would surely have agreed.

Ad A2 and A3: The conjunction of A2(a) and A2(b) is logically equivalent with the converse of A3, $\bigwedge x(\beta'(x) = \beta(x) \vee \beta'(x) = c(x) \supset \beta(x) + \beta'(x) = \beta(x))$; and from this we obtain A2' $\bigwedge x(\beta'(x) = \beta(x) \vee \beta'(x) = c(x) \vee \beta(x) = c(x) \supset \beta(x) + \beta'(x) = \beta(x) \vee \beta'(x) + \beta(x) = \beta'(x))$. From A3 on the other hand we get A3' $\bigwedge x(\beta(x) + \beta'(x) = \beta(x) \vee \beta'(x) + \beta(x) = \beta'(x) \supset \beta'(x) = \beta(x) \vee \beta'(x) = c(x) \vee \beta(x) = c(x))$. This means that from A2 and A3 follows a sentence-form stating the necessary and sufficient condition for improper composition. We had occasion to mention that object-aspects cannot be proper parts of each other; otherwise A2 and A3 could not be formulated in the given manner, but would have to take care of the possibility that '$\beta'(0)$ is a proper part of $\beta(0)$' is true. (Of course one may say '$m(g)$ is proper part of $w(g)$ ($= f(g) + m(g)$)'; but this is analogous to saying 'object a is proper part of proposition $f(a)$', not analogous to saying '$\lambda x(x = a)$ is proper part of $\lambda x(x = a \vee x = b)$ $(a \neq b)$'. If it were analogous to the latter, then we would have $w(g) + m(g) = w(g)$, although we have $\neg m(g) = w(g)$ and $\neg m(g) = c(g)$, where 'g' is referring to a material object.)

Ad B1: An object is composed of its essence and its being, and its essence is in turn composed of its pure (substantial) form and its matter. Aquinas states this explicitly for material objects (vide quotation 6). In view of the possibility of improper composition we can make the Aquinatic statement apply to all objects without obtaining consequences that contradict Aquinatic doctrine (as will be seen).

Ad B2: We have already given a justification of this axiom above (in the middle of section III.): the composition of essence and being is identical with the composition of actuating form and matter. Therefore, Aquinas sometimes says that a material object is composed of its form *(actuating form)* and its matter (vide for instance quotation 2), sometimes that there is a double composition in a material object: its essence is composed of its form *(pure form)* and its matter, and the material object itself is composed of its essence and its being (vide quotation 6). Again the possibility of improper composition allows us to make an insight primarily reached for material objects apply to all objects.

Ad B3: The content of B3 is evident in the light of the ontology of Aquinas. Neither the pure form nor the actuating form nor the being of an object is its matter. It will be proved below that no object is its matter and that the essence of no object is the matter of the object.

Ad B4: This axiom characterizes the function *the empty aspect of,* which Aquinas does not consider, in relation to the other aspect-functions and in relation to objects in the following manner: with respect to identity between the empty aspect of an object and 'another' aspect of that object or the object itself, it only leaves open the possibility that its matter is its empty aspect.

Ad B5: Under the intended interpretation B5 says that if an object is its pure form, then it is an immaterial object; which completely agrees with what Aquinas says about objects which are forms.

Ad B6, B7, B8: The axiom B6 has already been justified above (at the end of section III.); it expresses the stipulation there proposed. The axioms B7 and B8 fulfill the same role as B6 of completing the definition of the composition-function for cases in which it is initially not defined. We have no information as to what Aquinas considered to result by the composition of the essence and the pure form, or the essence and the matter of a material object; and we have no information as to what Aquinas considered to result by the composition of the actuating form and the pure form, or the actuating form and the being of an object whose pure form and being are different. Hence we must consider the composition-function to be initially not defined for these cases. (Concerning the composition of the matter and being of a material object we have *positive* evidence that Aquinas regarded it as impossible.) B6, B7 and B8 may be called 'the reduction-axioms' from their important role in the reduction of all AEs 'around' a certain OD to basic AEs around it and the OD itself. This reduction, programmaticly described in section IV., will be carried out in section X. The uses of B7 and B8 in the logical reconstruction of Aquinatic ontology *are not* exhaustively described by these remarks. The impression of their ad-hoc-character will be dispelled as we move on to the proving of theorems.

VIII.

T1 $\bigwedge x(m(x) = c(x) \supset x = a(x))$

(An immaterial object is its actuating form)

Proof: assume $m(x) = c(x)$; by B1 $x = (f(x) + m(x)) + s(x)$; hence $x = (f(x) + c(x)) + s(x)$; by A2(b) $f(x) + c(x) = f(x)$; hence $x = f(x) + s(x)$, hence by D2 $x = a(x)$.

T2 $\bigwedge x(x = a(x) \supset m(x) = c(x))$

(An object that is its actuating form is immaterial)

Proof: assume $x = a(x)$, hence by D2 $x = f(x) + s(x)$; by B1 $x = (f(x) + m(x)) + s(x)$; hence $f(x) + s(x) = (f(x) + m(x)) + s(x)$; by B2 $(f(x) + m(x)) + s(x) = (f(x) + s(x)) + m(x)$; hence $f(x) + s(x) = (f(x) + s(x)) + m(x)$, hence $(f(x) + s(x)) + m(x) = f(x) + s(x)$, hence by A3 $m(x) = f(x) + s(x) \vee m(x) = c(x)$; by B3(c) $\neg m(x) = f(x) + s(x)$; hence $m(x) = c(x)$.

T3 $\bigwedge x(m(x) = c(x) \supset w(x) = f(x))$

(The essence of an immaterial object is its (pure) form)

Proof: assume $m(x) = c(x)$; by D1 $w(x) = f(x) + m(x)$; hence $w(x) = f(x) + c(x)$; by A2(b) $f(x) + c(x) = f(x)$; hence $w(x) = f(x)$.

T4 $\bigwedge x(w(x) = f(x) \supset m(x) = c(x))$

(An object whose essence is its form is an immaterial object)

Proof: assume $w(x) = f(x)$, hence by D1 $f(x) + m(x) = f(x)$, hence by A3 $m(x) = f(x) \vee m(x) = c(x)$; by B3(a) $\neg m(x) = f(x)$; hence $m(x) = c(x)$.
Concerning T3 and T4 compare quotation 1. Concerning T1 consider the following quotation:

11. In his igitur quae non sunt composita ex materia et forma, in quibus individuatio non est per materiam individualem, id est per hanc materiam, sed ipsae formae per se individuantur, oportet quod ipsae formae sint supposita subsistentia. Unde in eis non differt suppositum et natura. (Summa theologiae, I,3,3)

This evidence for T1 is somewhat vitiated by the fact that Aquinas in this passage confuses what is valid of pure form with what is valid of actuating form. The context makes it clear that he intends to assert 'All immaterial objects are their pure forms'. (By the way, in the article from which quotation 11 is taken Aquinas identifies essence – 'natura vel essentia' – and pure form '‹quae› comprehendit in se illa tantum quae cadunt in definitione speciei', which is contradictory to what he says in other places; vide quotation 5.) However, by his own lights, this is false. A *created* immaterial object (an angel, for example) is not its pure form, and consequently – the essence of an immaterial object being its pure form – it is not its essence. Aquinas, however, deduces from 'All immaterial objects are their pure forms' – the essence of an immaterial object being its pure form – 'All immaterial objects are their essence' ('Unde in eis non differt suppositum et natura'). A created immaterial object is not its pure form, because its being is distinct from its essence, viz. its pure form:

12. *Secundo modo* invenitur essentia in substantiis creatis intellectualibus, in quibus est aliud esse quam essentia earum, quamvis essentia sit sine materia.
(De ente et essentia, 5, 31)
13. oportet quod in intelligentiis sit esse praeter formam; et ideo dictum est quod intelligentia est forma et esse (De ente et essentia, 4, 26)

(This last assertion does not hinder Aquinas from asserting a few lines further on: 'intelligentiae quidditas est ipsamet intelligentia, ideo quidditas vel essentia eius est ipsum quod est ipsa' (*De ente et essentia,* 4, 28))
This means that a created immaterial object is properly composed of its being and its essence, viz. its pure form; hence it is not identical with its pure form. Thus Aquinas by the equivocation in his use of the word 'forma' is led to imagining a proposition valid relative to *pure form* which is not valid relative there to, but rather valid relative to *actuating form:* 'In his igitur quae non sunt composita ex materia et forma ... oportet quod ipsae formae sint supposita subsistentia'.

