Logic and **L**ogical **P**hilosophy
Volume 2 (1994), 23–33

Uwe Meixner

EVENTS AND THEIR REALITY

CONTENTS

Received August 15, 1994

This paper presents a set-theoretical conceptual framework for theorizing about *(possible) events*, and states some analytical and synthetical principles which describe the way in which the concept of *reality* (or *actuality*) applies to them. The conceptual framework has few primitives, but is nevertheless of great definitional power; the demonstration of this will fill the first part of the paper.

1. Conceptual Framework

1.1. Primitives

T – the set of time-points; it is assumed to be non-empty.

S – the set of total momentary states*; it is assumed to be non-empty. The asterisk indicates that the elements of S are states *which are individuals, not sets*; below we will mainly consider states *which are sets, not individuals*, and "state" without asterisk will always refer to those. You may think of total momentary states* as obtainable by a procedure of abstraction and variation: take a complete (zero-extended) slice of the world (which is considered to be a temporally extended entity), abstract from its temporal position, that is, consider merely its contents, and vary its contents *ad libitum*, but consistently; this gives you every total momentary state*.

$\prec$ – the before-relation (a set of ordered pairs); it is assumed to be non--empty, and to be a relation in T, which is irreflexive, transitive and linear in T. (Linearity of $\prec$ in T: for all elements t and t' of T: $t \prec t'$ or $t' \prec t$ or $t = t'$.)

The assumptions presented in connection with these three primitives are minimal. They may, of course, be extended. No objections to this.

1.2. Defined concepts

Monadic concepts:

D1. *S is a momentary state iff S is a non-empty subset of* S.

D2. *S is a total momentary state iff S is a singleton subset of* S.

D3. *R is a temporal region iff R is a non-empty subset of* T.

D4. *R is a minimal temporal region iff R is a singleton subset of* T.

D5. *R is a connected temporal region iff R is a temporal region, and for all elements t, t' of R: all t'' with $t \prec t'' \prec t'$ are elements of R.*

A connected temporal region is a set of time-points that has no lacunae with respect to $\prec$, T and every minimal temporal region are trivially connected temporal regions.

D6. *R is an interrupted temporal region iff R is a temporal region, but not a connected one.*

D7. f *is an event iff* f *is a (set-theoretical) function, the domain of* f *is a temporal region, and for all elements t of the domain of* f: $\mathsf{f}(t)$ *is a momentary state.*

The motivation behind this definition is that anything whose intrinsic make is described by stating what is going on at each moment of a certain stretch of time (interrupted or not) is an event, and that an event is always something whose intrinsic make is described by stating what is going on at each moment of a certain stretch of time. This conception of events is at least as legitimate as any of the many others that have been put forward in recent years. Events are, of course, not literally certain set-theoretical functions; but they can be *conceptualised* as being such functions.

Who would like to reserve the term 'event' for something more specific than what is described by the definiens of D7 is free to do so; he may take the definiens of D7 to define the predicate 'f is a pre-event'; then he can hold that only some pre-events are events, namely pre-events having such and such properties.

D8. f *is a momentary event iff* f *is an event whose domain is a minimal temporal region.*

D9. f *is a process iff* f *is an event, but not a momentary one.*

Processes form a species of events; this is quite in harmony with ordinary discourse. However, there is also some tendency in ordinary discourse to reserve the term 'event' for momentary events: 'event' has in some uses the connotation of suddenness.

D10. f *is an uninterrupted process iff* f *is a process whose domain is a connected temporal region.*

D11. f *is a stable state iff* f *is an uninterrupted process, and for all* t, t' *in the domain of* f*:* $\mathsf{f}(t) = \mathsf{f}(t')$.

A stable state, which is an event, is quite a different entity from a momentary state, which is not an event. The (uninterrupted) standing of a specific vase on a specific table between two specific moments of time is an example for a stable state; it is the temporally uninterrupted repetition of one and the same momentary state: the specific vase standing on the specific table.

