

ON NEGATIVE AND DISJUNCTIVE PROPERTIES

In *Universals and Scientific Realism*, II, D.M. Armstrong argues against the existence of negative and disjunctive universals, in particular against the existence of negative and disjunctive properties – monadic universals. He admits, however, conjunctive universals. My paper is a defense of negative and disjunctive properties.

By the word “term” I shall always mean *singular term*; property-terms are special singular terms; *prima facie* they refer to properties. In the present context \wedge , \neg , \vee are term-forming operators; they form more complex property-terms out of simpler property-terms; for example, if f and g are property-terms, $(f \wedge g)$, $\neg f$, and $(f \vee g)$. These complex property-terms can be easily eliminated in certain contexts. We have:

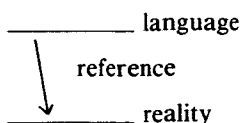
$$\begin{aligned} (f \wedge g)(x) &\text{ iff } f(x) \text{ and } g(x) \text{ [} f(x) : x \text{ exemplifies } f \text{]} \\ (f \vee g)(x) &\text{ iff } f(x) \text{ or } g(x) \\ (\neg f)(x) &\text{ iff not } f(x) \end{aligned}$$

These rules suffice to make any occurrence of a conjunctive, disjunctive or negative property-term disappear in *contexts of first-order predication* in favor of simple property-terms. So far there is no need to take complex property-terms ontologically seriously. Unfortunately contexts of first-order predication are not the only contexts in which complex property-terms occur. Take for example “Not to lie is sometimes hard”, “He intends to fly or to go by train”. And in deference to Armstrong’s distrust of the ontological authority of ordinary language I offer another example. Let “ f ” and “ g ” refer to two impeccable monadic universals, which play a central role in the higher regions of quantum mechanics; let us stipulate that there are some particulars to which they both apply, and some particulars to which only one or the other applies; these universals

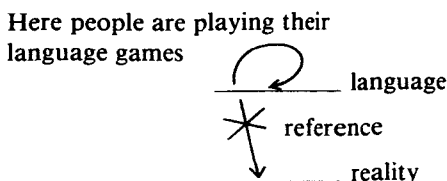
will serve as our examples throughout this paper. Then we can formulate the sentences “ $\neg f$ is not a property”, “ $(f \vee g)$ is not a property”.

Since “ $\neg f$ ”, “ $(f \wedge g)$ ”, and “ $(f \vee g)$ ” will be seen to be definable by definite descriptions, there is a general procedure for eliminating them from contexts like those mentioned; take Carnap’s or Russell’s method. Nevertheless we shall see that they are *genuinely referring* singular terms. The singular term “the father of John” is also eliminable from any context; nevertheless it is a genuinely referring singular term: since there is exactly one father of John, it genuinely refers to John’s father. “ $\neg f$ ”, “ $(f \wedge g)$ ”, “ $(f \vee g)$ ” will analogously be seen to genuinely refer to something, that is, *not* to some entity artificially stipulated as their reference. The only question is whether this something is indeed a property.

Let me make a parenthetical remark. Abstract singular terms in subject-position are the stronghold of the Platonists against the nominalists. This stronghold is unconquerable as long as both parties assume, as I do here, the *reference-paradigm* of language and its corresponding concept of truth:



Nominalists had better give up this paradigm. Yet many nominalists will find it hard to swallow the relativist consequences of giving it up in favor of the non-referential paradigm advocated, for example, by Wittgenstein in the *Philosophische Untersuchungen*; this paradigm might be sketched like this:



Moreover, the referential paradigm of language is very well entrenched. Motives of ontological parsimony aside, there is hardly any reason to relinquish it.

I proceed on the assumption, which is to be justified later, that “ $(f \wedge g)$ ”, “ $(f \vee g)$ ”, and “ $\neg f$ ” genuinely refer to something.

