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It is NOW

by UWE MEIXNER

Franz von Kutschera has recently proved the completeness of "TxW-logic": a combination of tense and modal logic for worlds or histories with the same time order[1]. In order to obtain this result, he strengthened the object-language logic by (a modified version of) Gabbay's irreflexivity rule, and the completeness proof itself makes crucial use of Gabbay's irreflexivity lemma. This suggests that the main obstacle to proving TxW-logic complete is the well known limited power of expressibility of *P,F*-logic, ordinary tense logic – a condition that can in some measure be remedied by introducing Gabbay's irreflexivity rule.

Since this rule is a somewhat unwieldy item, this paper introduces a tense-logical operator which – if added to *P* ("it was") and *F* ("it will be"), and provided the concept of a tense-logical valuation is modified in a certain way – allows to present a perspicuous formula that is characteristic for the irreflexivity of the temporal ordering in ordinary temporal frames. It is well known that there is no such formula, if merely *F* and *P* are considered, and tense-logical valuations are standardly defined.

It also allows to present a formula which is characteristic for the linearity of the temporal ordering in ordinary temporal frames (the formula is a material implication that, rather pleasingly, is the converse of a material implication characteristic for irreflexivity). Note that $PFA \rightarrow PA \vee A \vee FA$ is characteristic for right-linearity, and $FPA \rightarrow PA \vee A \vee FA$ characteristic for left-linearity; but linearity is not simply the conjunction of right- and left-linearity, and therefore $(PFA \rightarrow PA \vee A \vee FA) \wedge (FPA \rightarrow PA \vee A \vee FA)$ is not characteristic for linearity. Consider the following temporal frame: two isolated time-points not connected by a temporal relation; in other words, the temporal frame

[1] Cf. Kutschera [1996]

⟨T,R⟩ consists of a set of two time-points $T=\{t_1,t_2\}$, and a relation R which is taken to be the empty set (the emptiness of R is not essential for proving what is to be shown, but it makes the proof particularly simple). It is easily seen that R is not linear on T: neither t_1Rt_2, nor t_2Rt_1, nor $t_1=t_2$; but, trivially, it is both right- and left-linear on T (both "For all t,t′,t″ in T: if t″Rt and t″Rt′, then tRt′ or t′Rt or t=t′" and "For all t,t′,t″ in T: if tRt″ and t′Rt″, then tRt′ or t′Rt or t=t′" are trivially true), and according to standard semantics both $FPA \rightarrow PA \vee A \vee FA$ and $PFA \rightarrow PA \vee A \vee FA$ are true in ⟨T,R⟩ for any A.

How then is linearity expressed? The truth of the matter is that there simply is no *P,F*-formula that expresses (full) linearity (the idea for a simple proof of this can be can be gotten from Rescher/Urquhart [1971, 120]). This seems to be much less generally known than the inexpressibility of irreflexivity in *P,F*-logic. In any case, there is no general feeling of a deficiency in *P,F*-logic due to the inexpressibility of linearity, although linearity can hardly be said to be a less important feature of time than irreflexivity.

Let L be a propositional language with the tense operators *F*, *P* and N^* (and the basic truth-functional operators ¬ and →). N^* is to be read as "It is now, at this special moment t*, the case that"; $N^*(A \rightarrow A)$, in particular, is to be read as "It is now this special moment t*", or in other words "It is NOW" (and correspondingly N^*p may also be read as "It is NOW the case that"). Thus N^* says neither merely "it is now the case that", nor merely "it is at the special moment t* the case that". N^* is the operator of implicitly dated presence. (Note that someone who says "It is now, at this special moment t*, the case that p" need not be able to specify t*, to identify it explicitly as a specific date.) This operator is in fact used in ordinary language: we sometimes say sentences of the following form: "It is NOW, and A", which is logically equivalent to N^*A. More often we don't actually say such sentences, but rather are convinced of what they say: that it is now, at this special moment t*, that such and such beautiful (or terrible) things happen – the conviction of which is part and parcel of a mode of feeling that might be termed "the feeling of the specialness of the presence" (a feeling that has a basis in the ontology of time, although nobody will deny that there are countless moments of time, and that they all are, have been or will be present).