T2 says about objects that are their actuating forms what B5 says about objects that are their pure forms: that they are immaterial. If B5 agrees with Aquinatic doctrine, so does T2.

T5 $\bigwedge x(m(x) = c(x) \wedge \neg w(x) = s(x) \supset \neg x = f(x))$
(An immaterial created object is not its pure form)

Proof: assume $m(x) = c(x) \wedge \neg w(x) = s(x)$, hence by T3 $w(x) = f(x)$; by B1 and D1 $x = w(x) + s(x)$; hence $x = f(x) + s(x) \wedge \neg f(x) = s(x)$; assume $x = f(x)$; hence $f(x) = f(x) + s(x)$, hence $f(x) + s(x) = f(x)$, hence by A3 $s(x) = f(x) \vee s(x) = c(x)$, hence by B4(c) $s(x) = f(x)$ contradicting $\neg f(x) = s(x)$.

T6 $\bigwedge x(\neg x = f(x) \wedge m(x) = c(x) \supset \neg x = w(x))$
(An immaterial object that is not its pure form is not its essence)

Proof: assume $\neg x = f(x) \wedge m(x) = c(x)$, hence by T3 $w(x) = f(x)$; hence $\neg x = w(x)$.

T5 and T6 formally state the principles we have just now informally made use of.

T7 $\bigwedge x(x = f(x) \supset x = a(x) \wedge x = s(x) \wedge x = w(x))$
(An object that is its pure form is its actuating form, its being and its essence)

Proof: assume $x = f(x)$, hence by B5 $m(x) = c(x)$, hence by T1 $x = a(x)$, hence by D2 $x = f(x) + s(x)$, hence $f(x) + s(x) = f(x)$, hence by A3 $s(x) = f(x) \vee s(x) = c(x)$, hence by B4 (c) $s(x) = f(x)$, hence $x = s(x)$, hence by B1 $s(x) = (f(x) + m(x)) + s(x)$, hence by A1 $s(x) + (f(x) + m(x)) = s(x)$, hence by A3 $f(x) + m(x) = s(x) \vee f(x) + m(x) = c(x)$, hence by B4 (e) and D1 $w(x) = s(x)$, hence $x = w(x)$.

T7 contains a principle Aquinas can be said to be using in quotation 11 to deduce from the invalid sentence $\bigwedge x(m(x) = c(x) \supset x = f(x))$ the invalid sentence $\bigwedge x(m(x) = c(x) \supset x = w(x))$, namely $\bigwedge x(x = f(x) \supset x = w(x))$. Other principles Aquinas can instead be said to be using in this deduction are $\bigwedge x(m(x) = c(x) \supset w(x) = f(x))$ (the most likely candidate) and $\bigwedge x(x = f(x) \wedge m(x) = c(x) \supset x = w(x))$. Relative to B5 this latter principle is equivalent with $\bigwedge x(x = f(x) \supset x = w(x))$; consequently, since it is very easy to prove, it opens an easier way than the way via the proof of T7 for arriving at $\bigwedge x(x = f(x) \supset x = w(x))$:

T8 $\bigwedge x(x = f(x) \wedge m(x) = c(x) \supset x = w(x))$
(An immaterial object that is its pure form is its essence)

Proof: assume $x = f(x) \wedge m(x) = c(x)$, hence by T3 $w(x) = f(x)$, hence $x = w(x)$.

T7 shows that objects that are their pure form are in a way simple objects; we shall have occasion to come back to this.

T9 $\bigwedge x(x = w(x) \supset w(x) = s(x))$
(An object that is its essence is uncreated)

Proof: assume $x = w(x)$; by B1 $x = w(x) + s(x)$; hence $w(x) + s(x) = w(x)$, hence by A3 $s(x) = w(x) \vee s(x) = c(x)$, hence by B4 (c) $w(x) = s(x)$.

T10 $\bigwedge x(w(x) = s(x) \supset x = w(x))$
(An uncreated object is its essence)

Proof: assume $w(x) = s(x)$; by B1 $x = w(x) + s(x)$; hence $x = s(x) + s(x)$, hence by A2 (a) $x = s(x)$; hence $x = w(x)$.

The proof of T10 contains the proof of

T11 $\bigwedge x(w(x) = s(x) \supset x = s(x))$
(An uncreated object is its being)

And we also have

T12 $\bigwedge x(x = s(x) \supset w(x) = s(x))$
(An object that is its being is uncreated)

Proof: assume $x = s(x)$; by B1 $x = w(x) + s(x)$; hence $w(x) + s(x) = s(x)$, hence by A1 $s(x) + w(x) = s(x)$, hence by A3 $w(x) = s(x) \vee w(x) = c(x)$, hence by B4 (e), D1 $w(x) = s(x)$.

We have all the time been reading $m(x) = c(x)$ as 'x is an immaterial object' and $w(x) = s(x)$ as 'x is an uncreated object'. According to stipulation the matter of an object is its empty aspect if the object is immaterial; if, on the other hand, the object is material, then, clearly, its matter is not its empty aspect. This justifies the reading of $m(x) = c(x)$ as 'x is an immaterial object'. According to Aquinas the totality of objects is divided into the one uncreated object, God, and the many created objects. God is the only object whose essence is its being:

14. Hinc est quod proprium nomen Dei ponitur esse QUI EST (*Exodus*, III,14), quia eius solius est quod sua substantia no sit aliud quam suum esse.
(Summa contra gentiles, 2,52)

Consequently, the essence of every uncreated object is its being; consequently, every object whose essence is its being is uncreated:

15. cuilibet rei creatae suum esse est ei per aliud; alias non esset creatum. Nullius igitur substantiae creatae suum esse est sua substantia.
(Summa contra gentiles, 2,52)

This justifies the reading of $w(x) = s(x)$ as 'x is an uncreated object'.

For convenience we introduce these two definitions:

D3 $M(0) := \neg m(0) = c(0)$ (for all ODs 0 of T)
D4 $C(0) := \neg w(0) = s(0)$ (for all ODs 0 of T)

With the help of the predicates M and C we now define the four principal Aquinatic categories of objects:

D5 $D(0) := \neg M(0) \wedge \neg C(0)$ (for all ODs 0 of T)
D6 $I(0) := \neg M(0) \wedge C(0)$ (for all ODs 0 of T)
D7 $E(0) := M(0) \wedge \neg C(0)$ (for all ODs 0 of T)
D8 $B(0) := M(0) \wedge C(0)$ (for all ODs 0 of T)

According to Aquinatic doctrine the third category is empty. There are no material uncreated objects (for example, *elementa* in the sense of the Pre-Socratics, having a quasi-divine character):

16. Per hoc autem ‹quod omnia quae sunt, a Deo sunt› excluditur antiquorum naturalium error, qui ponebant corpora quaedam non habere causam essendi; et etiam quorumdam, qui dicunt Deum non esse causam substantiae caeli, sed solum motus.
(Summa contra gentiles, 2,15)

In TO we can prove

T13 $\neg \bigvee x E(x)$
(There are no material uncreated objects)

Proof: By D7 $\neg \bigvee x E(x)$ is equivalent with $\neg \bigvee x(M(x) \wedge \neg C(x))$, that is, with $\bigwedge x(M(x) \supset C(x))$, which is equivalent by D3 and D4 with $\bigwedge x(\neg m(x) = c(x) \supset \neg w(x) = s(x))$, that is, with $\bigwedge x(w(x) = s(x) \supset m(x) = c(x))$; the latter is proved as follows: assume $w(x) = s(x)$, hence $s(x) + f(x) = w(x) + f(x)$, hence by D1 $s(x) + f(x) = (f(x) + m(x)) + f(x)$; by B4 (d) $\neg f(x) + s(x) = c(x)$, hence by A1 $\neg s(x) + f(x) = c(x)$; hence $\neg (f(x) + m(x)) + f(x) = c(x)$, hence by B7 $m(x) = c(x)$.