Armstrong says that there are no negative and disjunctive properties. This implies that he considers the sentences “ $\neg f$ is not a property”, “ $(f \vee g)$ is not a property” to be true in their straightforward meaning. If he did not consider them true in their straightforward meaning, I, for my part, would not know what he means by negative and disjunctive properties. I assume he means this:

- h is a negative property := h is a property and $\exists g'(g' \text{ is a property and } h = \neg g')$ (\exists : there is)
- h is a disjunctive property := h is a property and $\exists g' \exists f'(g' \text{ is a property and } f' \text{ is a property and } h = (f' \vee g'))$

If $\neg f$ and $(f \vee g)$ are not properties, what are they? Of course we can stipulate that the negation of a property and the disjunction of two properties never is a property, but is rather a so-and-so (a mere *concept*, for example); but why should we make such a stipulation? Why not admit $\neg f$ and $(f \vee g)$ as properties along with f and g ?

Armstrong offers several arguments. I cannot go over all of them here. I shall restrict my attention to two of them, which seem to me to be central. Armstrong says: “Properties should be such that it at least makes sense to attribute causal powers to objects in virtue of these properties. But how could a mere lack or absence endow anything with causal powers?” (*Universals and Scientific Realism*, II, p. 25). This is directed against negative properties; no negation of a property is a property, since it is “a mere lack or absence”, which cannot confer causal powers, something every property ought to be able to do. I will not at this point impugn the conception of properties implied. Just one global remark: the concepts of causality and privation are in my eyes heavily epistemological and evaluative in character, and hence out of place in ontology. But the negation of property f is *not* “a mere lack or absence”, whatever that may be. For then the negation of this negation would have to be a mere lack or absence, too. “Nothing will come of nothing”, as Armstrong says. But the negation of the negation of f is *not* a mere lack or absence; rather it is the property f itself: $\neg \neg f = f$.

Armstrong also says: “disjunctive properties offend against the principle that a genuine property is identical in its different particulars”

(*Universals and Scientific Realism*, II, p. 20). I do not impugn the conception of properties implied. But do disjunctive properties really offend against the mentioned principle? What, indeed, have two particulars in common that both exemplify $(f \vee g)$, because one of them exemplifies f , but not g , the other g , but not f ? What is identical in them because of this? This seems a hard question. But in fact there is no problem at all. They have in common the greatest entity that is both a part of f and a part of g . (Armstrong, it will be remembered, allows us to speak of parts of properties.) This entity shared by them is no other than $(f \vee g)$ itself. Assume a first-order language (variables f, g, h, \dots) including a part-predicate \leq (sentences $f \leq g, \dots$); call the entities in its domain “concepts”; let us *at this point* assume that some, but not all, concepts are properties.¹ Then we can define:

$$(f \vee g) := \iota k [k \leq f \wedge k \leq g \wedge \forall m (m \leq f \wedge m \leq g \text{ imp. } m \leq k)]$$

(\wedge : and; imp. : implies; \forall : for all; ι : the)

Now, it seems very plausible that every part of a property is a property. Armstrong himself admits this; on page 35 of *Universals and Scientific Realism*, he writes: “a part of a property will be a property”. Hence, since $(f \vee g)$ is a part of property f (and of property g), it is a property itself. Hence there are disjunctive properties.

For comparison I also offer the definitions of conjunction and negation (of concepts):

$$(f \wedge g) := \iota k [f \leq k \wedge g \leq k \wedge \forall m (f \leq m \wedge g \leq m \text{ imp. } k \leq m)]$$

(The conjunction of f and g is the smallest concept of which both f and g are parts.)

$$\neg f := \iota k [\forall h (h \leq k \wedge \text{not } \forall h' (h \leq h') \text{ imp. not } h \leq f) \wedge \forall g' (\forall h (h \leq g' \wedge \text{not } \forall h' (h \leq h') \text{ imp. not } h \leq f) \text{ imp. } g' \leq k)]$$

(The negation of f is the greatest concept that has no non-trivial part in common with f .)