D12. f *is a pure change iff* f *is an uninterrupted process, and for all* t, t' *in the domain of* f *with* t *different from* t'*: the intersection of* $\mathsf{f}(t)$ *and* $\mathsf{f}(t')$ *is empty.*

The momentary states that are the functional values of a pure change are all incompatible with one another: their intersection is empty: there is no total momentary state* in which they both obtain. The (uninterrupted) movement of a specific particle from one specific place to another specific place between two specific moments of time (in a specific manner, for example, in a straight line and unaccelerated), is an example for a pure change. The momentary states that are succeeding each other in it are all incompatible with one another, because at each moment of the change the particle is at a different place and because the momentary states *the particle being at place* p and *the particle being at place* p' are incompatible if p and p' are different.

D13. f *has an end iff* f *is a process, and there is a* t *which is element of the domain of* f*, such that any* t' *with* $t \prec t'$ *is not element of the domain of* f *(in short: the domain of* f *has a last element).*

D14. f *has a beginning iff* f *is a process whose domain has a first element.*

D15. f *is bounded iff* f *has a beginning and an end.*

D16. f *is a temporally maximal event iff* f *is an event whose domain is* T.

D17. f *is an event maximal in contents iff* f *is an event, and for all elements* t *of the domain of* f*:* $\mathsf{f}(t)$ *is a total momentary state.*

D18. f *is a world-history iff* f *is an event that is maximal temporally and in contents.*

Non-monadic concepts:

D19. R *is the temporal extension of* f *iff* f *is an event, and* R *is the domain of* f.

D20. *S is the state of* f *at* t *iff* f *is an event,* t *is an element of the domain of* f*, and* $S = \mathsf{f}(t)$.

D21. *S is a sub-state of S' iff S and S' are momentary states, and $S' \subseteq S$.*

It would be inadequate to use the clause "S is a subset of S'" instead of the clause "S' is a subset of S" in order to express that momentary state S is a sub-state of momentary state S'. Since the contents of momentary states becomes greater the fewer elements they have (total momentary states have maximal contents and are singleton sets), it is clear that only "S' is a subset of S" can adequately express that S is a sub-state of S': because this is taken to imply that the contents of S is contained in the contents of S'.

D22. f *is a sub-event of* g *iff* f *and* g *are events, and the domain of* f *is a subset of the domain of* g*, and for all elements* t *of the domain of* f*:* $\mathsf{g}(t) \subseteq \mathsf{f}(t)$.

According to this definition f is a sub-event of g just in the case: the temporal extension of f is contained in the temporal extension of g, and for every moment t in the temporal extension of f the state of f at t is a sub-state of the state of g at t.

It is easy to prove the following results: Every event is a sub-event of itself; any sub-event of a sub-event of x is a sub-event of x; if x and y are subevents of each other, then they are identical. This means that the sub-event-relation has the basic properties of a part-relation.

D23. f *is a phase of* g *iff* f *is a momentary event that is a sub-event of* g*, and for all elements* t *of the domain of* f*:* $\mathsf{f}(t) \subseteq \mathsf{g}(t)$.

From the definiens can be deduced: $\mathsf{g}(t) = \mathsf{f}(t)$ for the single time-point t in the domain of f.

D24. f *is a section of* g *iff* f *is a sub-event of* g*, and for all elements* t *of the domain of* f*:* $\mathsf{f}(t)$ *is a subset of* $\mathsf{g}(t)$*, and for all elements* t, t' *of the domain of* f *such that* $t \prec t'$*: there is no element* t'' *of the domain of* g *with* $t \prec t'' \prec t'$ *which is not a member of the domain of* f.

From the definiens can be deduced: $\mathsf{g}(t) = \mathsf{f}(t)$ for all elements t of the domain of f.

If f is a section of g, then the domain of f captures without residuum a certain stretch of the temporal extension of g, whether this stretch is a connected temporal region or not. Thus an uninterrupted process f is a section of g, if its domain is a subset of the domain of event g and if we have for every element t of its domain $\mathsf{g}(t) = \mathsf{f}(t)$; but if g is itself an interrupted process, it also has sections which are *interrupted* processes; moreover every phase of g is also a section of g.

D25. *f and g are compossible events iff f and g are events, and for every t which is an element both of the domain of f and the domain of g: the intersection of $\mathsf{g}(t)$ and $\mathsf{f}(t)$ is non-empty.*

Events whose domains have no time-point in common are trivially compossible.