From quotation 15 we may gather: *If the being of an object is caused by another object, then this being is different from the essence of the object.* The converse *If the being of an object is different from the essence of the object, then this being is caused by another object* is also valid according to Aquinas:

17. oportet quod omnis talis res, cuius esse est aliud quam natura sua, habet esse ab alio.
(De ente et essentia, 4,27; *vide also* Summa theologiae, I,3,4)

Hence we can read C(x), that is, $\neg w(x) = s(x)$ as 'the being of x is caused by another object'; $\neg C(x)$, that is, $w(x) = s(x)$ as 'the being of x is not caused by another object', which for Aquinas is equivalent to 'the being of x is not caused by *any* object', self-causation being impossible according to Aquinas:

18. nec tamen invenitur, nec est possibile, quod aliquid sit causa efficiens sui ipsius; quia sic esset prius seipso, quod est impossibile.
(Summa theologiae, I,2,3)
19. Non autem potest esse quod ipsum esse sit causatum ab ipsa forma vel quidditate rei, dico sicut a causa efficiente; quia sic aliqua res esset causa sui ipsius, et aliqua res seipsam in esse produceret, quod est impossibile.
(De ente et essentia, 4,27)

From T13 we can easily deduce:

T14 $\bigwedge x(M(x) \equiv B(x))$
(The material objects are the contingent bodies)

Proof: from T13 by D7 $\bigwedge x(M(x) \supset C(x))$, hence $\bigwedge x(M(x) \equiv M(x) \wedge C(x))$, hence by D8 $\bigwedge x(M(x) \equiv B(x))$.

T15 $\bigwedge x(\neg C(x) \equiv D(x))$
(The uncreated – uncaused – objects are the divine objects)

Proof: from T13 $\bigwedge x(\neg C(x) \supset \neg M(x))$, hence $\bigwedge x(\neg C(x) \equiv \neg M(x) \wedge \neg C(x))$, hence by D5 $\bigwedge x(\neg C(x) \equiv D(x))$.

D5 mirrors the Aquinatic conception of divinity: a divine object is an uncreated (uncaused) immaterial object. This conception is the Judaeo-Christian conception of divinity, but with a special Aristotelian interpretation resulting from taking 'uncreated object' to mean an object whose essence is its being.

It is problematic whether there are divine objects. It is at least equally problematic whether there are *intelligences* ('substantiae creatae intellectuales ‹immateriales›', 'intelligentiae'). In accordance with prevailing agnosticism neither $\bigvee xD(x)$ nor $\bigvee xI(x)$ (nor their negations) can be proved in TO, although the truth of $\bigvee xD(x)$ and $\bigvee xI(x)$ (under the given interpretation of T) would have been indubitable for Aquinas.

In the ontological doctrines here referred to Aquinas does not consider so-called *abstract objects,* numbers, for example, or geometrical figures (which one might think of subsuming under category I); hence they do not belong to the universe of discourse. There is also a positive justification for this:

20. corpus mathematicum non est per se existens, ut Philosophus probat (Summa contra gentiles, 1,20)

Abstract 'objects' are not objects in the full sense ('substantiae') for Aquinas.

In a sequel to this paper extensions of TO are considered in which $\bigvee x I(x)$ and $\bigvee x D(x)$ *are* provable. TO, indeed, is very weak in its existential assumptions; not even $\bigvee x M(x)$, which corresponds to the entirely(?) unproblematic assertion that there are material objects, can be deduced in it. However, TO, while implying no further existential commitment than that there is at least one object, implies (under the given interpretation of T) that there are no material uncreated (uncaused) objects. Here TO is following Aquinas.

The second part of the proof of T13 can be re-ordered in the following manner: $\bigwedge x \neg s(x) + f(x) = c(x)$ by B4 (d) and A1, hence $\bigwedge x(w(x) = s(x) \supset \neg w(x) + f(x) = c(x))$; $\bigwedge x(\neg m(x) = c(x) \supset w(x) + f(x) = c(x))$ by B7, D1; hence $\bigwedge x(\neg m(x) = c(x) \supset \neg w(x) = s(x))$. In this way it becomes easier to informally rephrase the proof in order to bring out the intuitive idea behind it: The pure form (as well as the essence) of an object enters into composition with the being of the object; here we may cite

21. esse est actualitas omnis formae vel naturae (Summa theologiae, I,3,4)

Hence, if the essence of an object is identical with the being of the object, then its pure form enters into composition with its essence. But the pure form of a material object does not enter into composition with its essence; *there is nothing in a material object constituted by both of them.* Hence the essence of a material object is different from its being.

IX. By D5, T11 and T10 follow theorems about divine objects that correspond to Aquinatic dicta about God:

T16 (a) $\bigwedge x(D(x) \supset \neg M(x))$ – 22. Deus non est corpus (Summa contra gentiles, 1,20)

(b) $\bigwedge x(D(x) \supset w(x) = s(x))$ – 23. In Deo non est aliud essentia vel quidditas, quam suum esse
(Summa contra gentiles, 1,22)

(c) $\bigwedge x(D(x) \supset x = w(x))$ – 24. Deus est sua essentia
(Summa contra gentiles, 1,21)

(d) $\bigwedge x(D(x) \supset x = s(x))$ – 25. Deus non solum est sua essentia, ut ostensum est, sed etiam suum esse
(Summa theologiae, I,3,4)

By T13 and T9 we obtain

T17 $\bigwedge x(M(x) \supset \neg\, x = w(x))$

And Aquinas says accordingly:

26. in rebus compositis ex materia et forma, necesse est quod differant natura vel essentia et suppositum ‹seu substantia individua›
(Summa theologiae, I,3,3)

In reverse order, Aquinas says:

27. Si autem sint aliquae formae creatae non receptae in materia, sed per se subsistentes, ut quidam de angelis opinantur, erunt quidem infinitae secundum quid, inquantum huiusmodi formae non terminantur neque contrahuntur per aliquam materiam: sed quia forma creata sic subsistens habet esse, et non est suum esse, necesse est quod ipsum eius esse sit receptum et contractum ad determinatam naturam.
(Summa theologiae, I,7,2)

And in accordance with this quotation we have

T18 (a) $\bigwedge x(I(x) \supset x = a(x))$ (by D6 and T1)
(b) $\bigwedge x(I(x) \supset \neg\, x = s(x))$ (by D6 and T12)

All this amply shows that our theorems and definitions mirror Aquinatic doctrine.

In *Summa theologiae, I,3,3* Aquinas deduces T16(c) from the invalid sentence $\bigwedge x(m(x) = c(x) \supset x = f(x))$ (S) (‘Every immaterial object is its pure form') with the help of the principle $\bigwedge x(m(x) = c(x) \supset w(x) = f(x))$ (T3) and the principle $\bigwedge x(D(x) \supset m(x) = c(x))$ (T16(a)). From S and T3 he first gets $\bigwedge x(m(x) = c(x) \supset x = w(x))$ (which is invalid) and then, in continuation of quotation 11, he writes:

28. Et sic, cum Deus non sit compositus ex materia et forma ‹T16 (a)›, ut ostensum est, oportet quod Deus sit sua deitas ‹id est, sua essentia›, sua vita, et quidquid aliud sic de Deo praedicatur.