Where there are parts, there are sums, products, and complements. The sums, products, and complements of individuals are individuals. Why shouldn't the sums, products, and complements of properties be properties? From the mereological point of view Armstrong offers no good reason why $\neg f$ and $(f \vee g)$ are not properties, but $(f \wedge g)$ is.

It should be noticed that for the purposes of this defense it is not

necessary to assume that every property has a complement, that every two properties have a product. As we know, this is not true of individuals. I am inclined to say that it is true of concepts, and hence of properties (since every property is a concept). But be that as it may, what good reason is there for denying that *some* property has a complement, or for denying that *some* pair of properties has a product? None. Rather, there is good reason for accepting the points in question, if we want to speak of parts of properties at all and apply the common laws of mereology. Since this is so, and since complement and product are unique if they exist, we may unproblematically assume that f has exactly one complement, referred to by " $\neg f$ ", and f and g exactly one product, referred to by " $(f \vee g)$ ". And why shouldn't these also be properties? Clearly, that they are *not* properties cannot be seen from their definitions.

But assume that $\neg f$ and $(f \vee g)$ are not properties. Strangely enough $\neg \neg f$ is a property, since $\neg \neg f = f$ and f is a property; this is like saying that $(2:0)$ is not a number, but $(2:0):0$ is. This is not an important point. But the next one is. Let us for a moment assume that g is a part of f ; then we have $(f \vee g) = g$; hence $(f \vee g)$ is a property, since g is a property. This means that if Armstrong wants to uphold the position that every disjunction of two properties is not a property, he must also hold that no property g ever is part of another property f . But the latter position seems to be clearly false, and contradicts moreover Armstrong's position on conjunctive properties. If there are conjunctive properties, then, of course, they have properties as parts.

Given Armstrong's criteria for propertyhood, like causal efficacy and identity in different particulars, can we be a priori sure that in every case, if h and k fulfill these criteria, $\neg h$ and $(h \vee k)$ don't? Why shouldn't it be possible that besides h and k also $\neg h$ and $(h \vee k)$ are causally efficacious and identical in different particulars? Let me give three examples, that are not mere possibilities.

(1) *Even* and *not-even*, that is, *odd* partition the domain of natural numbers (the entities they can be a predicated of) into two halves of equal size. Even numbers are the values of the function $2x$, odd numbers are the values of the function $2x + 1$ (with respect to the domain of natural numbers). Can we say that being odd is a mere absence or privation? We cannot. Rather, oddness is a substantial quality present in all odd numbers, just as evenness is a substantial quality present in all even numbers.

(2) *Animate* and *inanimate* partition the domain of material objects (the entities they can be predicated of) into two halves. Can we say that being inanimate is a mere absence or privation? We cannot. It certainly is a substantial quality just as being animate is. This is shown in the fact that only inanimate objects can serve as nourishment for living beings:

hence there is a causal efficacy that only inanimate objects can exert. To die is not a mere event of privation, but also a release of forces so far locked up inside the organism, forces that will serve to uphold life in other organisms.

It is not possible to explain the specific causal efficacy of inanimate objects ultimately by their so-called positive properties, since only being inanimate causes the object to have these positive properties. Armstrong, however, thinks that *prima facie* “negative” causation can always be seen to be “positive” causation, for example in the case of death by dehydration. I quote: “To say that lack of water caused his death reflects not a metaphysics of the causal efficacy of absences but merely ignorance. Certain (positive) processes were going on in his body, processes which, in the absence of water, resulted in a physiological condition in virtue of which the predicate ‘dead’ applied to his body” (*Universals and Scientific Realism*, II, p. 44). What is revealing about this passage is that the author finds it necessary to insert into his purported positive account of death by dehydration the phrase “in the absence of water”. Apparently even the author darkly feels that absence of water caused the positive processes he invokes to occur; they could not have occurred without it; it, and not they, is ultimately responsible for death by dehydration.

