Causation in a New Old Key

Abstract. I argue (1) that it is not philosophically significant whether causation is linguistically represented by a predicate or by a sentence connective; (2) that there is no philosophically significant distinction between event- and states-of-affairs-causation; (3) that there is indeed a philosophically significant distinction between agent- and event-causation, and that event-causation must be regarded as an analog of agent-causation. Developing this point, I argue that event-causation’s being in the image of agent-causation requires, mainly, (a) that the cause is temporally prior to the effect, (b) that the cause necessitates (is sufficient with necessity) for the effect. Causal necessity is explained as a derivative of nomological necessity, and finally, via a definition of the causal sentence connective, the logic of event-causation is shown to be a part of temporal modal logic.

Keywords: event-causation, agent-causation, events, states of affairs, causal priority, causal necessity, causal predicate, causal sentence connective, logic of causation, temporal modal logic.

I. Introduction

In this paper I will argue for the following conclusions:

(1) There has been a controversy about the linguistic representation of causation: whether it is represented by a predicate or by a sentence connective. The controversy is misguided, since insofar as causation is linguistically represented in common speech by a sentence connective, it is also — at the same time — represented by a predicate.

(2) There has been a controversy about the ontological nature of causal relata: whether they are events, or rather states of affairs. The controversy is misguided, since events either are states of affairs, or, if not, are at least one-to-one representable by states of affairs. Hence there is no significant distinction between event-causation and state-of-affairs-causation.

(3) Contrary to the views of many, there is indeed a significant distinction between event-causation (or state-of-affairs-causation) and agent-causation. The latter is the form of causation that truly deserves to
be called "efficient causation", whereas event-causation and state-of-affairs-causation can only be regarded as being analogically efficient. Since causation must be at least analogically efficient in order to be causation at all, any analysis of event-causation (or state-of-affairs-causation) must seek, in order to be adequate, to come as close as possible to agent-causation. Only to the extent that it succeeds in doing so, it can be regarded as an adequate analysis of causation in the realm of events (respectively, the realm of states of affairs).

(4) An analysis of event-causation that is fairly in the image of agent-causation — of truly efficient causation — must establish three main features of event-causation: (a) the cause is sufficient for the effect; (b) the effect is connected to the cause by a form of necessity; (c) the cause precedes the effect in time. No analysis of event-causation is adequate that does not produce all three of these features (since, if it does not produce all three of them, it is not an analysis of analogically efficient causation, and hence not an analysis of causation at all).

(5) Feature (a) combined with feature (b) means that the cause necessitates the effect. The necessity involved in causal necessitation must be regarded as being based on laws of nature (and thus is objective only to the extent that the concept of law of nature is objective). Nevertheless, there need not be any explicit reference to laws of nature in the analysis of event-causation. The theory of event-causation here advocated is an implicit nomological regularity theory, which has, by its avoidance of explicit reference to laws of nature, a certain epistemological advantage over an orthodox, explicit, nomological regularity theory of causation.

Besides arguing for the five conclusions just listed, I will define a causal predicate, "e Ecauses e'", for events which has, among other features, the three features mentioned under (4) and which, in the spirit of (5), only indirectly refers to laws of nature. In view of (2), "e Ecauses e'" automatically induces a causal predicate for states of affairs, "p Scauses p'", which is its mirror image. (In view of (2), I could of course also start with the causal predicate for states of affairs and have it induce the causal predicate for events.) To the extent that states of affairs which are events or represent events can be expressed by sentences, "p Scauses p'", in its turn, is mirrored by a causal sentence connective, "C(A, B)'", that can be defined by using only well-known modal and temporal sentence connectives. Thus the logic of causation that is not agent-causation becomes a part of temporal modal logic.
II. Causal Predicate or Causal Sentence Connective?

Ever since Donald Davidson’s “Causal Relations” it has been an issue in the philosophical discussion of causation whether causation is linguistically represented by a sentence connective, or rather by a predicate (i.e., by a name connective). The piece of linguistic phenomenology that may lead one to regard causation as linguistically represented by a sentence connective is quite precisely this: We often utter sentences that have the following general form: that \( A \) caused it to be the case that \( B \). It is natural to think of sentences which have this form as formed by the sentence connective “that ___ caused it to be the case that ___” by connecting the sentences “\( A \)” and “\( B \)”.