Thus, starting from an invalid premise, he obtains a valid conclusion. The argument in *Summa theologiae, I,3,3* may also be taken to reach $\bigwedge x(D(x) \supset x = f(x))$, starting out from S and using T16 (a). This sentence ('Every divine object is its pure form'), too, inspite of the invalid premise from which it is derived, is Aquinatically valid, as is $\bigwedge x(D(x) \supset x = a(x))$ ('Every divine object is its actuating form'), which we get from T16 (a) by T1:

29. unumquodque agens agit per suam formam: unde secundum quod aliquid se habet ad suam formam, sic se habet ad hoc quod sit agens. Quod igitur primum est et per se agens, oportet quod sit primo et per se forma. Deus autem est primum agens, cum sit prima causa efficiens, ut ostensum est. Est igitur per essentiam suam forma; et non compositus ex materia et forma.
(Summa theologiae, I,3,2)

In this quotation Aquinas certainly did not intend to refer to pure form rather than to actuating form, or vice versa, since he did not distinguish between them. Indeed, with respect to divine objects Aquinas is right in this. The actuating and the pure form of a divine object are identical. Hence quotation 29 supports $\bigwedge x(D(x) \supset x = f(x))$ as well as $\bigwedge x(D(x) \supset x = a(x))$, these sentences being provably equivalent. We have:

T19 $\bigwedge x(D(x) \supset a(x) = f(x))$

Proof: assume $D(x)$, hence by D5 $\neg M(x) \wedge \neg C(x)$, hence by D3 and D4 $m(x) = c(x) \wedge w(x) = s(x)$, hence by D1 $m(x) = c(x) \wedge f(x) + m(x) = s(x)$, hence $f(x) + c(x) = s(x)$, hence by A2 (b) $f(x) = s(x)$, hence $f(x) + s(x) = f(x) + f(x)$, hence by D2 and A2 (a) $a(x) = f(x)$.

We can also prove the converse of T19:

T20 $\bigwedge x(a(x) = f(x) \supset D(x))$
(If the actuating form of an object is its pure form, then the object is divine)

Proof: assume $a(x) = f(x)$, hence by D2 $f(x) + s(x) = f(x)$, hence by A3 $s(x) = f(x) \vee s(x) = c(x)$, hence by B4 (c) $s(x) = f(x)$; by B1 $x = (f(x) + m(x)) + s(x)$, hence by B2 $x = (f(x) + s(x)) + m(x)$; hence $x = (f(x) + f(x)) + m(x)$, hence by A2 (a) $x = f(x) + m(x)$, hence by D1 $x = w(x)$, hence by T9 $w(x) = s(x)$, hence by D4 $\neg C(x)$, hence by T15 $D(x)$.

T19 and T20 make precise what is meant by 'the pure form of an object is normally distinct from its actuating form'. The pure form of an object is distinct from its actuating form if and only if it is not a divine object, what, clearly, is normally the case.

There are many equivalences regarding the predicate D which are provable in TO, beside the equivalence T15 and the trivial definitional equivalence:

T21 (a) $\bigwedge x(D(x) \equiv a(x) = f(x))$ (by T19, T20)
(b) $\bigwedge x(D(x) \equiv x = w(x))$ (by T15, D4, T9, T10)
(c) $\bigwedge x(D(x) \equiv x = s(x))$ (by T15, D4, T11, T12)
(d) $\bigwedge x(D(x) \equiv s(x) = f(x))$

Proof: from $a(x) = f(x)$ $s(x) = f(x)$ (vide the proof of T20); from $s(x) = f(x)$ $a(x) = f(x)$ (vide the proof of T19); hence by T21 (a) T21 (d).

(e) $\bigwedge x(D(x) \equiv x = f(x))$

Proof: from $D(x)$ $x = s(x) \wedge s(x) = f(x)$ by T21 (c) and T 21 (d), hence $x = f(x)$; from $x = f(x)$ by T7 $x = s(x)$, hence by T21 (c) $D(x)$.

(f) $\bigwedge x(D(x) \equiv s(x) = a(x))$

Proof: from $D(x)$ $a(x) = f(x) \wedge s(x) = f(x)$ by T21 (a) and T21 (d), hence $s(x) = a(x)$; from $s(x) = a(x)$ by D2 $s(x) = f(x) + s(x)$, hence by A3 $f(x) = s(x) \vee f(x) = c(x)$, hence by B4 (b) $f(x) = s(x)$, hence by T21 (d) $D(x)$.

(g) $\bigwedge x(D(x) \equiv w(x) = a(x))$

Proof: from $D(x)$ $x = f(x) \wedge x = w(x) \wedge a(x) = f(x)$ by T21 (e), T21 (b) and T21 (a), hence $w(x) = a(x)$; from $w(x) = a(x)$ by D1 and D2 $f(x) + m(x) = f(x) + s(x)$, hence by B1 $x = (f(x) + s(x)) + s(x)$, hence by B4 (a) $\neg (f(x) + s(x)) + s(x) = c(x)$, hence by B8 $f(x) = s(x)$, hence by T21 (d) $D(x)$.

We have a shorter sequence of in TO provable equivalences regarding the predicate $\neg M$:

T22 (a) $\bigwedge x(\neg M(x) \equiv x = a(x))$ (by T1, T2, D3)
(b) $\bigwedge x(\neg M(x) \equiv w(x) = f(x))$ (by T3, T4, D3)

It is interesting to compare T22 (a) with T21 (e), and T22 (b) with T21 (g), and the two pairs with each other. In the pair T22 (b), T21 (g) the role of f(x) and a(x) is inverse to the role of f(x) and a(x) in the pair T22 (a), T 21 (e).

From T21 and T22 follows the absolute simplicity of a divine object. An object is said to be *absolutely simple* if all its (essential) aspects that are different from its empty aspect are identical with the object itself (in other words, if it has no real components).

T23 $\bigwedge x(D(x) \supset x = f(x) \wedge x = w(x) \wedge x = s(x) \wedge x = a(x))$
(A divine object is absolutely simple)

Proof: assume D(x), hence $\neg M(x)$ by D5; hence the succedent of T23 by T21 and T22; hence every aspect of x that is different from its empty aspect is identical with x. (The theorem warranting the conclusion that f(x), w(x), s(x), a(x) are *all* the non-empty aspects of x is given below; vide T28.)

It can easily be seen that the converse of T23 is also provable. T23 corresponds to the Aquinatic doctrine of the total simplicity of God:

30. quod Deum omnino esse simplicem, multipliciter potest esse manifestum. Primo quidem per supradicta. Cum enim in Deo non sit compositio, neque quantitativarum partium, quia corpus non est; neque compositio formae et materiae: neque in eo sit aliud natura et suppositum; neque aliud essentia et esse ... manifestum est quod Deus nullo modo compositus est, sed est omnino simplex. ... Unde, cum Deus sit ipsa forma, vel potius ipsum esse, nullo modo compositus esse potest (Summa theologiae, I,3,7)

The degree of composition of an object is the number of its real components, that is, the number of its (essential) aspects that are different from its empty aspect and different from the object itself. Evidently the degree of composition of a divine object is zero.

An object is said to be *absolutely composite* if its degree of composition is maximal. Material objects are absolutely composite, as we shall see. We first prove the following two theorems:

T24 $\bigwedge x \neg x = m(x)$
(No object is its matter)

Proof: assume $x = m(x)$; by B1 $x = (f(x) + m(x)) + s(x)$, hence by B2 $x = (f(x) + s(x)) + m(x)$; hence $m(x) = (f(x) + s(x)) + m(x)$, hence by A1

$m(x) + (f(x) + s(x)) = m(x)$, hence by A3 $f(x) + s(x) = m(x) \vee f(x) + s(x) = c(x)$, what contradicts B3 (c) and B4 (d).

Aquinas says:

31. esse autem non dicitur de materia, sed de toto; unde materia non potest dici quod est, sed ipsa substantia est id quod est
(Summa contra gentiles, 2,54)

T25 $\bigwedge x \neg w(x) = m(x)$
(The essence of no object is its matter)

Proof: assume $w(x) = m(x)$, hence by D1 $f(x) + m(x) = m(x)$, hence by A1 $m(x) + f(x) = m(x)$, hence by A3 $f(x) = m(x) \vee f(x) = c(x)$, what contradicts B3 (a) and B4 (b).