(3) Finally, *male* and *female* have a greatest common part: *male-or-female*, which is identical with *sexed*. Clearly, there is a causal efficacy that only sexed objects can exert, there is a common quality in all sexed objects.

For those not prejudiced in favor of the concepts of advanced natural science these examples serve to show that besides a property *h* and a property *k* we can have a property $\neg h$ and a property $(h \vee k)$ (on Armstrong’s own criteria for propertyhood!), hence that there are negative and disjunctive properties. For others they pose a question: Might not what they exemplify also occur in the domain of quantum mechanics?

The criticism of Armstrong’s position I am now going to offer is more fundamental than anything said so far. Armstrong thinks the semantical tradition in ontology to be on the whole discreditable. I am not of this opinion. But be that as it may, there is a profound reason why the semantical tradition has reigned in ontology and – we may safely assume – will continue to reign. Ontology will not come to lean on natural science for the reason that it seeks exclusively *metaphysically necessary*² truths, which are not germane to natural science, but rather to semantics. (In what follows I take the word “necessary” to mean *metaphysically necessary*.) A classical task of ontology is to classify entities on the most general level, that is, to decide the truth of questions of the form “Is *b* a *C*?”, where *C* is any ontological category, like “universal”, “relation”, “individual”, “state of affairs”, and so on. Hence the following principle must be true:

- (P) For every ontological category C: “b is a C” is necessarily true or “b is not a C” is necessarily true

If it were not true, then for some ontological category K “Is b a K?” would not be an ontological question, since the correct answer to it, whether positive or negative, would not be necessarily true, and ontology exclusively seeks answers that are necessarily true, as I have said. But it is a task of ontology to decide truthfully *any* question of the form “Is b a C?”, where C is any ontological category. Consequently “Is b a K?” turns out to be an ontological question after all, and we have a *reductio* argument for the truth of (P).

From (P) (P') follows:

- (P') “b is a property” is necessarily true or “b is not a property” is necessarily true,

if “property” is an ontological category, that is, without epistemological or evaluative connotations, which still seems to me to be the most satisfactory view of the meaning of that word. From (P') we can see that natural science is irrelevant for the question whether an entity *y* is a property or not. The truths that belong to natural science are all *metaphysically contingent*; but the question at hand has an answer that is necessarily true. If we already know that an entity *y* is a property, then natural science may tell us contingent facts about it: that it is exemplified, that it plays a fundamental role in the make-up of the world, while its negation does not; that it lends a certain causal efficacy to the particulars that exemplify it, while its negation does not. All this is valuable information, but it is of no concern for ontology. For, from the metaphysical point of view, all this might be different, *y* might not be exemplified; or it might not play a fundamental role in the make-up of the world, but at the same time its negation might; it might lend no causal efficacy to the particulars that exemplify it, but at the same time its negation might lend causal efficacy to the particulars that exemplify it in their turn. And yet the entity *y* would remain a property, since it is a property by metaphysical necessity. Its being a property is independent of what natural science can tell us about it; in particular there is no link between being a property and causality. But rather its being a property (“property” taken in a purely ontological sense) is founded by the role expressions corresponding to it play in our descriptively meaningful discourse, which in order to be effective must mirror not the particular

structure of reality, but the general structure of any possible reality. Being a property is one feature of that general structure, which is the structure ontology seeks to describe; the particular structure of reality is the concern of physics.

Now, expressions corresponding to $\exists f$ and $(f \vee g)$ play the very same role in our descriptively meaningful discourse as do expressions corresponding to f and g . Language is not an infallible guide in ontological matters (by no means!), but it is the best guide we have. Hence $\exists f$ and $(f \vee g)$ are properties along with f and g . Hence there are negative and disjunctive properties.