But it is equally natural to think of those sentences as formed by the predicate “___ caused it to be the case ___” by connecting the singular terms “that \( A \)” and “that \( B \).” Thus, sentences of the form that \( A \) caused it to be the case that \( B \) are syntactically ambiguous.\(^1\) And in general we have: There is no evidence for causation being linguistically represented by a sentence connective which is not syntactically ambiguous in the manner just described and hence not also evidence for causation being linguistically represented by a predicate.\(^2\) In other words, every causal sentence that can be regarded as formed by a causal sentence connective can also be regarded as formed by a causal predicate. In view of this, no great importance can attach to the mere question whether causation is linguistically represented by a sentence connective or by a predicate. Since states of affairs are standard designata of “that”-phrases, causation is linguistically represented by a sentence connective if, and only if, it is linguistically represented by a predicate for states of affairs. Hence the real issue that the question, sentence connective or predicate?, is merely hinting at is whether the relata of the causal relation are states of affairs, or rather something else.

III. Causal Predicate for States of Affairs or Causal Predicate for Events?

Whether states of affairs are the causal relata, or, on the contrary, events, (these, normally, are the only alternatives seriously considered) is also a

---

1. This ambiguity, of course, is a special case of the general syntactical ambiguity of all sentences involving “that”-phrases.

2. Contrary to this assertion, one might think that sentences of the form “\( B \) because \( A \)” provide precisely such evidence. But sentences of the form “\( D \) because \( A \)” are not ipso facto causal sentences. In the cases where such sentences are causal sentences, they are synonymous to sentences of the form “that \( A \) caused it to be the case that \( B \)”, and hence (indirectly) syntactically ambiguous as described.
much debated issue of the theory of causation. But this issue can only exist as long as an event is taken to be something significantly different from a state of affairs: something that neither is nor can be represented by a state of affairs. We are not forced to regard events in this light. On the contrary, events can plausibly be taken to be states of affairs, or to be at least one-to-one representable by states of affairs. (Not the least virtue of this approach is that we can finally get rid of a totally fruitless dispute.) This being the case, we necessarily arrive at the conclusion that the causal predicate for events (howsoever defined) is, by the very nature of events, a causal predicate for states of affairs, or at least corresponds isomorphically to a causal predicate for states of affairs; in any case, there is not conflict, but intrinsic correspondence between causation for events and causation for states of affairs.

This pleasing result obviously requires a certain amount of preparation. The thing to start out with is the following intuitively satisfactory conception of eventhood: An event is what happens in time, and therefore: an event is a temporally located sequence of states. I allow two ways in which such a sequence can be degenerate: (1) it may involve only one, constant, state; (2) it may involve only one moment of time. (Who thinks that momentary events and, generally, events that are not changes should be excluded, may exclude them\(^3\) by making the appropriate adjustments in what follows; nothing in this paper hinges on this issue.) I take it that the above characterization of events is at least as good as other characterizations of them, and certainly as good as Davidson's, according to whom they are "unrepeatable, dated individuals" ("Mental Events", Essays on Actions and Events, p. 209).

This said, consider the following set-theoretically formulated theory of events.

*Set-theoretical basis:*

Z is the set of (possible) total basis-states; T is the set of moments of time.

We can fruitfully identify T with any large set of real numbers (whichever such set of real numbers seems most appropriate). The "earlier than"-relation can then be simply identified with the "is a smaller real number than"-relation (that is, with <); it is automatically guaranteed to be a linear total ordering of T.\(^4\)

\(^3\) I do not think that this is recommendable, but I will not dwell on this here.

\(^4\) This means we have: \(\forall t t' t''(\text{if } t < t' \text{ and } t' < t'', \text{then } t < t'')\) (transitivity), \(\forall t t' (\text{if } t < t', \text{then not } t' < t)\) (asymmetry), \(\forall t t' (t \in T \text{ and } t' \in T: t < t' \text{ or } t' < t \text{ or } t' = t)\) (linearity over T).
A total basis-state is a broadly possible momentary state of the entire world, but taken without an intrinsic temporal location. Metaphorically speaking, a total basis-state is a snapshot of what, in a broad sense, might happen or might have happened in the world at some moment of time, but leaving that moment entirely unspecified.

We assume that the set $Z$ is at least as big as the set $T$.$^5$

*Set-theoretical constructions:* Any non-empty set of total basis-states is a (possible) state; any non-empty set of moments of time is a time. The singleton sets of total basis-states are the total states.