Let ⟨T,R⟩ be a temporal frame: that is, T is a non-empty set of time-points, R is a relation on T. The concept of a *centered valuation* of L on ⟨T,R⟩ is defined as follows:

DEFINITION 1:
V is a centered valuation of L on ⟨T,R⟩ := V is a function whose domain is the union of TxL and {T}, and for which the following conditions hold:

(i) V(T)∈T;
(ii) for every formula A of L and every t in T: V(t,A)∈{0,1};
(iii) for all formulae A and B of L and every t in T:
V(t,¬A)=1 iff V(t,A)=0;
V(t,A→B)=1 iff V(t,A)=0 or V(t,B)=1;
V(t,*F*A)=1 iff there is a t′ such that: tRt′ and V(t′,A)=1;
V(t,*P*A)=1 iff there is a t′ such that: t′Rt and V(t′,A)=1;
V(t,*N**A)=1 iff t=V(T) and V(t,A)=1.

Centered valuations of L on ⟨T,R⟩ differ from normal valuations in having one extra element in their domain, namely T itself, from which they select one element: their center; this special time-point is then used for giving the truth condition for *N**A.

With respect to centered valuations the formula *N**p → ¬*PN**p can be seen to be characteristic for temporal frames ⟨T,R⟩ where R is irreflexive:

(1) Let ⟨T,R⟩ be a temporal frame, R being irreflexive; assume there is a centered valuation V of L on ⟨T,R⟩ such that there is a t∈T and V(t,*N**p → ¬*PN**p)=0; hence V(t,*N**p)=1 and V(t,*PN**p)=1; hence t=V(T) and V(t,p)=1, and there is a t′ such that t′Rt and t′=V(T) and V(t′,p)=1; from this we obtain tRt – contradicting the irreflexivity of R.

(2) Let ⟨T,R⟩ be a temporal frame, R being not irreflexive; hence there is a t (in T) such that tRt; obviously there is a centered valuation V of L on ⟨T,R⟩ such that V(T)=t and V(t,p)=1; hence V(t,*N**p)=1 and V(t,*PN**p)=1 (since tRt); hence V(t,*N**p → ¬*PN**p)=0.

Thus we have:

THEOREM 1:
For every temporal frame ⟨T,R⟩: R is irreflexive iff every centered valuation of L on ⟨T,R⟩ verifies *N**p → ¬*PN**p at every t in T.

*N**p → ¬*FN**p and *N**(p→p) → (¬*FN**(p→p)∧¬*PN**(p→p)) would do as well for characterizing irreflexivity. The converse of the latter formula, however, is characteristic (with respect to centered valuations) for temporal frames ⟨T,R⟩ where R is linear on T:

(1) Let ⟨T,R⟩ be a temporal frame, R being linear on T; assume there is a centered valuation V of L on ⟨T,R⟩ such that there is a $t \in T$ and $V(t,\neg FN^*(p\rightarrow p))=1$ and $V(t,\neg PN^*(p\rightarrow p))=1$ and $V(t,N^*(p\rightarrow p))=0$; by the linearity of R on T: R(V(T),t) or R(t,V(T)) or t=V(T); since $V(t,p\rightarrow p)=1$ and $V(t,N^*(p\rightarrow p))=0$: $V(T)\neq t$; since $V(V(T), N^*(p\rightarrow p))=1$ and $V(t,FN^*(p\rightarrow p))=0$ and $V(t,PN^*(p\rightarrow p))=0$, not R(V(T),t) and not R(t,V(T)); hence the linearity of R on T is being contradicted.

(2) Let ⟨T,R⟩ be a temporal frame, R not being linear on T; hence there are t,t′ in T such that neither tRt′ nor t′Rt nor t=t′; obviously there is a centered valuation V of L on ⟨T,R⟩ with V(T)=t; hence $V(T)\neq t'$ [else t=t′], hence $V(t',N^*(p\rightarrow p))=0$; hence for every t″: if t″Rt′, then $t''\neq V(T)$ [else tRt′], hence $V(t',\neg PN^*(p\rightarrow p))=1$; hence for every t″: if t′Rt″, then $t''\neq V(T)$ [else t′Rt], hence $V(t',\neg FN^*(p\rightarrow p))=1$.

Thus we have:

THEOREM 2:
For every temporal frame ⟨T,R⟩: R is linear on T iff every centered valuation of L on ⟨T,R⟩ verifies $(\neg PN^*(p\rightarrow p)\wedge\neg FN^*(p\rightarrow p)) \rightarrow N^*(p\rightarrow p)$ at every t in T.

