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Propensity-theory of Causality

(0) Since causality is essentially a temporal and modal notion, its analysis in terms of temporally structured possible worlds has recently been most prominent. Ontological scruples about possible worlds may lead one to seek a different route of analysis that nevertheless captures both the modal and the temporal aspects of causality. I present an eligible alternative that is based on the concept of *propensity* (from one moment of time to another), which in turn is definable by the concept of *difficulty of realization* (from one moment of time to another). In addition to being technically interesting and straightforward in application, this approach has the ontological advantage of referring to no unactualized possibilities ("counterfactualities"). It includes the analysis of objective probabilistic causality, which, however, will here appear in a form quite different from that it is usually presented in.

(1) There are two basic ideas of the propensity-theory of causality:

First: There are two kinds of temporal realization ("temporal being") *temporally centralized* (or *temporally manifest*) realization: *x is g at t*, or pleonastically *x at t is g at t*; and *temporally decentralized* (or *temporally latent*) realization: *x at t is g at t'*, where $t \neq t'$. Temporally decentralized realization in its turn comes in two kinds: *antedated realization: x at g is g at t'*, where $t \prec t'$; and *postdated realization: x at g is g at t'*, where $t' \prec t$.

Second: Temporal realization coincides analytically with *temporal necessity*. This simply means what is expressed by the following biconditional (intended to be analytical), and does in no manner entail determinism (as we shall see): *It is necessary for x at t to be g at t' iff x at t is g at t'*. Thus, "it is necessary for x at t to be g at (the same moment) t" is synonymous to "x at t is g at t", that is, synonymous to "x is g at t"; *temporally centralized necessity* coincides with temporally centralized realization: with temporal realization as usually conceived. In order to make the identification of temporal necessity and temporal realization intuitively palatable, take "it is necessary at t" in the sense of "it is *unchangeable* at t"; obviously being g at t is unchangeable for x at t iff x is g at t, and in general: being g at t' is unchangeable for x at t iff x at t *is* (already, though latently) g at t'.

(2) The basic ideas of the propensity-theory of causality that have just been

described have, on the face of them, nothing to do with either propensities or causality. But they are now being generalized in two respects: In the first place I allow *degrees of (temporal) realization, that is, of (temporal) necessity*; and in the second place I allow the relativization of graded realization (necessity) to *particular* aspects of the total situation to which it is temporally referred (in a centralized or decentralized manner). This will give us the *propensity*-theory of *causality* (PTC) as follows:

The basic concept of the PTC is "$d(x,t,f,g,t')$": "the difficulty for x qua being f at t to be g at t'" (or "the distance of x qua being f at t from being g at t'"). I then define:

Definition 1
(a) If $d(x,t,f,g,t') = \infty$, *then* $pro(x,t,f,g,t') = 0$;
(b) if $0 \leq d(x,t,f,g,t')$, *then*

$$pro(x,t,f,g,t') = \frac{1}{d(x,t,f,g,t')+1} .$$

(The truth of "$\nu < \nu'$" demands that both ν and ν' are real numbers, the truth of "$\nu = \nu'$" does not; "$pro(x,t,f,g,t')$" is to be read as "the propensity of x qua being f at t is to be g at t'" or "the degree in which it is necessary for x qua being f at t to be g at t'".)

Definition 2
x qua being f at t is g in the degree r at t'
$\Longleftrightarrow_{df}$ $pro(x,t,f,g,t') = r$.

Definition 3
It is in the degree r necessary for x qua being f at t to be g at t' $\Longleftrightarrow_{df}$ *x qua being f at t is g in the degree r at t'.*

Definition 4
x qua being f at t is g at t' $\Longleftrightarrow_{df}$ *x qua being f at t is g in the degree 1 at t'.*

Definition 5
It is necessary for x qua being f at t to be g at t' $\Longleftrightarrow_{df}$ *it is in the degree 1 necessary for x qua being f at t to be g at t'.*

Definition 6
x at t is g at t' $\Longleftrightarrow_{df}$ *x qua being* $con\, f(x \text{ is } f \text{ at } t)$ *at t is g at t'.*

("$con\, f(x \text{ is } f \text{ at } t)$" is to be read as "the conjunction of all f that x is at t".)