And Aquinas says:

32. materia non est ipsa substantia rei; nam sequeretur omnes formas esse accidentia, sicut antiqui naturales opinabantur; sed materia est pars substantiae
(Summa contra gentiles, 2,54)
33. Quod enim materia sola rei non sit essentia, planum est, quia res per essentiam suam et cognoscibilis est, et in specie ordinatur vel in genere; sed materia neque cognitionis principium ‹est (?)›, neque secundum eam aliquid ad genus vel speciem determinatur, sed secundum id quo ‹?: quod?› aliquid actu est
(De ente et essentia, 2,5)

We then have

T26 (a) $\bigwedge x(M(x) \supset \neg x = m(x) \wedge \neg x = f(x) \wedge \neg x = w(x) \wedge \neg x = s(x) \wedge \neg x = a(x))$
(b) $\bigwedge x(M(x) \supset \neg m(x) = f(x) \wedge \neg m(x) = w(x) \wedge \neg m(x) = s(x) \wedge \neg m(x) = a(x) \wedge \neg f(x) = w(x) \wedge \neg f(x) = s(x) \wedge \neg f(x) = a(x) \wedge \neg w(x) = s(x) \wedge \neg w(x) = a(x) \wedge \neg s(x) = a(x))$

Proof: (a) assume $M(x)$; then $\neg x = m(x)$ by T24; $\neg x = f(x)$ by B5, D3; $\neg x = w(x)$ by T21 (b), T16 (a); $\neg x = s(x)$ by T21 (c), T16 (a); $\neg x = a(x)$ by T2, D3;
(b) assume $M(x)$; then $\neg m(x) = f(x)$ by B3 (a); $\neg m(x) = w(x)$ by T25; $\neg m(x) = s(x)$ by B3 (b); $\neg m(x) = a(x)$ by B3 (c), D2; $\neg f(x) = w(x)$ by T4, D3; $\neg f(x) = s(x)$ by T21 (d), T16 (a); $\neg f(x) = a(x)$ by T21 (a), T16 (a); $\neg w(x) = s(x)$ by T15, T16 (a), D4; $\neg w(x) = a(x)$ by T21 (g), T16 (a); $\neg s(x) = a(x)$ by T21 (f), T16 (a).

From T26 (and B4) it is apparent that a material object has at least five real components; there cannot be more than five real components in an object (vide T28); hence the degree of composition of a material object is maximal, and hence it is absolutely composite.

The intelligences are neither absolutely simple nor absolutely composite. While the degree of composition of divine objects is zero, and that of material objects five, the degree of composition of created immaterial substances is two:

T27 (a) $\bigwedge x(I(x) \supset \neg x = f(x) \wedge \neg x = w(x) \wedge \neg x = s(x) \wedge x = a(x))$
(b) $\bigwedge x(I(x) \supset f(x) = w(x) \wedge \neg f(x) = s(x))$

Proof: (a) assume $I(x)$, hence by D6 $\neg M(x) \wedge C(x)$; then by T15, T21(e) $\neg x = f(x)$; by T15, T21 (b) $\neg x = w(x)$; by T18 (b) $\neg x = s(x)$; by T18 (a) $x = a(x)$.
(b) assume $I(x)$, hence by D6 $\neg M(x) \wedge C(x)$; then by T3, D3 $f(x) = w(x)$; by T15, T21 (d) $\neg f(x) = s(x)$;
by T27 (a) (and B4) $f(x)$, $w(x)$ and $s(x)$ are real components of x, and they are the only real components of x (vide T28); of these only two are distinct (by T27 (b)).

Occasionally Aquinas calls intelligences as well as God 'substantiae simplices' (vide quotation 1). However:

34. Non est autem opinandum, quamvis substantiae intellectuales non sint corporeae nec ex materia et forma compositae nec in materia existentes sicut formae immateriales ‹?; materiales?›, quod propter hoc divinae simplicitati adaequentur.
(Summa contra gentiles, 2,52; see also quotation 13)

X. *Definition:* An AE of T α is in TO reducible to the EDs of T $\beta_1, \ldots, \beta_n$ if and only if $\alpha = \beta_1 \vee \ldots \vee \alpha = \beta_n$ is provable in TO.

Reduction Theorem: If 0 is an OD of T and $\beta(0)$ is an AE of T, then $\beta(0)$ is reducible in TO to 0, $f(0)$, $m(0)$, $s(0)$, $f(0) + m(0)$, $f(0) + s(0)$ and $c(0)$ (for convenience this sequence is named Y).

Proof: let 0 be an OD of T; there are four PAEs of T around 0: $f(0)$, $m(0)$, $s(0)$ and $c(0)$; with these 16 SAEs of T around 0 can be formed, and 128 TAEs of T around 0; there are no other AEs of T around 0.

Because of A1 6 of the 16 SAEs of T around 0 are each reducible in TO to the respective converse; hence they are each reducible in TO to Y if the remaining 10 are each reducible in TO to Y; let the remaining 10 be for example:

(i) c(0) + c(0), m(0) + m(0), f(0) + f(0), s(0) + s(0);
(ii) m(0) + c(0), f(0) + c(0), s(0) + c(0);
(iii) m(0) + s(0);
(iv) f(0) + s(0), f(0) + m(0);

by A2(a) the AEs in row (i) are each reducible in TO to a PAE of T around 0, hence to Y;
by A2(b) the AEs in row (ii) are each reducible in TO to a PAE of T around 0, hence to Y;
by B6, A1, A2(b) the AE in row (iii) is reducible in TO to c(0), s(0), hence to Y;
the AEs in row (iv) are each trivially reducible in TO to Y.
We have now established
Lemma: Every SAE of T around 0 is reducible in TO to Y.

Half of the TAEs of T around 0 are by A1 each reducible in TO to the respective converse, for example in such a manner that each converse has the form (β(0) + β'(0)) + β''(0); hence they are each reducible in TO to Y, if each respective converse is reducible in TO to Y; for these converses, each having the form (β(0) + β'(0)) + β''(0), we obtain:

if β(0) + β'(0) is α(0) + α(0), then the AE is reducible in TO to a SAE of T around 0 by A2(a), hence to Y by *Lemma;*

if β(0) + β'(0) is α(0) + c(0) or c(0) + α(0), then the AE is reducible in TO to a SAE of T around 0 by A2(b) and A1, hence to Y by *Lemma;*

if β(0) + β'(0) is f(0) + m(0) or m(0) + f(0);
then, if β''(0) is s(0), the AE is reducible in TO to 0 by B1 and A1, hence to Y;
then, if β''(0) is c(0), the AE is reducible in TO to f(0) + m(0) by A2(b) and A1, hence to Y;
then, if β''(0) is f(0), the AE is reducible in TO to c(0), f(0) by B7 , A1 and A2, hence to Y;
then, if β''(0) is m(0), the AE is reducible in TO to c(0), f(0) by B7, A1 and A2(b), hence to Y;

if β(0) + β'(0) is m(0) + s(0) or s(0) + m(0);
then, if β''(0) is s(0), the AE is reducible in TO to s(0) by B6, A1 and A2, hence to Y;
then, if β''(0) is c(0), the AE is reducible in TO to a SAE of T around 0 by A2(b), hence to Y by *Lemma;*

then, if $\beta''(0)$ is $f(0)$, the AE is reducible in TO to $f(0)$, $f(0) + s(0)$ by B6, A2(b), A1, hence to Y;
then, if $\beta''(0)$ is $m(0)$, the AE is reducible in TO to $m(0)$, $s(0)$ by B6, A2(b), A1, hence to Y;

if $\beta'(0) + \beta''(0)$ is $f(0) + s(0)$ or $s(0) + f(0)$;
then, if $\beta''(0)$ is $s(0)$, the AE is reducible in TO to $c(0)$, $f(0)$ by B8, A2(a), A1, hence to Y;
then, if $\beta''(0)$ is $c(0)$, the AE is reducible in TO to $f(0) + s(0)$ by A2(b) and A1, hence to Y;
then, if $\beta''(0)$ is $f(0)$, the AE is reducible in TO to $c(0)$, $f(0)$ by B8, A2(a), A1, hence to Y;
then, if $\beta''(0)$ is $m(0)$, the AE is reducible in TO to 0 by B2, B1, A1, hence to Y.