It is in accordance with the above ontological characterization of events as temporally located sequences of states if (possible) events are set-theoretically construed as functions whose domains are times and which assign states to all moments of time in their domains. But it is immediately evident that the times of such events are essential to them. Since time-essentiality is not in accordance with a general conception of events, the functions which assign states to moments of time cannot simply be the events, but can only constitute a special, important class of events: the primary events.

An important subset of the primary events is formed by the maximal primary events: by those primary events that exclusively assign total states to the moments of time in their domains. And again, an important subset of the maximal primary events is formed by those maximal primary events which are temporally complete (that is, which have $T$ as their domain): the (possible) worlds. (The set of them is $W$.) Note that all worlds have the same time and the same order of time. This is a feature of them that is well-motivated if possibility is to be derived from, and hence secondary to, actuality;$^5$ then the time-order of each possible world should be the time-order of the actual world, and therefore be the same in all possible worlds.

The following relational concept for primary events will play an important role: $e$ is a primary subevent of $e'$ iff (1) $e$ and $e'$ are primary events such that the domain of $e$ is set-theoretically included in the domain of $e'$, and (2) for every moment of time $t$ in the domain of $e$: the state that $e'$

---

$^5$ This implies that there are possible worlds without repetition of total states; see below for the definition of possible worlds.

$^6$ This *actualistic* view of possibility (which, however, can be taken to various lengths; I, for my part, do not endorse that the same individuals exist in every possible world that exist in the actual world) already motivated my characterization of the set $Z$ of total basic states: I did not draw into consideration entirely *alien* total basis-states that could not be or have been the totality of what happens in *the actual world* at some moment of time.
assigns to \( t \) is set-theoretically included in the state that \( e \) assigns to \( t \).\(^7\) The concept just defined can be used immediately to define what it means for a primary event to happen (or occur or take place) in a world: \( e \) happens in \( w \) iff \( w \) is a world and \( e \) is a primary subevent of \( w \).

Primary events are **singular events**: events that happen only once in every world in which they happen. The same is true of **secondary events**, which are defined on the basis of primary events as follows: A **secondary event** is a non-empty set of maximal primary events such that no two elements in the set are primary subevents of the same world. The primary events that are the elements of a secondary event are to be considered as the realizations of that event in various possible worlds. The definition of secondary events guarantees that a secondary event has at most one realization in each possible world, and therefore: a secondary event happens only once in each world in which it happens.

Just like primary events, secondary events are temporally located sequences of states. But the time of a secondary event is **normally** not essential to it. Indeed, in contrast to primary events, it is not automatically meaningful to speak of the time of a secondary event. This is meaningful only if all the elements of the secondary event have the same time, or if the secondary event is (simpliciter) real, that is: has a realization in the actual world (or in other words: has an element which is a primary subevent of the actual world, has an element that happens in the actual world). For then the time of the secondary event can be identified with the time of all of its elements, or with the time of the one element of it which is a (simpliciter) real primary event.

Obviously, if a secondary event has elements that differ in their times, and it is not a real event (first alternative), then there is no time that is its time, and hence its time is, in a manner; not essential to it; if, however, the secondary event is real (second alternative), then indeed, there is a time which is its time, but again that time is not essential to it, since in another possible world the secondary event has a realization which differs in time from its realization in the actual world.

In conclusion of this selection of set-theoretical constructions, we need the set-theoretical construal of states of affairs, since states of affairs are the entities that we intend to consider in their relation to events. A **state of affairs** is any subset of \( \mathbb{W} \times T \), that is, any subset of the Cartesian product of the set of possible worlds and the set of moments of time. This gives states

---

\(^7\) The set-theoretical inclusion of \( e'(t) \) in \( e(t) \) means that, with respect to richness of content, \( e(t) \) is a substate of \( e'(t) \).
of affairs an intrinsic temporal dimension and leaves them coarse-grained, as is appropriate if they are to represent events. (*Atemporal* (coarse-grained) states of affairs — states of affairs as usually understood: simple subsets of the set of possible worlds — can be represented by *temporal* (coarse-grained) states of affairs — states of affairs as here understood — in the obvious manner: if $p$ is an atemporal state of affairs, then $p \times T$ is the corresponding temporal one.)