Definition 7
It is necessary for x at t to be g at t' $\Longleftrightarrow_{df}$ *it is necessary for x qua being* $con\, f(x \text{ is } f \text{ at } t)$ *at t to be g at t'.*

The following two definitions establish the connection between the present article and my earlier article "Propensity and Possibility" (see the end of the paper):

Definition 8
$d(x,t,g,t')$ *("the difficulty for x at t to be g at t'")*
$\Longleftrightarrow_{df}$ $d(x,t,con\,f(x \text{ is } f \text{ at } t), g, t')$

Definition 9
$pro(x,t,g,t')$ *("the propensity of x at t to be g at t'")*
$\Longleftrightarrow_{df}$ $pro(x,t,con\,f(x \text{ is } f \text{ at } t), g, t')$

(3) I now head straight for causality:

Definition 10
x being g at t' is a consequence of x being f at t $\Longleftrightarrow_{df}$
i. $t \prec t'$;
ii. it is necessary for x qua being f at t to be g at t'.

Defining

Definition 11
x being f at t anticipates x being g at t' $\Longleftrightarrow_{df}$
i. $t \prec t'$;
ii. x qua being f at t is g at t',

we can prove:

Theorem 1
x being g at t' is a consequence of x being f at t $\Leftrightarrow$ *x being f at t anticipates x being g at t'.*

Various clauses may be added to the definiens of definition 10 in order to obtain various concepts of causation fitting various intuitions. (If we do want to exclude *instantaneous causation*, "$t \prec t'$" is to be changed to "$t \preceq t'$".) I won't go into this detail. Just a few notes: Frequently "cause" is understood to mean *first cause*:

Definition 12
x being f at t (first) causes x being g at t' $\Longleftrightarrow_{df}$ *x being g at t' is a consequence of x being f at t, and* $\forall t*(t* \prec t$: *it is not necessary for x at t* to be g at t').*

Defining

Definition 13
x being g at t' is anticipated at t $\Longleftrightarrow_{df}$
i. $t \prec t'$;
ii. x at t is g at t'.

we can prove:

Theorem 2
x being f at t causes x being g at t' $\Leftrightarrow$ *x being f at t anticipates x being g at t', and x being g at t' is not anticipated before t.*

By adding the clause "not $\exists h(h$ is intensional part of f and $h \neq f$ and x being g at t' is a consequence of x being h at $t)$" we obtain the concept of *precisely sufficient causality.* By adding the clause "not $\exists h$(g is intensional part of h and $h \neq g$ and x being h at t' is a consequence of x being f at t)" we obtain the concept of *maximal causation.* (Concerning the notion of intensional part see below, section 5.)

(4) The import of all these definitions is of course vague as long as the concept on which they all are ultimately based has not been axiomatically characterized. Here are its axioms:

A1 (a) If x is not f at t, then $d(x,t,f,g,t') = \infty$.

(b) If x is f at t, then $d(x,t,f,g,t') = \infty$ or $0 \leq d(x,t,f,g,t')$.

An immediate consequence of A1 is:

Theorem 3

$d(x,t,f,g,t') = \infty$ *or* $0 \leq d(x,t,f,g,t')$.

A2 If x is f at t, then $d(x,t,f,f,t) = 0$.

(The reverse is obtained by A1(a), since $0 \neq \infty$.)

We immediately obtain from this:

Theorem 4

If $d(x,t,f,f,t) = \infty$, *then* x *is not* f *at* t.

(The reverse is obtained by A1(a).)

A3 (a) If $d(x,t,f,g,t') = 0$ and x is h at t and f is *intensional part* of h, then $d(x,t,h,g,t') = 0$.

(b) If $d(x,t,f,g,t') = 0$ and $t \prec t*$, then $d(x,t*,h \vee \neg h,g,t') = 0$.