D26. *M is a set of collectively compossible events iff M is a set of events such that for all non-empty subsets M' of M: for all elements t of the intersection of the domains of all g in M': the intersection of the values $\mathsf{g}(t)$ for all g in M' is non-empty.**

D26 is a (massive) generalization of D25. It can easily be proved: f and g are compossible events iff $\{\mathsf{f}, \mathsf{g}\}$ is a set of collectively compossible events.

Note that we have:

Fact 1. *The definiens of* D26 *is equivalent to:*

*M is a set of events such that for all elements t of the union of the domains of all g in M: the intersection of the values $\mathsf{g}(t)$ for all g in M of whose domain t is an element is non-empty.***

Proof. Given that M is a set of events.

"⇒" Assume that for all non-empty subsets M' of M, for all elements t of the intersection of the domains of all g in M': the intersection of the values $\mathsf{g}(t)$ for all g in M' is non-empty. Let t be an element of the union of the domains of all f in M. Then $\{\mathsf{g} \in \mathsf{M} : t \in \text{the domain of } \mathsf{g}\}$ is a non-empty subset M' of M, and t (trivially) is an element of the intersection of the domains of all g in M'. Hence, by the assumption, the intersection of the values $\mathsf{g}(t)$ for all g in M' is non-empty. Hence the intersection of the values $\mathsf{g}(t)$ for all g in M of whose domain t is an element is non-empty.

* Editors note: In symbolic language – M is a set of collectively compossible events iff M is a set of events and for all M', t: if $\emptyset \neq \mathsf{M}' \subseteq \mathsf{M}$ and $t \in \bigcap\{\mathrm{dom}(\mathsf{g}) : \mathsf{g} \in \mathsf{M}'\}$, then $\bigcap\{\mathsf{g}(t) : \mathsf{g} \in \mathsf{M}'\} \neq \emptyset$.

** Editors note: In symbolic language – The definiens of D26 is equivalent to: M is a set of events and for any t: if $t \in \bigcup\{\mathrm{dom}(\mathsf{g}) : \mathsf{g} \in \mathsf{M}\}$, then $\bigcap\{\mathsf{g}(t) : \mathsf{g} \in \mathsf{M}\ \&\ t \in \mathrm{dom}(\mathsf{g})\} \neq \emptyset$.

"$\Leftarrow$" Let M' be a non-empty subset of M. Assume that for all elements t of the union of the domains of all g in M: the intersection of the values $\mathsf{g}(t)$ for all g in M of whose domain t is an element is non-empty. Let t be an element of the intersection of the domains of all g in M'. Then t is an element of the domain of every g in M'. Hence — since M' is non-empty — there is an element g of M' of whose domain t is an element. Hence there is an element g of M of whose domain t is an element (since M' is a subset of M); hence t is an element of the union of the domains of all g in M. Hence, by the assumption, the intersection of the values $\mathsf{g}(t)$ for all g in M of whose domain t is an element is non-empty. Hence there is a total momentary state* s which is an element of the value $\mathsf{g}(t)$ for all g in M of whose domain t is an element. Hence there is a total momentary state* s which is an element of the value $\mathsf{g}(t)$ for all g in M' of whose domain t is an element (since M' is subset of M). Hence there is a total momentary state* s which is an element of the value $\mathsf{g}(t)$ for all g in M' (since t is an element of the domain of every g in M'). Hence the intersection of the values $\mathsf{g}(t)$ for all g in M' is non-empty. *QED*

Functions: In defining event-functions matters could be simplified by allowing the empty set to be a momentary state ("the impossible momentary state"), and — in consequence — by countenancing events which have the empty set as value for some t in their domain. However, this seems too far a departure from the ordinary meaning of 'event'.

Let M be a non-empty set of collectively compossible events:

D27. *The event which is the sum of the* M*-events* $\stackrel{\text{df}}{=}$ *the function* f*: the domain of* f *is the union of the domains of all* g *in* M*, and for every element* t *in the domain of* f*:* $\mathsf{f}(t) =$ *the intersection of the values* $\mathsf{g}(t)$ *for all* g *in* M *of whose domain* t *is an element.*

If the condition of D27 is fulfilled, then the event which is the sum of the M-events is an event of which all M-events are sub-events, and it is itself sub-event of every event of which all M-events are sub-events, in short: it is the smallest event of which all M-events are sub-events.