**Representation results:**

(1) Secondary events are generalizations of primary events, but of course not in the simple sense that every primary event is also a secondary event (and not vice versa). Rather, all primary events are provably represented one-to-one by certain secondary events (but not vice versa). This is effected by the mapping $\Phi_1$. If $e$ is any primary event, then $\Phi_1(e)$ is \{ $f : \text{Fct}(f)$ and $d(f) = d(e)$ and for all $t$ in $d(f) : \exists z \in e(t)$ and $f(t) = \{ z \}$ \}. (Here “Fct($f$)” means as much as “$f$ is a (set-theoretical) function” “$d(f)$” means as much as “the domain of $f$”.) $\Phi_1$ assigns to $e$ the set of all maximal primary events $f$ which have the same time (domain) as $e$ and of which $e$ is a primary subevent. It is easily verified that if $e$ is a primary event, $\Phi_1(e)$ is a secondary event, and if $e$ and $e'$ are different primary events, $\Phi_1(e)$ is different from $\Phi_1(e')$. — Since secondary events appear to have a sufficiently comprehensive nature, they will here be considered to be the events, that is, I identify the concept of event with the concept of secondary event. Some readers may perhaps believe that the concept of eventhood thus obtained is too comprehensive; they are at liberty to do so, and may add to the concept of secondary event whatever restricting conditions they find necessary. Nothing in what follows depends on not adding anything to the concept of secondary event in order to specify eventhood simpliciter.

(2) States of affairs in their turn are generalizations of secondary events, although not in the simple sense that every secondary event is a state of affairs (and not vice versa). Rather, all secondary events are provably

---

8 Nevertheless, there is an event-concept in the immediate vicinity of the concept of secondary event which is even more general than this concept: A tertiary event is a non-empty set of primary events (not necessarily of maximal primary events, as is demanded for secondary events) which contains at most one primary subevent from each world. However, the usefulness of tertiary events is questionable. Most of them are highly fine-grained entities, and it is hard to see where such fine-grainedness would be necessary. Moreover, there is such a multitude of tertiary events that they cannot be represented one-to-one by states of affairs (but only by sets of states of affairs).
represented one-to-one by certain states of affairs (but not vice versa). This is effected by the mapping \( \Phi_2 \). If \( E \) is any secondary event, then \( \Phi_2(E) = \{ (w, t) : w \in W \text{ and } t \in T \text{ and } \exists e (e \in E \text{ and } e \text{ is a primary subevent of } w \text{ and } t \in d(e)) \} \). It is easily verified that if \( E \) is a secondary event, \( \Phi_2(E) \) is a state of affairs — precisely the state of affairs that is expressed by the sentence “\( E_1 \) is going on (just at the moment)”, where “\( E_1 \)” is a name of \( E \). That is, we have \( \Phi_2(E) = \{ (w, t) : w \in W \text{ and } t \in T \text{ and } “E_1 \text{ is going on}” \text{ is true in } w \text{ at } t \} \). Also, it is easily verified that if \( E \) und \( E' \) are different secondary events, \( \Phi_2(E) \) is different from \( \Phi_2(E') \).

(3) It is an immediate consequence of (1) and (2) that primary events, too, are represented one-to-one by states of affairs.

(4) The states of affairs that represent events can be independently characterized, i.e., without referring to the events that they represent: In each possible world \( w \) in which a state of affairs \( p \) obtains at some moment of time, \( p \) can be assigned the non-empty set \( \tau \) of moments of time at which it obtains in \( w \). And a state of affairs \( p \) is temporally local iff it is non-empty (not the empty set) and its obtaining in a world \( w \) at precisely the moments \( t \) in a time \( \tau \) depends exclusively on the (total) states of \( w \) at those moments, that is, on the \( w(t) \) for all \( t \in \tau \). Now, temporal locality can be taken to be a distinguishing mark of eventlike states of affairs. Imposing no further restrictions, I define: \( p \) is an eventlike state of affairs := \( p \) is a state of affairs that is temporally local, i.e., a state of affairs that besides being non-empty (“non-selfcontradictory”) is such that for all possible worlds \( w \) and \( w' \) and times \( \tau \): if \( \forall t (\langle w, t \rangle \in p \iff t \in \tau) \) and \( \forall t (t \in \tau : w'(t) = w(t)) \), then \( \forall t (\langle w', t \rangle \in p \iff t \in \tau) \). (This condition, obviously, expresses that the obtaining of \( p \) in a world \( w \) at precisely the moments of time in a time \( \tau \) is purely a matter of the states of \( w \) at those moments of time.) That eventlike states of affairs in this sense, which does not presuppose the specified concept of eventhood, are rightly called “eventlike” is further corroborated by the following results: (I) For every secondary event \( E \): \( \Phi_2(E) \) is an eventlike state of affairs. (A proof of this can be found in the Appendix.) (II) If \( E \) and \( E' \) are different secondary events, then \( \Phi_2(E) \) and \( \Phi_2(E') \) are different eventlike states of affairs. (III) For every eventlike state of affairs \( p \) there is a secondary event \( E \) such that \( \Phi_2(E) = p \). (A proof of this can be found in the Appendix.)