A3(a) and A3(b) are the *conservation-principles of complete realization*; for "$d(x,t,f,g,t') = 0$" is analytically equivalent to "x qua being f at t is g at t'" (because we can derive $d(x,t,f,g,t') = 0 \Leftrightarrow pro(x,t,f,g,t') = 1$, using theorem 3 and definition 1). Complete realization (realization of degree 1) is preserved in lengthening the temporal sequence (A3(b)), and in strengthening its basis — the property qua having which at t x has another (or the same) property at t' (A3(a)).

(5) A3 provides the opportunity for interpolating what has to be said about the logical background of the PTC.

The language in which it is formulated has four kinds of variables (is a four-sorted language): *variables for properties*: $f, g, h, f' \ldots$; *variables for individuals*: $x, y, z, x' \ldots$; *variables for moments (points in time)*: $t, t', t* \ldots$; *variables for quantities*: $r, r', r* \ldots$.

Other quantity-terms besides quantity-variables are standard real number names and standard arithmetical functional expressions (addition, division, subtraction, multiplication). A special quantity-term is ∞ ("infinity"). There are quantity-terms that are not recursively based on other quantity-terms, but are nevertheless non-atomic: all terms $d(\nu, \tau, \varphi, \psi, \tau')$ (where ν is an individual-term, τ, τ' are moment-terms, and φ, ψ are property-terms).

The following basic predicates are meaningful for quantity-terms, (ϱ, ϱ' indicate the places of substitution for them): $\varrho = \varrho'$, $\varrho < \varrho'$, $R(\varrho)$ ("ϱ is a real number"): Other predicates for quantity-terms, like $\varrho \leq \varrho'$ and $\varrho > \varrho'$, are defined as usual.

I presuppose the arithmetic of real numbers (plus ∞) as part of the background logic. The list of axioms starts with $\forall r(R(r)$ or $r = \infty)$; not $R(\infty)$; $\forall r \forall r'$(if $r < r'$, then $R(r)$ and $R(r')$); etc.

Other property-terms besides property-variables are *abstraction-terms*: $\lambda\nu A[\nu]$ (where ν is an individual-variable, and all other terms in $A[\nu]$ *that do not occur within other terms in* $A[\nu]$ are individual-terms or property-terms); and *conjunction-terms*: $con\,\varphi A[\varphi]$ (where φ is a property-variable and $A[\varphi]$ a temporally definite predicate: it does not contain an expression $\psi(\nu)$ [see below], or an expression defined using such an expression). Properties are assumed to be *momentary* and *non-modal*: neither other moments of time nor alternative courses of events have to be considered for answering the question whether x is f at t.

The following basic predicates are meaningful for property-terms (φ, ψ indicate the places of substitution for them, ν the place of substitution for individual-terms, τ that for moment-terms): $\varphi(\nu)$ ("ν is φ"), ν is φ at τ, $\psi = \varphi$.

The definitions of important functional property-terms are: $\neg\varphi := \lambda\nu'$ not$\varphi(\nu')$, $(\varphi \wedge \psi) := \lambda\nu'$ ($\varphi(\nu')$ and $\psi(\nu')$), $(\varphi \vee \psi) := \lambda\nu'$ ($\varphi(\nu')$ or $\psi(\nu')$) (where ν' does not occur in φ, ψ). A special property-term is $\kappa := \lambda x$not$(x = x)$. A defined predicate meaningful for property-terms is: φ is intensional part of $\psi := (\varphi \wedge \neg\psi) = \kappa$.

I presuppose as a *logical axiom(-schema)* $\forall\nu(\lambda\nu'\, A[\nu'](\nu)$ iff $A[\nu])$ (ν' not in $A[\nu]$, ν not in $A[\nu']$), and as a *logical rule*: $\vdash \forall\nu(\varphi(\nu)$ iff $\psi(\nu)) \models \vdash \varphi = \psi$ (ν not in φ, ψ). (The rule is weaker than an axiom of extensionality!)