We have now established
Lemma:* Every TAE of T around 0 is reducible in TO to Y.
Since every PAE of T around 0 is trivially reducible in TO to Y, and since every AE of T around 0 is either a PAE or a SAE or a TAE around 0, we obtain by *Lemma* and *Lemma*:*
Every AE of T around 0 is reducible in TO to Y.
This result establishes the *Reduction Theorem.*

From the *Reduction Theorem* follows (by making use of D1 and D2)

($\beta(x)$ is any AE of T around x)
T28 $\bigwedge x(\beta(x) = x \vee \beta(x) = f(x) \vee \beta(x) = m(x) \vee \beta(x) = s(x) \vee \beta(x) = w(x) \vee \beta(x) = a(x) \vee \beta(x) = c(x))$

T28 is logically equivalent with $\bigwedge x(\neg\beta(x) = x \wedge \neg\beta(x) = c(x) \supset \beta(x) = f(x) \vee \beta(x) = m(x) \vee \beta(x) = s(x) \vee \beta(x) = w(x) \vee \beta(x) = a(x))$, which says that there are at most five real components in an object, namely $f(x)$, $m(x)$, $s(x)$, $w(x)$ and $a(x)$.

A PSL of T is called 'undecided in TO' if and only if neither itself nor its negation are provable in TO. It can easily be shown that of the PSLs of T which are formed out of irreducible AEs of T (and 0) and which contain only the OD of T 0, at most (and very probably *exactly*) the following are undecided in TO (and PSLs equivalent with them by A1 and the symmetry of identity): $m(0) = c(0)$, $f(0) = 0$, $f(0) = s(0)$, $f(0) = f(0) + m(0)$, $f(0) = f(0) + s(0)$, $s(0) = 0$, $s(0) = f(0) + m(0)$, $s(0) = f(0) + s(0)$, $f(0)$

+ s(0) = 0, f(0) + m(0) = 0, f(0) + s(0) = f(0) + m(0). These SLs can be grouped in two equivalence-lists based on the proved theorems:

Divinity	*Immateriality*
$f(0) = 0$	
$s(0) = 0$	
$f(0) = s(0)$	$m(0) = c(0)$
$f(0) + m(0) = 0$	$f(0) + s(0) = 0$
$f(0) = f(0) + s(0)$	$f(0) = f(0) + m(0)$
$s(0) = f(0) + m(0)$	
$s(0) = f(0) + s(0)$	
$f(0) + s(0) = f(0) + m(0)$	

Every SL in the left list implies every SL in the right list.

If 0 is an OD of T and $\beta(0) = \beta'(0)$ a PSL of T ($\beta(0)$, $\beta'(0)$ may be the same expression as 0) we can show:

T29 $\beta(0) = \beta'(0) \equiv \beta(0) = r(0) \wedge \beta'(0) = r'(0) \wedge r(0) = r'(0) \vee$
$\beta(0) = r(0) \wedge \beta'(0) = k'(0) \wedge r(0) = k'(0) \vee$
$\beta(0) = k(0) \wedge \beta'(0) = r'(0) \wedge k(0) = r'(0) \vee$
$\beta(0) = k(0) \wedge \beta'(0) = k'(0) \wedge k(0) = k'(0)$

(where r(0), k(0), r'(0), k'(0) belong to Y; r(0), k(0) are the ultimate reducts of $\beta(0)$, r'(0), k'(0) are the ultimate reducts of $\beta'(0)$; possibly some or all expressions out of r(0), k(0), r'(0), k'(0) are identical.)

Proof: the proof going from the right side of the equivalence to the left is trivial; assume $\beta(0) = \beta'(0)$; by the Reduction Theorem we have $(\beta(0) = r(0) \vee \beta(0) = k(0)) \wedge (\beta'(0) = r'(0) \vee \beta'(0) = k'(0))$, hence $\beta(0) = r(0) \wedge \beta'(0) = r'(0) \vee \beta(0) = r(0) \wedge \beta'(0) = k'(0) \vee \beta(0) = k(0) \wedge \beta'(0) = r'(0) \vee \beta(0) = k(0) \wedge \beta'(0) = k'(0)$, hence by $\beta(0) = \beta'(0)$ the right side of the equivalence.

For obvious reasons T29 may be called the *Normal Form Theorem*. Here is an example of its application:

$$\begin{aligned} m(x) + s(x) = m(x) \equiv\ & m(x) + s(x) = c(x) \wedge m(x) = m(x) \wedge c(x) = m(x) \vee \\ & m(x) + s(x) = c(x) \wedge m(x) = m(x) \wedge c(x) = m(x) \vee \\ & m(x) + s(x) = s(x) \wedge m(x) = m(x) \wedge s(x) = m(x) \vee \\ & m(x) + s(x) = s(x) \wedge m(x) = m(x) \wedge s(x) = m(x) \end{aligned}$$

hence

$$m(x) + s(x) = m(x) \equiv m(x) + s(x) = c(x) \wedge c(x) = m(x) \vee m(x) + s(x) = s(x) \wedge s(x) = m(x),$$

hence by B3(b)

$m(x) + s(x) = m(x) \equiv m(x) + s(x) = c(x) \wedge c(x) = m(x)$,

hency by A1, A2(b), B4(c) $\neg m(x) + s(x) = m(x)$.

XI. We now proceed to the proof of the consistency of TO. The consistency of TO will be proved by providing a model for it in an interpreted semiformal language T' that contains the language T.

The ODs of T are the second-order ODs of T'; these refer to the circles in a plane (which have finite positive radius); they are identified with certain sets of points in that plane. (The points inside a circle belong to a circle.) The first-order ODs of T' refer to the points in the plane. Moreover there are designators referring to real numbers. While the second-order OVs of T' are x, x', x'', ..., the first-order OVs of T' are y, y', y'', ...

In a circle x conceived to be a set of points there can be distinguished certain proper subsets; for example, the set to which belongs only the centre of x, the set of all points in the periphery of x, the set of all points which lie between the centre and the periphery of x. We define:

For all second-order ODs 0 of T':

(a) $m(0) :=$ λy (y = the center of 0)
(The center of 0 is the point of 0 whose distance from any two points of 0 that have distance k is k/2, where k is the furthest distance between points of 0.)

(b) $f(0) :=$ λy (y lies between the center of 0 and the periphery of 0)
(y lies between the centre of 0 and the periphery of 0 iff $\bigvee y'$ (y' is in the periphery of 0 $\wedge$ y is between the centre of 0 and y').)

(c) $s(0) :=$ λy (y is in the periphery of 0)
(y is in the periphery of 0 iff $y \varepsilon 0 \wedge \bigvee y' (y' \varepsilon 0 \wedge$ distance $(y,y') = k)$.)

(d) $c(0) :=$ the empty set $(\lambda y(y \neq y))$

(e) α is connected with $\beta := \neg\alpha = \beta \wedge$
$\bigvee y \bigvee y'$ ($y \varepsilon \alpha \wedge y' \varepsilon \beta \wedge y'$ can be reached from y without touching a point that belongs neither to α nor to β)
$\vee$
$\neg\alpha = \beta \wedge (\alpha = \lambda y\, (y \neq y) \vee \beta = \lambda y (y \neq y))$

(for all designators α, β referring to sets of points in the plane)

(f) $\alpha + \beta := \lambda y$ ((α is connected with $\beta \wedge \alpha$ and β have no common element $\vee\ \alpha = \beta) \wedge (y\varepsilon\alpha \vee y\varepsilon\beta)$)

With the help of these definitions we can prove the axioms of TO on the basis of certain elementary geometrical facts about circles and fundamental principles of set theory (the only sets used are sets of individuals, i.e. points):

Proof of A1: $\beta(x)$ is connected with $\beta'(x) \equiv \beta'(x)$ is connected with $\beta(x)$, as can be seen from (e); hence $\bigwedge y$(($\beta(x)$ is connected with $\beta'(x) \wedge \beta(x)$ and $\beta'(x)$ have no common element $\vee\ \beta(x) = \beta'(x)) \wedge (y\varepsilon\beta(x) \vee y\varepsilon\beta'(x))$ $\equiv$ ($\beta'(x)$ is connected with $\beta(x) \wedge \beta'(x)$ and $\beta(x)$ have no common element $\vee\ \beta'(x) = \beta(x)) \wedge (y\varepsilon\beta'(x) \vee y\varepsilon\beta(x))$), hence by (f) and the principles of set-theory $\beta(x) + \beta'(x) = \beta'(x) + \beta(x)$.