We can say simply say "the conjunction of the M-events" instead of "the event which is the sum of the M-events".

Let g be an event such that for no element t in its domain $g(t) = \mathsf{S}$:

D28. *The negation-event of* g $\stackrel{\text{df}}{=}$ *the function* f*: the domain of* f $=$ *the domain of* g*, and for all elements* t *of the domain of* f*:* $\mathsf{f}(t) = \mathsf{S} \setminus g(t)$.

Examples of negation-events of given events are most easily specified for those events which are stable states. The (uninterrupted) *not-standing* of a specific vase on a specific table between two specific moments of time is the negation-event of the (uninterrupted) *standing* of that vase on that table between those moments of time.

Let M be a non-empty set of events:

D29. *The disjunction of the* M*-events* $\stackrel{\text{df}}{=}$ *the function* f*: the domain of* f *is the union of the domains of all* g *in* M*, and for all elements* t *of the domain of* f*:* $\mathsf{f}(t) =$ *the union of the values* $\mathsf{g}(t)$ *for all* g *in* M*.*

Let 29 definitions suffice for giving an impression of the fruitfulness of the chosen conceptual framework (one more definition follows further down). It is natural to head next for the definition of event-causation. But this is not my concern here.

2. The Reality of Events

For an event to be actual or real is *to happen*. Not all events happen (but only events happen); hence some events are not actual, but merely possible.

2.1. Analytical principles for *to happen*

H1. *For all* x*:* x *happens iff* x *is an actual (real) event.*

H2. *For all* x, y*: if* x *happens and* y *is a sub-event of* x*, then* y *happens.*

H3. *If* M *is a non-empty set of collectively compossible events, all of which happen, then the event which is the sum of the* M*-events happens.*

A consequence of H3 is for example: If y is an event every phase of which happens, then y happens (since every event is identical to the conjunction of its phases).

Using H2 it is easy to prove: If M is a non-empty set of collectively compossible events and the conjunction of the M-events happens, then every element of M happens.

The analytical nature of these three principles can hardly be doubted.

2.2. Synthetical principles

However, it is at least possible to hold that the following principles are synthetical:

H4. *The set of all events that happen is a set of collectively compossible events.*

Note that we have:

Fact 2. H4 *is equivalent to the principle:*
There is a world-history of which all events that happen are sub-events.

This principle may be termed "the Unity of Event-Reality". In going from H4 to *the Unity of Event-Reality* the Axiom of Choice is needed; elementary set-theory is not sufficient.

Proof. "⇒" Assume the set M of all events that happen is a set of collectively compossible events. By the Axiom of Choice there exists a function f whose domain is T and which satisfies the following description: in the case t is an element of the union of the domains of all g in M: $\mathsf{f}(t)$ is a singleton subset of the — by assumption non-empty — intersection of the values $\mathsf{g}(t)$ for all g in M to whose domain t belongs. For all other elements t of T $\mathsf{f}(t)$ is an arbitrary total momentary state. Clearly f is a world-history of which all events that happen are sub-events.

"⇐"Assume there is a world-history h of which all events that happen are sub-events. Let M' be a non-empty subset of the set M of all events that happen, and let t belong to the domain of every g in M'. Then every element g of M' is a sub-event of h. Hence $\mathsf{h}(t)$ — a singleton set — is a subset of $\mathsf{g}(t)$ for every g in M'. Hence the intersection of the values $\mathsf{g}(t)$ for all g in M' is non-empty. This means that M is a set of collectively compossible events. *QED*

Because of the proved equivalence of H4 and the Unity of Event-Reality, I use 'H4' also as a label for the latter principle.

H5. *There is a world-history which happens.*

This principle may be termed 'the Completeness of Event-Reality'. Since world-histories are events, H5 implies that there is an actual event: a world-history which happens; and by H2 we obtain that every sub-event of it is an actual event, too.

2.3. Theorems

On the basis of H4 and H5 we have:

Theorem 1. *There is precisely one world-history which happens.*

For the proof of this it is important that if a world-history h is sub-event of a world-history h', then h and h' are identical; given this, the proof is easy.