I use the following special principles for $con\,\psi A[\psi]$ (I will not bother about the general laws for $con\,\psi A[\psi]$ of which the special principles are consequences):

$\forall x \forall f \forall t(x$ is f at t iff f is intensional part of $con\,f(x$ is f at $t))$, $\forall x \forall f(f$ is intensional part of $con\,f(x$is$f$at$t)$ iff $\neg f$ is not intensional part of $con\,f(x$is$f$at$t))$. They are sufficient for deducing:

$\forall x \forall f \forall g(x$ is $f \vee g$ at t iff x is$f$ at t or x is g at $t)$, $\forall x \forall f(x$ is $\neg f$ at t iff x is not f at $t)$, $\forall x \forall f \forall g(x$ is $f \wedge g$ at t iff x is$f$ at t and x is g at $t)$.

The following basic predicates are meaningful for moment-terms (τ, τ' indicate the places of substitution for them): $\tau \prec \tau'$, $\tau = \tau'$. $\prec$ is assumed to have the characteristics of a temporal ordering in the classical sense. But I shall not here make use of these.

(6) The list of the main axioms is completed by:

A4 If x is f at t, then

(a) if $d(x,t,f,g,t') = 0$, then $d(x,t,f,\neg g,t') = \infty$;

(b) if $d(x,t,f,g,t') > 0$, then

$$d(x,t,f,\neg g,t') = \frac{1}{d(x,t,f,g,t')};$$

(c) if $d(x,t,f,g,t') = \infty$, then $d(x,t,f,\neg g,t') = 0$.

A5 If x is f at t and $(g \wedge h) = \kappa$, then

(a) if $d(x,t,f,g,t') = 0$ or $d(x,t,f,h,t') = 0$, then $d(x,t,f,g \vee h,t') = 0$;

(b_1) if $d(x,t,f,g,t') = \infty$, then $d(x,t,f,g \vee h,t') = d(x,t,f,h,t')$;

(b_2) if $d(x,t,f,h,t') = \infty$, then $d(x,t,f,g \vee h,t') = d(x,t,f,g,t')$;

(c) if $d(x,t,f,g,t') > 0$ and $d(x,t,f,h,t') > 0$, then

$$d(x,t,f,g \vee h,t') = \frac{d(x,t,f,g,t')d(x,t,f,h,t') - 1}{d(x,t,f,g,t') + d(x,t,f,h,t') + 2}.$$

Here are some important theorems that can be deduced from A1 – A5:

Theorem 5

$d(x,t,f,g \wedge \neg g,t') = \infty$

Theorem 6

If $d(x,t,f,g,t') = r$ *and* $R(r)$ *and* h *is intensional part of* g, *then* $d(x,t,f,h,t') \leq r$.

Theorem 7

If $d(x,t,f,g,t') = \infty$ *and* x *is* f *at* t *and* $t \prec t*$, *then* $d(x,t*,h \vee \neg h,g,t') = \infty$

Theorem 8

If $d(x,t,f,g,t') = \infty$ *and* f *is intensional part of* h, *then* $d(x,t,h,g,t') = \infty$.

Theorem 9

(a) $d(x,t,f \vee \neg f,g,t') = \infty$ *iff* $\forall h(d(x,t,h,g,t') = \infty)$;

(b) $d(x,t,f \vee \neg f,g,t') = 0$ *iff* $\forall h($ *if* x *is* h *at* t, *then* $d(x,t,h,g,t') = 0)$.

Theorem 10

(a) If $d(x,t,f,g,t') = 0$, *then* $d(x,t',g,g,t') = 0$ *[that is,* x *is* g *at* t'*];*

(b) If x *is* f *at* t *and* $d(x,t,f,g,t') = \infty$, *then* $d(x,t',g,g,t') = \infty$ *[that is,* x *is not* g *at* t'*].*

Theorem 11

If f *is intensional part of* g *and* x *is* g *at* t, *then* $d(x,t,g,f,t) = 0$.

Theorem 12
If $t \prec t'$, *then* $d(x,t',g,f,t) = 0$ *or* $d(x,t',g,f,t) = \infty$.

(7) A further principle with some intuitive credentials is the *Principle of Uniformity*: *If* $d(x,t,f,g,t') = r$ *and* x *is* f *at* t, *then* $\forall y($ *if* y *is* f *at* t, *then* $d(y,t,f,g,t') = r)$.