In view of these results, we arrive at the following conclusions regarding the question whether the causal predicate is a predicate for events or a predicate
for states of affairs:
(A) We could very well identify events with certain states of affairs (that is, with the eventlike state of affairs, as defined in (4) above), and then we have
\( x \text{ Ecauses } y := x \text{ Scauses } y \) (where the latter predicate is taken to imply: “\( x \) and \( y \) are eventlike states of affairs / events”).

Or vice versa:
\( x \text{ Scauses } y := x \text{ Ecauses } y \) (where the latter predicate is taken to imply: “\( x \) and \( y \) are events / eventlike states of affairs”).

Whether the predicate for event-causation is taken as basic, or the predicate for state-of-affairs-causation, is a matter of arbitrary choice. They have entirely equal rights.

(B) But we could also very well distinguish events from states of affairs (such that no event is a state of affairs). In that case we have
\( x \text{ Ecauses } y := x \text{ and } y \) are (secondary) events, and \( \Phi_2(x) \text{ Scauses } \Phi_2(y) \).

Or vice versa:
\( x \text{ Scauses } y := x \text{ and } y \) are eventlike states of affairs, and \( \Phi_2(x) \text{ Ecauses } \Phi_2(y) \). (Here \( \Phi_2 \) is the mapping inverse to \( \Phi_2 \), which inverse mapping does of course exist, since \( \Phi_2 \) is a one-to-one mapping of secondary events onto eventlike states of affairs.)

And again, whether the predicate for event-causation is taken as basic, or the predicate for state-of-affairs-causation, is a matter of arbitrary choice. Both predicates have entirely equal rights.

Note, however, that this complete parity of the two causal predicates does obtain only as long as state-of-affairs-causation is restricted, as we have assumed, to eventlike states of affairs. If causation could obtain between states of affairs that are not eventlike, then “\( x \text{ Scauses } y \)” is the more general predicate, and event-causation — expressed by “\( x \text{ Ecauses } y \)” — would become a special form, or would become isomorphic to a special form, of state-of-affairs-causation. We could merely define:
\( x \text{ Ecauses } y := x \text{ is an event / eventlike state of affairs and } y \text{ is an event / eventlike state of affairs and } x \text{ Scauses } y \) [under (A)],
or\( x \text{ Scauses } y := x \text{ and } y \) are (secondary) events and \( \Phi_2(x) \text{ Scauses } \Phi_2(y) \) [under (B)].

The “vice versa”-definitions would have to be dropped.

I am inclined to hold that state-of-affairs-causation must indeed be restricted to eventlike states of affairs, and I will proceed on this basis. Yet,
be that as it may, the main point remains: there is no significant distinction between event-causation and state-of-affairs-causation (which, in particular, implies that there can be no philosophical merit in praising the one and damning the other\(^9\)). At most, event-causation can be distinguished from (general) state-of-affairs-causation as a special form, or as isomorphic to a special form, of state-of-affairs-causation (which fosters a distinction like the distinction that obtains between being a high mountain and being a mountain).

IV. Agent-Causation

But there is indeed a significant distinction between agent-causation and event-causation (or state-of-affairs-causation). Agent-causation is not a special form of event-causation (as most philosophers think, along the lines of the following definition: \(x\) Acauses \(y\) := there is an action \(z\) of \(x\) [not agent-caused by \(x\), of course] such that \(z\) Ecauses \(y\)). I have argued against the reduction of agent-causation to event-causation elsewhere.\(^{10}\) Here it must suffice that agent-causation is efficient causation, in the proper sense of the word; but no event or state of affairs can exert efficiency in the proper, full sense of the word: neither events nor states of affairs can make something actual. Perhaps nothing in the world can do this (as I have argued in Ereignis und Substanz), perhaps nothing at all can do this (this seems to be Hume’s position), but events and states of affairs certainly can’t. All they can do is to “exert” an analog of making actual, an analog of efficiency. Thus event- and state-of-affairs-causation can at best be analogs of efficient causation — and they ought to be explicated as such, since any analysis of causation must after all be intended as much as possible as an analysis of precisely this: efficient causation.