The next question is of course, how the logic of *P*,*F* and *N** is to be adequately axiomatized (with respect to centered valuations). Consider *minimally adequate temporal frames*, that is, temporal frames ⟨T,R⟩ where R is transitive, irreflexive and linear on T.

DEFINITION 2:
B is a valid* formula of L := B is a formula of L such that for all T,R and V: if ⟨T,R⟩ is a minimally adequate temporal frame and V is a centered valuation of L on ⟨T,R⟩, then for every t in T: V(t,B)=1.

The axiomatic system S* (specified below) can easily be seen to be sound with respect to valid* formulae of L (and I conjecture that it is also complete). S* is obtained by adding to the appropriate *P*,*F*-basis for linear time (including truth-functional propositional logic based on ¬ and →) the following axiom-schemata:

N*1	$N^*A \rightarrow A$
N*2	$N^*B \rightarrow (A\rightarrow N^*A)$
N*3	$N^*A \rightarrow \neg PN^*A,\ N^*A \rightarrow \neg FN^*A$
N*4	$\neg PN^*(A\rightarrow A) \rightarrow (\neg FN^*(A\rightarrow A)\rightarrow N^*(A\rightarrow A))$

The following schemata of formulae, for example, are then provable in S*:

$$(B \to A) \to (N^*B \to N^*A)$$
$$N^*A \leftrightarrow N^*N^*A$$
$$N^*A \leftrightarrow (A \land N^*(p \to p))$$ [2]
$$\neg N^*FN^*A$$
$$\neg N^*PN^*A$$ [3]
$$(\neg PN^*A \land \neg FN^*A \land \neg P\neg A \land \neg F\neg A \land A) \to N^*A$$
$$(\neg FN^*A \land \neg FN^*\neg A \land \neg PN^*A \land \neg PN^*\neg A \land \neg N^*\neg A) \leftrightarrow N^*A.$$

No formula N^*A is provable in S*. Else $N^*(p \to p)$ would also be provable in S*; but $N^*(p \to p)$ is not a valid* formula of L, and S* is sound with respect to validity* (only valid* formulae of L are provable in S*). It is interesting to see what happens if $N^*(p \to p)$ is added to S*. Then $N^*A \leftrightarrow A$ becomes provable (by N*1 and N*2); but given this, $p \to \neg Fp$ and $p \to \neg Pp$ become provable as well (by N*3). Both these formulae are characteristic of temporal frames ⟨T,R⟩ in which R is empty. Consider $p \to \neg Pp$:

(1) Let ⟨T,R⟩ be a temporal frame, R being empty; assume there is a centered valuation V of L on ⟨T,R⟩ such that there is a t in T with $V(t,p)=1$ and $V(t,\neg Pp)=0$; hence $V(t,Pp)=1$, hence there is a t′ with t′Rt – but this contradicts the emptiness of R.

(2) Let ⟨T,R⟩ be a temporal frame, R not being empty; hence there is a t and a t′ such that t′Rt; obviously there is a centered valuation V of L on ⟨T,R⟩ with $V(t,p)=1$ and $V(t',p)=1$; hence $V(t,Pp)=1$, hence $V(t,p \to \neg Pp)=0$.

$N^*(p \to p)$ itself is characteristic of temporal frames ⟨T,R⟩ in which T contains precisely one element:

(1) Let ⟨T,R⟩ be a temporal frame, T containing precisely one element; assume there is a centered valuation V of L on ⟨T,R⟩ such that there is a t in T with $V(t,N^*(p \to p))=0$; hence $V(T) \neq t$; but this, since $V(T) \in T$, contradicts T's containing precisely one element.

(2) Let ⟨T,R⟩ be a temporal frame, T not containing precisely one element; hence there are t, t′ in T and $t \neq t'$ (T is non-empty); obviously there is a centered valuation V of L on ⟨T,R⟩ with $V(T)=t$; hence $V(t',N^*(p \to p))=0$.

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[2] This schema makes it very easy to prove $N^*(A \land B) \leftrightarrow (N^*A \land N^*B)$, $N^*(A \lor B) \leftrightarrow (N^*A \lor N^*B)$.

[3] The latter two formulae express irreflexivity!