What holds for an x that is f at t *qua being* f *at* t, that must hold for all y that are f at t (qua being f at t). Or so it seems.

The principle seems to make merely explicit what is intended by the phrase "qua being f at t". But in fact it has absurd consequences: $\forall y(y$ *is* $h \vee \neg h$ *at* $t)$ and $d(x,t,h \vee \neg h, \lambda y'\,(y' = x),t') = 0$ are unobjectionable premises; hence by the Principle of Uniformity $\forall y(d(y,t,h \vee \neg h, \lambda y'\,(y' = x),t') = 0)$, from which we obtain by theorem 10 $\forall y(y$ is $\lambda y'\,(y' = x)$ at $t')$.

The fact of the matter is that the phrase "x qua being f at t" does not cancel the particularity of x. Therefore, instead of the Principle of Uniformity we ought rather to add to the axioms:

A6 $d(x,t,f,g,t') = d(x,t,f \wedge \lambda y'\,(y' = x),g,t') = d(x,t,f,g \wedge \lambda y'\,(y' = x),t')$

Hence: "x qua being f at t" means x *qua being* x *[so to speak] and* f *at* t.

(8) Special properties as bases (at t) for complete realization define special necessities. The weakest necessity (relative to t) is defined by the strongest property x has at t: *con* $f(x$ is f at $t)$; it is necessity *simpliciter*, realization *simpliciter*, since we have x at t is g at t' iff x qua being *con* $f(x$ is f at $t)$ at t is g at t' iff it is necessary for x qua being *con* $f(x$ is f at $t)$ at t to be g at t' iff it is necessary for x at t to be g at t'.

The strongest necessity is defined by the weakest property x has at t: $\lambda y(y = y)$ $[\forall f(\lambda y\,(y = y) = (f \vee \neg f))]$. Another property x is guaranteed to have at t is $\lambda y'\,(y' = x)$: *the essence of* x which, in strength, is inbetween *con* $f(x$ is f at $t)$ and $\lambda y\,(y = y)$ (it is a genuine intensional part of the former, and the latter is a genuine intensional part of it), and we may use it to define yet another special necessity: *essential necessity*. However, on the basis of A6 it turns out that essential necessity coincides with the strongest necessity, since we have:

Theorem 13
$d(x,t,\lambda y\,(y = y),g,t') = d(x,t,\lambda\,(y = x),g,t')$.

I include the definitions of essential necessity and essential realization:

Definition 14
It is essentially necessary for x *at* t *to be* g *at* t' $\Longleftrightarrow_{df}$ *it is necessary for* x *qua being* $\lambda y\,(y = x)$ *["qua being* x*"] at* t *to be* g *at* t'.

Definition 15
x *at* t *is essentially* g *at* t' $\Longleftrightarrow_{df}$ x *qua being* $\lambda y\,(y = x)$ *at* t *is* g *at* t'.

Definition 16
x is essentially g at t $\Longleftrightarrow_{df}$ *x at t is essentially g at t.*

(9) Something has to be said about our cognition of causation and necessity in general. Given the PTC, the problem of cognition boils down to the question in what manner we can verify and falsify basic statements about distances, that is, difficulties of realization, in particular, statements assigning 0 or ∞ distance. Suppose someone asserts "$d(x_0, t_0, f_0, g_0, t_0) = 0$". This we can verify by finding out that x_0 is f_0 at t_0 and that g_0 is an intensional part of f_0 (using theorem 11). And we can falsify it by finding out, either that x_0 is not f_0 at t_0, or that x_0 is not g_0 at t_0.

But suppose x_0 is both g_0 and f_0 at t_0, but g_0 is not an intensional part of f_0? The cognized presence of what objective features makes us then reject or accept "$d(x_0, t_0, f_0, g_0, t_0) = 0$"? The problem becomes even more severe if the statement is "$d(x_0, t_0, f_0, g_0, t_1) = 0$", where $t_0 \prec t_1$. The cognized presence of what objective features over and above that x_0 is f_0 at t_0 and that x_0 is g_0 at t_1, which is all that we can observe, makes us assert or deny this?