Agent-causation (or efficient causation, properly speaking) has certain distinguishing features that event-causation (or state-of-affairs-causation, but I will leave this reminder tacit from now on) must imitate as far as possible in order to count at least in an analogical sense as efficient causation (it cannot be efficient causation in the proper sense). First, in agent-causation there is a strong asymmetry between the cause and the effect. This asymme-

---

\(^9\) See for example Jonathan Bennett’s “event causation should be banned from philosophy” (Events and Their Names, p. 142), or Donald Davidson’s “slingshot”-campaign against state-of-affairs-causation (in “Causal Relations”). In chapters 8 and 12 of the second part of my Theorie der Kausalität, I show in great detail that there are no sufficient reasons to ban the one or the other form of causation from philosophy. On the contrary.

\(^{10}\) In my books Ereignis und Substanz and Theorie der Kausalität.
try is, in the first place, categorial in nature: cause and effect belong to separate ontological categories,\textsuperscript{11} and in the second place it is an asymmetry of activity and passivity:\textsuperscript{12} the cause is active, the effect passive (this may simply mean: \textit{incapable of causal efficiency}). \textit{Second}, agent-causation implies the \textit{sufficiency of the cause for the effect}: whatever makes something actual is clearly causally sufficient for it (for its actuality). \textit{Third}, agent-causation implies a \textit{transfer}: the transfer of actuality.\textsuperscript{13} \textit{Fourth}, agent-causation, due to the categorial asymmetry between cause (an agent: a substance) and effect (an event, which, qua event, is not a substance), \textit{does not allow the formation of causal chains}.

In the next section we will see, into which items these central features of agent-causation need to be turned on the side of event-causation if event-causation is to imitate agent-causation (and it has already been determined that event-causation \textit{must} imitate agent-causation, since it needs to be explicated as an \textit{analog of efficient causation properly speaking} if it is causation which we would analyze).

\textbf{V. Event-Causation in the Image of Agent-Causation}

If causation between events is considered as an analog of efficient causation (or agent-causation), then, in event-causation, there must be, \textit{firstly}, a strong asymmetry between the cause-event and the effect-event. Yet, how can such an asymmetry be obtained, since there is obviously no categorial difference between cause-event and effect-event and no activity-passivity-contrast between them? The traditional answer (although, traditionally, not conceived of as being an answer to \textit{that} question) is that on conceptual grounds the cause-event must \textit{happen before} the effect-event. The traditional answer has come under heavy attack from three quarters:

\textsuperscript{11} With agent-causation, the effect can very well be — or rather, it seems best, \textit{should always} be — an event; but the cause cannot, since an event cannot exert causal efficiency. Thus the agent-cause must be a non-event, most likely a \textit{substance}.

\textsuperscript{12} Richard Taylor has drawn attention to this in his book \textit{Action and Purpose}.

\textsuperscript{13} The transfer of actuality from the agent-cause to the effect requires the actuality of the agent-cause; but it should not be taken to require that there is a quantity of actuality, so to speak, in the agent which gets diminished by the transferral. Actuality is not a physical quantity. In fact, it might be better to speak of \textit{a conferral} (bestowal) of actuality by the agent-cause upon the effect, and better to say that the agent-cause \textit{confers actuality} upon the effect than that actuality is transferred from the agent-cause to the effect, \textit{if it were not for this}: “transferral” and “to transfer” carry the desired connotation of \textit{giving something that one has}, while “conferral” and “to confer” do not.
(a) It is objected that according to the traditional answer time appears to be conceptually prior to causation, and hence a causal theory of time would be out of the question. — But this is no evil, since, indeed, time is conceptually prior to causation. Said bluntly, there would still be time (and space) if there were no causation at all.

(b) It is objected that according to the traditional answer backwards causation is a conceptual impossibility. — Again, this is no evil, since, indeed, for conceptual reasons there can be no efficient causation that is directed backwards in time.\(^{14}\) The paradigm of efficient causation has to be respected — this is the basic conceptual decision here made —, and therefore: backwards causation is conceptually out of the question.

(c) It is objected that according to the traditional answer simultaneous causation is a conceptual impossibility. — This, too, is no evil, since, indeed, for conceptual reasons there can be no contemporaneous efficient causation.\(^ {15}\) And the paradigm of efficient causation has to be respected, etc.

Thus, the objections to achieving the asymmetry of event-causeation via a temporal asymmetry of cause-event and effect-event can, at least, be set aside.