Proof of A2(a): clearly $\bigwedge y$(($\beta(x)$ is connected with $\beta(x) \wedge \beta(x)$ and $\beta(x)$ have no common element $\vee\ \beta(x) = \beta(x)) \wedge (y\varepsilon\beta(x) \vee y\varepsilon\beta(x)) \equiv y\varepsilon\beta(x)$), hence by (f) and the principles of set-theory $\beta(x) + \beta(x) = \beta(x)$.

Proof of A2(b): (i) assume ($\beta(x)$ is connected with $c(x) \wedge \beta(x)$ and $c(x)$ have no element in common $\vee\ \beta(x) = c(x)) \wedge (y\varepsilon\beta(x) \vee y\varepsilon c(x))$, hence by (d) $(y\varepsilon\beta(x) \vee y\varepsilon\lambda y'\, (y' \neq y'))$, hence, since $\neg y\varepsilon\lambda y'\, (y' \neq y')$, $y\varepsilon\beta(x)$;

(ii) assume $y\varepsilon\beta(x)$, hence $(y\varepsilon\beta(x) \vee y\varepsilon\lambda y'\, (y' \neq y'))$; $c(x) = \lambda y'\, (y' \neq y')$ by (d); hence $\neg\beta(x) = c(x) \wedge (\beta(x) = \lambda y'\, (y' \neq y') \vee c(x) = \lambda y'\, (y' \neq y'))$, hence $\beta(x)$ is connected with $c(x)$; $\beta(x)$ and $c(x)$ have no common element, since $c(x) = \lambda y'\, (y' \neq y')$;
from (i) and (ii) by (f) $\beta(x) + c(x) = \beta(x)$.

Proof of A3: assume $\beta(x) + \beta'(x) = \beta(x)$; assume $\neg\beta'(x) = c(x)$; what must be deduced is $\beta'(x) = \beta(x)$;
(i) assume $\neg\bigvee y(y\varepsilon\beta(x))$, hence $\beta(x) = \lambda y(y \neq y)$; because of $\neg\beta'(x) = c(x)$ we have by (d) $\bigvee y'(y'\varepsilon\beta'(x))$; $y'\varepsilon\beta(x) + \beta'(x)$ by (f), since $(y'\varepsilon\beta(x) \vee y'\varepsilon\beta'(x)) \wedge \beta(x)$ is connected with $\beta'(x) \wedge \beta(x)$ and $\beta'(x)$ have no

common element $(\beta(x) = \lambda y(y \neq y), \quad \neg\beta'(x) = \lambda y(y \neq y))$; hence $\neg\beta(x) + \beta'(x) = \beta(x)$, contradicting the first assumption; (i) shows $\bigvee y (y\varepsilon\beta(x))$;
(ii) assume $y\varepsilon\beta(x)$, hence $y\varepsilon\beta(x) + \beta'(x)$ (since $\beta(x) + \beta'(x) = \beta(x)$), hence by (f) $\beta(x)$ is connected with $\beta'(x) \wedge \beta(x)$ and $\beta'(x)$ have no common element $\vee\ \beta(x) = \beta'(x)$; assume $\beta(x)$ is connected with $\beta'(x) \wedge \beta(x)$ and $\beta'(x)$ have no common element; because of $\neg\beta'(x) = c(x)$ we have by (d) $\bigvee y'(y'\varepsilon\beta'(x))$; $y'\varepsilon\beta(x) + \beta'(x)$ by (f), since $(y'\varepsilon\beta(x) \vee y'\varepsilon\beta'(x)) \wedge \beta(x)$ is connected with $\beta'(x) \wedge \beta(x)$ and $\beta'(x)$ have no common element; but $\neg y'\varepsilon\beta(x)$, since $\beta(x)$ and $\beta'(x)$ have no common element; hence $\neg\beta(x) + \beta'(x) = \beta(x)$, contradicting the first assumption; consequently $\beta'(x) = \beta(x)$;
(ii) shows $\bigvee y(y\varepsilon\beta(x)) \supset \beta'(x) = \beta(x)$;
from the results of (i) and (ii) we obtain $\beta'(x) = \beta(x)$.

Proof of B1: (i) assume $y\varepsilon x$, hence, since x is a circle, by (a), (b), (c) $y\varepsilon m(x) \vee y\varepsilon f(x) \vee y\varepsilon s(x)$;
in the first and second case: $y\varepsilon f(x) + m(x)$ by (f), since in the first and second case $(y\varepsilon f(x) \vee y\varepsilon m(x)) \wedge f(x)$ is connected with $m(x) \wedge f(x)$ and $m(x)$ have no element in common; hence $y\varepsilon(f(x) + m(x)) + s(x)$ by (f), since $(y\varepsilon f(x) + m(x) \vee y\varepsilon s(x)) \wedge f(x) + m(x)$ is connected with $s(x) \wedge f(x) + m(x)$ and $s(x)$ have no element in common (always considering that x is a circle);
in the third case: $y\varepsilon\ (f(x) + m(x)) + s(x)$ by (f), since $(y\varepsilon f(x) + m(x) \vee y\varepsilon s(x)) \wedge f(x) + m(x)$ is connected with $s(x) \wedge f(x) + m(x)$ and $s(x)$ have no common element;
(ii) assume $y\varepsilon(f(x) + m(x)) + s(x)$, hence by (f) $(y\varepsilon f(x) + m(x) \vee y\varepsilon s(x))$, hence by (f) $(y\varepsilon f(x) \vee y\varepsilon m(x) \vee y\varepsilon s(x))$, hence, since x is a circle, by (a), (b), (c) $y\varepsilon x$.

Proof of B2: (i) assume $y\varepsilon(f(x) + m(x)) + s(x)$, hence by (f) $y\varepsilon f(x) + m(x) \vee y\varepsilon s(x)$, hence by (f) $y\varepsilon f(x) \vee y\varepsilon m(x) \vee y\varepsilon s(x)$;
in the first and third case: $y\varepsilon f(x) + s(x)$ by (f), since $(y\varepsilon f(x) \vee y\varepsilon s(x)) \wedge f(x)$ is connected with $s(x) \wedge f(x)$ and $s(x)$ have no element in common; hence $y\varepsilon(f(x) + s(x)) + m(x)$ by (f), since $(y\varepsilon f(x) + s(x) \vee y\varepsilon m(x)) \wedge f(x) + s(x)$ is connected with $m(x) \wedge f(x) + s(x)$ and $m(x)$ have no common element;
in the second case: $y\varepsilon(f(x) + s(x)) + m(x)$ by (f), since $(y\varepsilon f(x) + s(x) \vee y\varepsilon m(x)) \wedge f(x) + s(x)$ is connected with $m(x) \wedge f(x) + s(x)$ and $m(x)$ have no common element;

(ii) assume $y\varepsilon(f(x)+s(x))+m(x)$, hence by (f) $y\varepsilon f(x) \vee y\varepsilon s(x) \vee y\varepsilon m(x)$;
in the first and third case: $y\varepsilon f(x)+m(x)$ by (f) since $(y\varepsilon f(x) \vee y\varepsilon m(x)) \wedge$ $f(x)$ is connected with $m(x) \wedge f(x)$ and $m(x)$ have no common element; hence $y\varepsilon(f(x)+m(x))+s(x)$ by (f), since $(y\varepsilon f(x)+m(x) \vee y\varepsilon s(x)) \wedge$ $f(x)+m(x)$ is connected with $s(x) \wedge f(x)+m(x)$ and $s(x)$ have no common element;
in the second case: $y\varepsilon(f(x)+m(x))+s(x)$, since $(y\varepsilon f(x)+m(x) \vee y\varepsilon s(x)) \wedge$ $f(x)+m(x)$ is connected with $s(x) \wedge f(x)+m(x)$ and $s(x)$ have no common element.