Hume, of course, would have contended that there are *no such* objective features whose presence can be cognized. If we agree with him, then we have to account for the strange fact that there are countless statements about objective difficulties of realization *in the future* — subjective probabilities in no manner enter into their meaning — that informed people unhesitatingly agree to, and others that they unhesitatingly deny. By virtue of what cognized fact? Should we say that there is a direct intuitive knowledge of difficulties of realization? I leave it at this question. I only add that every *objective* theory of causation and necessity — not only the propensity-theory — is troubled by the epistemological problem so forcefully pointed out by Hume. *Subjective* theories of causation and necessity, on the other hand, miss the entire point of these two concepts. Whatever is analyzed by these theories, *it is not* causation and necessity. A theory of causation and necessity is objective; or there is no such theory.

(10) I end this paper by giving some examples of the application of the PTC:

(a) *Formulations of determinism* (in the sequence of their strength):

Leibnizian determinism:

$$\forall x \forall f \forall t \forall t'(d(x, t, \lambda y(y = x), f, t') = 0 \text{ or } d(x, t, \lambda y(y = x), f, t') = \infty).$$

Laplacian determinism:

$$\forall x \forall f \forall t \forall t'(d(x, t, f, t') = 0 \text{ or } d(x, t, f, t') = \infty).$$

Causal determinism:

$$\forall x \forall f \forall t(\exists t'(t' \prec t \text{ and } d(x, t', f, t) = 0) \text{ or } \exists t'(t' \prec t \text{ and } d(x, t', f, t) = \infty)).$$

Causal determinism is actually a rather weak assertion; it does not conflict with the assumption of free decision *previous to the act*; it only says that an act is determined to occur or determined not to occur at *some* moment in the past; it does not say that an act is determined to occur or determined not to occur at *any* moment in the past.

None of these versions of determinism or its negation is implied by the PTC.

(b) *Definition of propositional causation*:

That x is f at t causes that y is g at t' $\Longleftrightarrow_{df}$ x being $\lambda z(f(x)$ and $z = z)$ at t causes x being $\lambda z(g(y)$ and $z = z)$ at t'.

The property $\lambda z(f(x)$ and $z = z)$ can represent the (momentary, non-modal) state-of-affairs *that x is f*. This is so, because $\forall y(\lambda z(f(x)$ and $z = z)(y)$ *iff* $f(x))$ is a logical theorem, and $\lambda z(f(x)$ and $z = z) = \lambda z(z = z)$ *or* $\lambda z(f(x)$ and $z = z) = \lambda z not(z = z)$ is not (in view of the absence of the principle of extensionality for properties). The logical background is to be strengthened by *x is f at t iff y is* $\lambda z(f(x)$ and $z = z)$ *at t*.

(c) *Definition of agent causality*:

Among the (momentary, non-modal) properties are the (momentary, non-modal) *action-properties*; among the individuals are the *persons*. The following definition of personal or agent causality can be formulated:

x at t causes that y is g at t' $\Longleftrightarrow_{df}$ x is a person and $\exists f(f$ is an action-property and x is f at $t)$ and $pro(x, t, con\, f(f$ is an action-property and x is f at $t), \lambda z(g(y)$ and $z = z), t') = 1$ and $not\exists t*(t* \prec t$ and $pro(x, t*, con\, f(f$ is an action-property and x is f at $t), t) = 1)$ and $not\exists t*(t* \prec t$ and $pro(x, t*, \lambda z(g(y)$ and $z = z), t') = 1)$.

According to this definition agent causality is always *free causality* [this is expressed by the clause "$not\exists t*(t* \prec t$ and $pro(x, t*, con\, f(f$ is an action-property and x is f at $t), t) = 1)$"] and *first causality* [this is expressed by the clause "$not\exists t*(t* \prec t$ and $pro(x, t*, \lambda z(g(y)$ and $z = z), t') = 1)$"]; it is *not*, however, *sole causality: x at t causes that y is g at t'* does not exclude according to the definition that *z at t causes that y is g at t'*, where $z \neq x$. ("$t \preceq t'$" follows from the definiens; if we want to exclude instantaneous agent causality, "$t \neq t'$" has to be added to it.)