The question remains how strong the temporal asymmetry is to be if there is to be a strong asymmetry between cause-event and effect-event. Is it enough if some part of the time of the cause-event is before the time of the effect-event, or need the entire time of the cause-event be before the time of the effect-event? Moreover, since events are here identified with secondary events, and therefore normally have temporally non-isomorphic realizations in different possible worlds, must the cause-event be before the effect-event only in the actual world, or must it be before it also in (at least) all other causally relevant possible worlds?

The asymmetry of cause-event and effect-event in event-causeation, imitating the strong asymmetry between an agent-cause and effect-event that

\(^{14}\) Making actual requires conceptually that what is made actual is not already completely actual, and all past events (implying: no part of them is present or future) are already completely actual. Therefore, no past event can be made actual by anything that is actual only now or later than now (to whatever moment of time after the event "now" is taken to refer to).

\(^{15}\) Making actual requires conceptually that what is made actual is not already completely actual, and all present events (implying: no part of them is past or future) are already completely actual. Therefore, no present event can be made actual by anything that is actual only now or later than now.
obtains in agent-causation, should be made as strong as it can be made, short of excluding causal chains. Thus the cause-event is required to be entirely before the effect-event, and not only in the actual world, but also in all other possible worlds that are causally relevant (for the instance of event-causation concerned). (Which possible worlds these are, i.e., in precisely what sense they are causally relevant, will be considered and determined below, in Section VI.\(^{16}\)) It would, however, result in too strong an asymmetry for event-causation to require additionally that no cause-event can be an effect-event at all, in other words: that if \(E\) Ecauses some event \(E'\), then there is no event \(E''\) that Ecauses \(E\). This would imply that every cause in event-causation is a first cause (precisely as it is in agent-causation) and that therefore there are no causal chains of event-causation, just as there are no causal chains of agent-causation. However, I do not believe that event-causation should imitate this particularly striking feature of agent-causation (the fourth one mentioned in Section IV),\(^{17}\) since the exclusion of causal chains is so closely connected to a categorial difference between cause and effect (how else but by that difference could that exclusion be sufficiently motivated?) that it should be given up when that difference is given up (as is necessarily done when moving from agent-causation to event-causation).

Concerning the asymmetry of event-causation, let this suffice. Now, if causation between events is considered as an analog of efficient causation (in the proper sense of the word), then, secondly, the cause in event-causation must be sufficient for the effect. The agent-cause in making an event actual is clearly sufficient for it (for its being actual), and so must be the cause-event for the effect-event. Obvious as this fundamental point in the theory of causation ought to be, denial or neglect of it is widespread: neither counterfactual (more precisely: counterfactual sine-qua-non) nor probabilis-

\(^{16}\) This strong temporal asymmetry between cause-event and effect-event effectively excludes not only self-causation simpliciter (as is done by any version of causal asymmetry, since asymmetry implies irreflexivity), but also partial self-causation (which must not be confused with one part of an event causing another part that is totally distinct from the former; that is entirely harmless and is not to be avoided): we have not only that no event causes itself, but also that no event causes any part of itself. (There is causally relevant overlap between secondary events \(E\) and \(E'\) iff in some causally relevant possible world a realization of \(E\) is partially identical to a realization of \(E'\). Causally relevant overlap between secondary events which are such that one of them causes the other is excluded by the strong temporal asymmetry between cause-event and effect-event, and therefore partial self-causation is also excluded.)

\(^{17}\) Nevertheless, there is at least one philosopher who has opted for this: Franz von Kutschera in his paper “Causation” from 1993.
tic theories of causation respect it.\textsuperscript{18} The necessary conclusion is: whatever concepts these theories of causation explicate, it is not a concept of causation, since causation is, or is the best analog that is possible of, efficient causation, and the cause in efficient causation is always sufficient for the effect. From this point of view, the mentioned “theories of causation” must be taken to systematically confuse being a cause with being a causal factor, thus making causes also out of mere causal factors. And it is true that common speech lends some support to this confusion, since in common speech mere causal factors are routinely called “causes”. But that mere causal factors are not called “causes” properly speaking can easily be detected: if the words “is a cause” are used in common speech for the designation of a mere causal factor \(x\), then we balk at concluding “\(x\) caused (or causes) \(y\)” from “\(x\) is a cause of \(y\)”. How can this be, since the two phrases ought to be obvious synonyms? The explanation of this inferential failure is that the words “is a cause” are indeed not being used on the occasion in their proper sense, in which they mean as much as “is a sufficient cause”, but are being used merely in an extended, analogical sense, where they mean as much as “is a causal factor”.