Proof of B3 and B4: on the basis of (a) – (f) and because of $\bigwedge x\text{Circle}(x)$, B3 and B4 are immediately evident.

Proof of B5: B5 results trivially, since $\bigwedge x \neg x = f(x)$.

Proof of B6: since $\bigwedge x \neg m(x) = c(x)$, B6 is equivalent to $\bigwedge x(m(x)+s(x)=c(x))$; assume $y\varepsilon m(x)+s(x)$, hence by (f) ($m(x)$ is connected with $s(x) \wedge m(x)$ and $s(x)$ have no common element $\vee$ $m(x)=s(x)) \wedge (y\varepsilon m(x) \vee y\varepsilon s(x))$; but according to (a), (c) and (e) and $\bigwedge x\text{Circle}(x)$, $m(x)$ is not connected with $s(x) \wedge m(x) \neq s(x)$; hence $\neg\bigvee y(y\varepsilon m(x)+s(x))$, hence $m(x)+s(x)=\lambda y(y \neq y)$, hence by (d) $m(x)+s(x)=c(x)$.

Proof of B7: since $\bigwedge x \neg m(x) = c(x)$, B7 is equivalent to $\bigwedge x((f(x)+m(x))+f(x)=c(x)) \wedge \bigwedge x((f(x)+m(x))+m(x)=c(x))$;
(i) assume $y\varepsilon(f(x)+m(x))+f(x)$, hence by (f) $(y\varepsilon f(x)+m(x) \vee y\varepsilon f(x)) \wedge$ ($f(x)+m(x)$ is connected with $f(x) \wedge f(x)+m(x)$ and $f(x)$ have no common element $\vee$ $f(x)+m(x)=f(x))$; but $f(x)+m(x)$ and $f(x)$ have a common element $\wedge$ $f(x)+m(x) \neq f(x)$: $\bigwedge y(y\varepsilon f(x) \supset y\varepsilon f(x)+m(x)) \wedge \bigvee y(y\varepsilon f(x)), \bigvee y(y\varepsilon m(x)) \wedge f(x)$ and $m(x)$ have no common element; consequently $\neg\bigvee y(y\varepsilon(f(x)+m(x))+f(x))$, hence $(f(x)+m(x))+f(x) = \lambda y(y \neq y)$, hence by (d) $(f(x)+m(x))+f(x)=c(x)$;
(ii) assume $y\varepsilon(f(x)+m(x))+m(x)$; continue mutatis mutandis as in (i).

Proof of B8: since $\bigwedge x \neg f(x) = s(x)$, B8 is equivalent to $\bigwedge x((f(x)+s(x))+f(x)=c(x)) \wedge \bigwedge x((f(x)+s(x))+s(x)=c(x))$;
(i) assume $y\varepsilon(f(x)+s(x))+f(x)$, hence $f(x)+s(x)$ is connected with $f(x)$ $\wedge$ $f(x)+s(x)$ and $f(x)$ have no common element $\vee$ $f(x)+s(x)=f(x)$; but $f(x)+s(x)$ and $f(x)$ have a common element $\wedge$ $\neg f(x)+s(x)=f(x)$;

consequently $\neg\bigvee y(y\varepsilon(f(x)+s(x))+f(x))$, hence $(f(x)+s(x))+f(x)=\lambda y(y \neq y)$, hence $(f(x)+m(x))+f(x)=c(x)$;
(ii) assume $y\varepsilon(f(x)+s(x))+s(x)$; continue mutatis mutandis as in (i).

The model given for TO is trivial only with respect to B5. But let the second-order ODs of T' refer to the spheres in an infinite space (which have positive radius), including the sphere in the space 'whose centre is everywhere and whose surface nowhere', that is, the sphere in the space which has infinite radius, that is, the space itself (called 'the super-sphere'). The spheres are certain sets of points in the space, and the first-order ODs of T' refer to points in the space. We define:

For all second-order ODs 0 of T':
(a') $m(0) := \lambda y$ (y is in the surface of 0)
(y is in the surface of 0 $\equiv y\varepsilon 0 \wedge \bigvee r$ (r is a maximal distance between points of 0 $\wedge \bigvee y'$ $(y'\varepsilon 0 \wedge \text{distance}(y,y')=r)))$
(b') $s(0) := \lambda y$ (y is a centre of 0)
(y is a centre of 0 $\equiv y\varepsilon 0 \wedge \bigwedge r$ (r is a maximal distance between points of 0 $\supset \bigwedge y' \bigwedge y''$ $(y'\varepsilon 0 \wedge y''\varepsilon 0 \wedge \text{distance}\ (y',y'')=r \supset \text{d.}\ (y',y)=r/2 \wedge \text{d.}\ (y'',y)=r/2)))$
(c') $f(0) := \lambda y\ (\bigvee y' \bigvee y''$ (y' is a centre of 0 $\wedge$ y'' is in the surface of 0 $\wedge$ y is between y' and y'') $\vee \neg\bigvee y''$ (y'' is in the surface of 0) $\wedge$ y is a centre of 0)

The rest is the same as in the previous definition. (Notice that the interpretations of m(0) and s(0) have interchanged.) Any sphere in the space is either a normal (finite) sphere or the super-sphere. For normal spheres x in the space we have: $\neg m(x)=c(x)$, $s(x)=\lambda y$ (y = the centre of x), λy (y = the centre of x) $\neq x$, $f(x)=\lambda y \bigvee y''$ (y'' is in the surface of x $\wedge$ y is between the centre of x and y''), $\neg s(x)=f(x)$, $\neg x=f(x)$. For the super-sphere in the space g on the other hand, we have: $m(g)=c(g)$, $s(g)=g$ (since there is no maximal distance r between points of g), $f(g)=s(g)$, $g=f(g)$. B5 is now valid in a non-trivial manner. With respect to g B1 is proved as follows: $g=s(g)$, hence $g=(s(g)+c(g))+s(g)$ by A2 (which is independent of the universe of discourse of T'), hence $g=(f(g)+m(g))+s(g)=s(g)$, $(f(g)=s(g),\ m(g)=c(g))$. With respect to normal spheres, it is proved as previously in the model of circles.

XII. As has already been mentioned, there exists a sequel to this paper. In it extensions of language and axiom-system are introduced, always in close correspondence to Aquinatic teachings; these extensions serve to strengthen the implicit definition of Aquinatic terms which is provided by the original axiom-system. The first extension consists in adding the predicates L(0) ('0 is a living object') and H(0) ('0 is a human object'); this allows to formulate new definitions, for example $A(0) := \bigvee v(L(v) \wedge M(v) \wedge 0 = a(v))$, where 0 can be replaced by any ED ('To be a soul is to be the actuating form of a living material object'), and new axioms, for example $\bigwedge x(H(x) \supset \bigvee x'(I(x') \wedge x' = a(x)))$ ('The actuating form of a human being is a created immaterial substance'). The second extension consists in adding individuation-axioms, for example $\bigwedge x \bigwedge x'(s(x) = s(x') \supset x = x')$ ('esse diversum est in diversis'). Finally the intuitive interpretation of Aquinatic terms is discussed in detail (what – formal developments aside – is to be intuitively understood by the pure form, the essence, the matter etc. of an object?), and it is found that to a surprising extent they can be intuitively elucidated; the distinction between universal and individual forms is seen to be very helpful for this. The sequel ends with a synopsis of Aquinas' theory of forms, and reaches the conclusion that Aquinas is not a pure Aristotelian, but – concerning God – a genuine Platonist.

Since the sequel comprises in manuscript another 32 pages, it could not be published in this journal. I will be happy to make the material available to anyone interested.