All this fits very well. For if $\langle T,R\rangle$ is a temporal frame with T containing precisely one element, then R is trivially linear on T; and then R is irreflexive iff R is empty. Correspondingly: by adding $N^*(p\rightarrow p)$ to S* we obtain a system in which N*4 is a trivial theorem, and in which N*3 – the axiom-schemata characteristic for irreflexivity – can be equivalently replaced by $A \rightarrow \neg PA$ and $A \rightarrow \neg FA$, which are the axiom-schemata characteristic for emptiness. Moreover, the emptiness of R trivially implies the transitivity of R, which is expressed by formulae of the form $FFA \rightarrow FA$, $PPA \rightarrow PA$; and correspondingly we have: since $\neg FA$ is provable in $S^* + N^*(p\rightarrow p)$ [assume FA, hence $F(p\rightarrow p)$, hence by the contraposition of $B \rightarrow \neg FB$: $\neg(p\rightarrow p)$ – which is a contradiction], $\neg FFA$ is also provable in $S^* + N^*(p\rightarrow p)$ [$\neg FFA$ is an instance of $\neg FA$]; hence, trivially, $FFA \rightarrow FA$ is provable in $S^* + N^*(p\rightarrow p)$.

Clearly, the addition of $N^*(p\rightarrow p)$ to S* trivializes the system. Yet, in a manner, $N^*(p\rightarrow p)$ could very well be regarded as a valid formula of L, and it is one, if we introduce an alternative definition of validity:

DEFINITION 3:
B is a valid+ formula of L := B is a formula of L such that for all T, R and V: if $\langle T,R\rangle$ is a minimally adequate temporal frame and V a centered valuation of L on $\langle T,R\rangle$, then $V(V(T),B)=1$.

Obviously any valid* formula of L is a valid+ formula of L, but not *vice versa*: $N^*(p\rightarrow p)$ is a case in point. But although S*, we may assume, is sound and complete with respect to the valid* formulae of L, there can be no adequate axiomatization of the valid+ formulae of L that is simply an extension of S*; for S* includes the rule $A \vdash \neg F\neg A$, and while this is a rule that preserves validity*, it is not a rule that preserves validity+: while $N^*(p\rightarrow p)$ is valid+, $\neg F\neg N^*(p\rightarrow p)$ is not: there are of course T, R and V such that $\langle T,R\rangle$ is a minimally adequate temporal frame and V a centered valuation of L on $\langle T,R\rangle$ and $V(V(T),F\neg N^*(p\rightarrow p))=1$.

By considering validity+ besides validity*, I have started a parade of alternatives to the semantical approach first presented by me. Notice next, that we can define in L: NOW := $N^*(p\rightarrow p)$ ("NOW" is not a term, but a formula which is to be read as "It is NOW"; compare the fifth paragraph of this paper). But taking "NOW" as basic in a language L′ which is otherwise like L, we might as well define in L′: N^*A := NOW$\wedge$A, and change the definition of centered valuations accordingly: "$V(t,\text{NOW})=1$ iff $t=V(T)$" instead of "$V(t,N^*A)=1$ iff t=V(T) and V(t,A)=1". Maybe the second way is even more natural than the first, but it certainly is more elegant: we can drop without replacement axioms N*1 and N*2, N*3 becomes NOW → ¬*F*NOW, NOW → ¬*P*NOW, and N*4 ¬*P*NOW → (¬*F*NOW → NOW). As a theorem we have NOW ↔ (¬*P*NOW ∧ ¬*F*NOW): "It is NOW iff it never was NOW, and never will be".[4]

I have used ordinary temporal frames, but centered valuations. Why not use ordinary valuations, but centered temporal frames?

DEFINITION 4:
A centered temporal frame is a triple ⟨T,R,x⟩ consisting of a non-empty set T (of time-points), a relation R on T, and a special element x of T.

Let ⟨T,R,x⟩ be a centered temporal frame.

DEFINITION 5:
V is a valuation of L on ⟨T,R,x⟩ := V is a function whose domain is TxL and for which the following conditions hold: [the rest is like definition 1, except that clause (i) is dropped, and that the condition for *N** now reads: "V(t,*N**A)=1 iff t=x, and V(t,A)=1"; if we refer to L′ instead of L, we have "V(t,NOW)=1 iff t=x"].

DEFINITION 6:
B is a valid formula of L := B is a formula of L, and for all T,R,x,V: if ⟨T,R,x⟩ is a minimally adequate centered temporal frame and V a valuation of L on ⟨T,R,x⟩, then for every t in T: V(t,B)=1.