(d) *Definition of causal influence and probabilistic causality*:

x being f at t exerts causal influence on x with respect to g and t' $\Longleftrightarrow_{df}$ $0 < pro(x, t, f, g, t') \neq pro(x, t, \lambda z(z = x), g, t')$.

We can distinguish positive and negative causal influence: replace "$\neq$" either by "$>$" or by "$<$". $pro(x, t, \lambda z(z = x), g, t')$ is *the essential (or inner) propensity of x at t to be g at t'*. It seems plausible to assume:*In case neither g nor $\neg g$ is an intensional part of $\lambda z(z = x)$ and $t \preceq t'$, the inner propensity of x at t to be g at t' is equal to the inner propensity of x at t to be $\neg g$ at t'* (contradicting Leibnizian determinism, since not for every property g and individual x: g is an intensional part of $\lambda z(z = x)$ or $\neg g$ is an intensional part of $\lambda z(z = x)$). Thus, (deterministic) causality (in the sense of *first causality*)

implies positive causal influence: From *x being f at t causes x being g at t'* we obtain $pro(x,t,f,g,t') = 1$, whereas $pro(x,t,\lambda z(z=x),g,t') = 0.5$, since $t \prec t'$, and neither g nor $\neg g$ is an intensional part of $\lambda z(z=x)$:
(1) suppose g is an intensional part of $\lambda z(z=x)$; let t^* be a moment before t; by theorem 5 and A4 we have

$$d(x,t^*,g\vee\neg g,g\vee\neg g,t') = 0 \; [x \text{ is } g\vee\neg g \text{ at } t^*,\ \neg(g\wedge\neg g) = g\vee\neg g],$$

hence by A6

$$d(x,t^*,g\vee\neg g,\lambda z(z=x),t') = 0;$$

hence by theorem 6 and 3

$$d(x,t^*,g\vee\neg g,g,t') = 0,$$

hence by A3(a)

$$d(x,t^*,\text{con}\,f(x \text{ is } f \text{ at } t*),g,t') = 0 \text{ —}$$

in contradiction to first causation.

(2) Suppose $\neg g$ is an intensional part of $\lambda z(z=x)$,
hence by theorem 11

$$d(x,t',\lambda z(z=x),\neg g,t') = 0,$$

hence via theorem 10 x is not g at t'; but x is g at t' by $pro(x,t,f,g,t') = 1$ and theorem 10.

Probabilistic causality is defined in terms of causal influence:

x being f at t is a (probabilistic) cause of x being f at t' $\Longleftrightarrow_{df}$ x is g at t', and x being f at t exerts positive causal influence on x with respect to g and t'.

"$t \preceq t'$" follows from the definiens of causal influence (hence there can be no causal influence into the past!): Suppose $t' \prec t$;
by theorem 12

$$d(x,t',f,g,t) = 0 \text{ or } d(x,t',f,g,t) = \infty;$$

if the last, then by definition 1

$$pro(x,t',f,g,t) = 0 \text{ —}$$

which excludes causal influence with respect to g and t (see the definition); if, however, the first, then

$$pro(x,t',f,g,t) = 1$$

by definition 1, and by theorem 12 we also have because of $t' \prec t$:

$$d(x,t',\lambda z(z=x),g,t) = 0 \text{ or } d(x,t',\lambda z(z=x),g,t) = \infty;$$

the latter contradicts $d(x,t',f,g,t,) = 0$ because of theorem 10; hence

$$d(x,t',\lambda z(z=x),g,t) = 0,$$

hence by definition 1

$$pro(x,t',\lambda z(z=x),g,t) = 1;$$

hence

$$pro(x,t',f,g,t,) = pro(x,t',\lambda z(z=x),g,t) \text{ —}$$

which excludes causal influence with respect to g and t (see the definition).

Literature

This paper is a generalization of my "Propensity and Possibility", *Erkenntnis* 38 (1993), pp. 323–341. "Propensity and Possibility" contains no material on causation and relativized propensity, but is more explicit on other points.