NOTES

¹ All properties, however, are concepts. If we want to make a distinction between properties and concepts (as I am doing here, but *vide infra*), then, roughly, properties are concepts which are "important" for us; there is, however, no purely ontological distinction between properties and concepts. Concepts can be thought of as the possible intensions (not *meanings*) of one-place predicates. They form ontological families with respect to the entities they can be predicated of. (Being inanimate, for example, does not belong to the family of concepts that can be predicated of natural numbers, but rather to the family of concepts that can be predicated of material objects.) \leq , applied to concepts of a certain family, stands for the intensional part-relation between them; being an animal, for example, is an intensional part of being human. This relation (with respect to a domain consisting of the concepts of a certain family) *can* be taken to satisfy the following axioms:

- A1 $f \leq g \text{ a. } g \leq h \text{ imp. } f \leq h$
 A2 $f \leq f$
 A3 $f \leq g \text{ a. } g \leq f \text{ imp. } f = g$
 A4 $\exists g[\forall f(A[f] \text{ imp. } f \leq g) \text{ a. } \forall k(\forall f(A[f] \text{ imp. } f \leq k) \text{ imp. } g \leq k)]$
 A5 $\forall k(A_t(k) \text{ a. } k \leq f \text{ imp. } k \leq g) \text{ imp. } f \leq g$
 $[A_t(k) := \forall h(h \leq k \text{ a. } \text{not } \forall h'(h \leq h') \text{ imp. } h = k); k \text{ is an atom}$
 $\text{iff every non trivial part of } k \text{ is identical with } k]$
 A6 $f \leq U_g A[g] \text{ a. not } \forall g(f \leq g) \text{ imp. } \exists k[k \leq f \text{ a. not } \forall g(k \leq g) \text{ a.}$
 $\exists h(k \leq h \text{ a. } A[h])]$
 $[U_g A[g] = \neg g[\forall f(A[f] \text{ imp. } f \leq g) \text{ a. } \forall k(\forall f(A[f] \text{ imp. } f \leq k) \text{ imp. } g \leq k)];$
 the conjunction of all A-concepts]

This formalism (a first-order approximation to a complete and atomistic Boolean algebra) is an extension of the calculus of concepts first developed by Leibniz (see W. Lenzen, "Leibniz und die Boolesche Algebra", *Studia Leibniziana* (1984), 16, p. 187-203). According to it every concept has a unique complement (negation), and every two concepts have a unique product (disjunction). One may define in a Leibnizian vein: $f(g) = \forall h(\text{not } h \leq g \text{ equ. } \neg h \leq g) \text{ a. } f \leq g$ [equ. : equivalent; g exemplifies f iff g is a maximal-consistent concept and f is part of g], which enables one to prove the predication-rules stated at the beginning of this paper. (All this is treated in detail in my forthcoming book *Axiomatische Ontologie*.)

Moreover, add these axioms:

- B1** $\text{Prop}(f) \text{ a. } g \leq f \text{ imp. Prop}(g)$ [$\text{Prop}(f) : f$ is a property]
B2 $\exists f \text{Prop}(f)$
B3 $\forall f(\text{A}[f] \text{ imp. Prop}(f)) \text{ imp. Prop}(\text{U}f\text{A}[f])$

Then one can prove: $\text{Prop}(f) \text{ imp. Prop}((f \vee g))$, $\text{Prop}(f) \text{ a. Prop}(g) \text{ equ. Prop}((f \wedge g))$.

² Metaphysical necessity is in between *analytical necessity* and *nomical necessity*. A sentence is analytically necessary iff it is true, and its meaning and syntactical form are sufficient for its truth; a sentence is nomically necessary iff it is true, and its logical relation to natural laws is sufficient for its truth; a sentence is metaphysically necessary iff it is true in every possible reality (or "world"). Analytical necessity implies metaphysical necessity, which in its turn implies nomic necessity. Let c be the number of concepts; then "There are precisely c concepts" is an example of a sentence that is metaphysically necessary, but not analytically necessary.

REFERENCE

Armstrong, D.M., 1978. *Universals and Scientific Realism*, Cambridge University Press, Vol. 1: *Nominalism and Realism*; Vol. 2: *A Theory of Universals*.