Indeed, why not base the whole affair on these three definitions instead? For we have:

THEOREM 3:
B is a valid formula of L iff B is a valid* formula of L.

The proof of this is rather obvious. However, on the basis of definitions 4 and 5 we cannot retrieve the expressibility results obtained

[4] Clearly, "It is NOW the case that" says something different from Kamp's "It is Now the case that": for Kamp's "Now" we have as a logical truth "If it is Now the case that A, then it always will be Now the case that A" (compare Burgess [1984, 124]); this is not a logical truth for "NOW". On the other hand, "If it is NOW the case that A, then it never will be NOW the case that A" is a logical truth for "NOW", but not for "Now". Both uses of "now" – in the sense of "Now" and in the sense of "NOW" – occur in ordinary language, but "NOW" seems not to have been noticed so far by logical semantics.)

above. It is true that $N^*p \rightarrow \neg FN^*p$ is valid in every centered temporal frame $\langle T,R,x \rangle$ where R is irreflexive; but it is not true that R is irreflexive for every centered temporal frame $\langle T,R,x \rangle$ in which $N^*p \rightarrow \neg FN^*p$ is valid. Consider the frame $\langle T,R,x \rangle := \langle \{t,x\},\{\langle t,t \rangle\},x \rangle$ (x is a time-point that is not identical to t). It is easily verified that $\langle T,R,x \rangle$ is a centered temporal frame for which R is not irreflexive; but nevertheless for every valuation V of L on $\langle T,R,x \rangle$ and every t' in T: $V(t',N^*p \rightarrow \neg FN^*p)=1$: t' must be either t or x; if it is x, then $V(t',\neg FA)=1$, since there is no t'' such that xRt'', hence $V(t',N^*p \rightarrow \neg FN^*p)=1$; if it is t, then $V(t',N^*A)=0$, since $t \neq x$, hence $V(t',N^*p \rightarrow \neg FN^*p)=1$.

Also: it is true that $(\neg P\text{NOW} \wedge \neg F\text{NOW}) \rightarrow \text{NOW}$ is valid in every centered temporal frame $\langle T,R,x \rangle$ where R is linear on T; but it is not true that R is linear on T for every centered temporal frame $\langle T,R,x \rangle$ in which $(\neg P\text{NOW} \wedge \neg F\text{NOW}) \rightarrow \text{NOW}$ is valid. Consider the frame $\langle T,R,x \rangle := \langle \{t,t',x\},\{\langle t,x \rangle,\langle t',x \rangle\},x \rangle$ (t and t' are time-points differing from each other and from x). It is easily seen that $\langle T,R,x \rangle$ is a centered temporal frame for which R is not linear on T, but that nevertheless $(\neg P\text{NOW} \wedge \neg F\text{NOW}) \rightarrow \text{NOW}$ is valid in it.

This shows that centered valuations of L *plus* ordinary temporal frames are equivalent to ordinary valuations of L *plus* centered temporal frames with respect to *logical truth*, but not with respect to *the expression of frame-properties*. Other things being equal, it is clear that centered valuations of L *plus* ordinary temporal frames should be preferred on account of their greater ability in expressing these properties.

But centered valuations of L *plus* ordinary temporal frames are *entirely equivalent* to ordinary temporal frames *plus* double-indexed valuations. Let $\langle T,R \rangle$ be a temporal frame:

DEFINITION 7:
V is a double-indexed valuation of L on $\langle T,R \rangle$:= V is a function whose domain is TxTxL and for which the following conditions hold:

(i) for all $t,t' \in T$ and every formula A of L: $V(t,t',A) \in \{1,0\}$;
(ii) for all $t,t' \in T$ and all formulae A and B of L:
$V(t,t',\neg A)=1$ iff $V(t,t',A)=0$,
$V(t,t',A \rightarrow B)=1$ iff $V(t,t',A)=0$ or $V(t,t',B)=1$,
$V(t,t',FA)=1$ iff for some t'': t'Rt'' and $V(t,t'',A)=1$;

$V(t,t',PA)=1$ iff for some t″: t″Rt′ and $V(t,t'',A)=1$,
$V(t,t',N^*A)=1$ iff $t'=t$ and $V(t,t',A)=1$.[5]

It is easily checked that the expressibility results available for centered valuations of L are also available for double-indexed valuations. (For example: Let ⟨T,R⟩ be a temporal frame, such that R is not linear on T; hence there are time-points t and t′ in T such that neither tRt′ nor t′Rt nor $t=t'$. Then $V(t,t',NOW)=0$, since $t'\neq t$; $V(t,t',FNOW)=0$, since there is no t″ with t′Rt″ and $V(t,t'',NOW)=1$ [else $t''=t$ and t′Rt]; $V(t,t',PNOW)=0$, since there is no t″ with t″Rt′ and $V(t,t'',NOW)=1$ [else $t''=t$ and tRt′].)