We can define:

D30. TWH *("The World History")* $\stackrel{\text{df}}{=}$ *the world-history which happens (which is real).*

"The World History" is not to be confused with "the world-history" — a definite description which would refer (as meant) iff there were precisely one world-history.

On the basis of H4 and H5 it can be proved:

Theorem 2. *For all* x*:* x *happens iff* x *is a sub-event of* **TWH**.

Proof. Assume x happens. Hence, by H4, x and **TWH** (since, according to Theorem 1, **TWH** is a world-history which happens) are sub-events of a world-history h. Hence h is identical to **TWH**. Hence x is a sub-event of **TWH**. The reverse is a simple consequence of H2 (since, according to Theorem 1, **TWH** happens). *QED*

The conjunction of H4 and H5 cannot be replaced by 'There is precisely one world-history which happens': H4 cannot be obtained from this. Nor can the conjunction of H4 and H5 be replaced by another corollary of it: 'There is precisely one world-history of which all events that happen are sub-events': H5 cannot be obtained from this. But the impossibility is marginal indeed:

Assume there is a precisely one world-history h of which all events that happen are sub-events; assume h does not happen, hence a phase $\{\langle t, \mathsf{h}(t)\rangle\}$ of h does not happen.

In the case there is no momentary actual event m with the domain $\{t\}$ such that $\mathsf{h}(t)$ is a subset of $\mathsf{m}(t)$ [that is, such that $\mathsf{m}(t)$ is a sub-state of $\mathsf{h}(t)$]: replace $\mathsf{h}(t)$ by *any other total momentary state* in $\mathsf{h} = \{\ldots, \langle t, \mathsf{h}(t)\rangle, \ldots\}$. This gives you a world-history h' different from h. All events that happen are also sub-events of h' — contradicting the assumption.

In the case there is a momentary actual event m with the domain $\{t\}$ such that $\mathsf{h}(t)$ is a subset of $\mathsf{m}(t)$: the set M of all such events is a non-empty set of collectively compossible events. Hence, by H3, the conjunction of the M-events f is an event that happens. Take any total momentary state of which $\mathsf{f}(t)$ is a sub-state and which is different from $\mathsf{h}(t)$, and replace $\mathsf{h}(t)$ in h by it (there must be at least one such total momentary state; else m would not be different from $\{\langle t, \mathsf{h}(t)\rangle\}$, what it must be, since it happens, but $\{\langle t, \mathsf{h}(t)\rangle\}$ does not). This gives you a world-history h'' which is different from h. All events that happen are also sub-events of h'' — contradicting the assumption.

From this it is clear that we need only to add to the basic assumptions that there is more than one total momentary state* (what is utterly plausible) in order to obtain H5 from 'There is a precisely one world-history of which all events that happen are sub-events'.

But the conjunction of H4 and H5 is without further assumptions equivalent to: *There is a world-history which happens of which all events that happen are sub-events*. If this is deemed to describe a contingent fact, that needs explaining, then H4 and H5 are synthetical principles.

They can, however, be made analytical principles by turning Theorem 2 into a definition: x happens $\stackrel{\text{df}}{=}$ x is a sub-event of **TWH**, and — treating **TWH** as a proper name for some world-history, not as a definite description — by adding the analytical principle: **TWH** *is a world-history*. H4 and H5 (and H3 and H2) can then be obtained as analytical theorems.

This procedure is very elegant; but there is a considerable problem attached to it: Which world-history — presumably there is more than one (and there is more than one, if there is more than one total momentary state*) — is to be accorded the honor of being The World History such that 'to happen' means *being a sub-event of it*? This seems to be a question which cannot be settled by stipulation. By what then? — Hard to say.

Thus it appears after all that the best treatment of 'The World History' is to regard it as short for 'the world-history which happens' (what is done in D30). But then it is impossible to regard Theorem 2 as a definition of 'to happen', and the possibility of regarding the unity and completeness of the reality of events as a remarkable metaphysical fact remains open.

UWE MEIXNER
Department of Philosophy
University of Regensburg
93040 Regensburg, GERMANY