Moreover we have:

THEOREM 4:
For any formula B of L: If ⟨T,R⟩ is a minimally adequate temporal frame and V a double-indexed valuation of L on ⟨T,R⟩ and t,t′ elements of T with $V(t,t',B)=0$, then there is a temporal frame ⟨T*,R*⟩ which is isomorphic to ⟨T,R⟩ [hence minimally adequate] and a centered valuation V* of L on ⟨T*,R*⟩ such that for ⟨t,t′⟩ in T*: $V^*(\langle t,t'\rangle,B)=0$.

Proof:
Let B be a formula of L, ⟨T,R⟩ a minimally adequate temporal frame, t and t′ elements of T, V a double-indexed valuation of L on ⟨T,R⟩ with $V(t,t',B)=0$. $T^* := \{t\}xT$; $xR^*y :=$ there are t_1,t_2 in T such that $x=\langle t,t_1\rangle$ and $y=\langle t,t_2\rangle$ and t_1Rt_2; there is no difficulty in showing that ⟨T*,R*⟩ is a temporal frame that is isomorphic to ⟨T,R⟩. We stipulate V* to be a function whose domain is T*xL, and such that for any ⟨t,t″⟩ in T* and any A of L: $V^*(\langle t,t''\rangle,A) = V(t,t'',A)$, and such that $V^*(T^*)=\langle t,t\rangle$; there is no difficulty in showing that V* is a centered valuation of L on ⟨T*,R*⟩; and we have ⟨t,t′⟩ in T* and $V^*(\langle t,t'\rangle,B)=0$.

Finally:

THEOREM 5:
For any formula B of L: if ⟨T,R⟩ is a minimally adequate temporal frame, $t\in T$ and V a centered valuation of L on ⟨T,R⟩ with $V(t,B)=0$, then there is a double-indexed valuation V* of L on ⟨T,R⟩ with $V^*(V(T),t,B)=0$.

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[5] Compare the truth-condition for N^* with that for Kamp's Now, for which double-indexed valuations were first introduced: "$V(t,t',JA)=1$ iff $V(t,t,A)=1$".

Proof:
Let B be a formula of L, $\langle T,R\rangle$ a minimally adequate temporal frame, t an element of T, V a centered valuation of L on $\langle T,R\rangle$ with V(t,B)=0. We associate any t′ in T with a centered valuation V[t′] of L on $\langle T,R\rangle$ such that V[t′](T)=t′, and such that V[V(T)]=V. We stipulate V* to be a function whose domain is TxTxL, and such that for any t′,t″ in T and any formula A of L: V*(t′,t″,A) = V[t′](t″,A). V* is a double-indexed valuation of L on $\langle T,R\rangle$:

Assume t′,t″ are in T, A a formula of L; consider the interesting cases:

V*(t′,t″,*F*A)=1 iff V[t′](t″,*F*A)=1 iff there is a t‴ such that t″Rt‴ and V[t′](t‴,A)=1 iff there is a t‴ such that t″Rt‴ and V*(t′, t‴,A)=1;

V*(t′,t″,*N**A)=1 iff V[t′](t″,*N**A)=1 iff t″=V[t′](T) and V[t′](t″,A)=1 iff t″=t′ and V(t′,t″,A)=1.

And we have V*(V(T),t,B)=V[V(T)](t,B)=V(t,B)=0.

Theorems 4 and 5 also hold true if we add, as is surely desirable, Kamp's Now (*J*) to L (for centered valuations V of L on a temporal frame $\langle T,R\rangle$ we then have for any $t \in T$: V(t,*J*A)=1 iff V(V(T),A)=1, and $N^*A \rightarrow JA$ is valid* for any formula A of L). This shows that centered valuations do as well as double-indexed valuations in the semantical treatment of Kamp's Now.

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