Given that event-causation requires the sufficiency of the cause, the question arises: sufficiency in what sense? Now, agent-causation incorporates the feature (the third one mentioned in Section IV) of the transfer of actuality from the cause to the effect. In imitation of this feature, the sufficiency of the cause for the effect in event-causation must be made a necessitating sufficiency: the cause-event necessitates the effect-event. This appears to be the only way in which the transfer-feature of agent-causation can be imitated by event-causation. There certainly cannot be a transfer of energy (or of any other conserved physical quantity) between events: events cannot have energy (or momentum, or whatever other conserved physical quantity), and therefore energy cannot be transferred from one event to another.\textsuperscript{19}

\textsuperscript{18} For a detailed discussion of the literature (Suppes, Lewis, Mellor, etc.) see my Theorie der Kausalität. The sufficiency of the cause is taken by many to be necessarily connected with the so-called “regularity theory of causation” — the theory of causation against which counterfactual and probabilistic theories of causation militate. But the connection is in fact only a historically generated, customary one. Conceptually, one can have the sufficiency of the cause without the regularity theory, and the regularity theory without the sufficiency of the cause (consider the following pseudo-Humean definition: \(E\) is a cause of \(E' := E\) preceded \(E\), and events like \(E'\) are always preceded by events like \(E\)).

\textsuperscript{19} Energy is transferred between physical objects, and the transfer of energy from one physical object to another is itself an event that — normally — has a cause. Therefore, causation cannot itself consist in the transfer of energy (or of momentum, or of whatever other conserved physical quantity). This at once refutes all attempts (by Fair, Kistler,
The strong temporal priority of the cause-event to the effect-event (in imitation of the asymmetry of agent-causation) and the necessitating sufficiency of the cause-event for the effect-event (in imitation of the actuality conferring sufficiency of the agent-cause) will be the hallmarks of the preliminary predicate of event-causation defined in the second to next section (and of the final predicate of event-causation defined in Section VIII). The nature of causal necessity, however, needs special attention; one section of this paper is dedicated to it.

VI. The Nature of Causal Necessity

Note first that the transfer of actuality from the agent-cause to the effect-event need not involve necessity. This is one of the advantages agent-causation has over event-causation, which, in striking contrast, must be considered to be burdened with the problem of causal necessity ever since Hume has made that problem apparent to everyone who cares about clarity and justification. On pains of losing its status of being a form of causality, although an analogical one, event-causation cannot do without causal necessity, that “necessary connection” between cause-event and effect-event that links the actuality of the latter to the actuality of the former. But the nature of causal necessity, since it certainly cannot be a species of logical necessity, seems to be utterly mysterious.

The (relatively) best thing to do, I submit, is to consider the causal necessity involved in event-causation as a necessity that is nomologically derived. This does not mean that the cause-event and effect-event must be subsumable in each instance of event-causation under some law of nature or other (as is Davidson’s position in “Causal Relations”); it means something else, as will be seen step by step from what is said in this section and the next.

In Section III above the concept of possible world was defined. Now, the set W of all (broadly) possible worlds has a proper non-empty subset, NW, of all the worlds which are nomologically possible, where a world is defined as being nomologically possible iff it is compatible with (does not violate) the

\[\text{naturalization}, \ i.e., \ physicalization \ of \ causation \ via \ turning \ it \ into \ a \ relation \ of \ physical \ transfer.\]

20 "Causal realism" should not be a euphemism for the neglect of clarity and justification in the theory of causality. Yet how can it avoid being precisely such a euphemism if the basic attitude involved in causal realism is taken to be the refusal to offer an analysis of causation, and specifically an analysis of causal necessity — an analysis which is, however, rationally unrefusable?"
laws of nature. Thus, which worlds are nomologically possible is conceptually dependent on what are the laws of nature. But the concept I will work with is the concept of nomologically possible world, not the — more fundamental — concept of law of nature. This makes the theory of event-causation that is going to be presented in this paper indeed a nomological regularity theory, but an implicit one, since the nomological regularities themselves — the laws of nature — merely stand "backstage": they will make no appearance on the stage of analysis. In particular, they are not invoked as direct mediators of causal relationships, as is done in the traditional, explicit regularity theories of causation.21

Yet, although laws of nature will here be kept in the background, the objectivity of the causal necessity which is ineluctably involved in event-causation depends precisely on their objectivity, since this causal necessity will be defined by making essential use of the concept of nomologically possible world, which concept, in its turn, has been defined by making essential use of the concept of law of nature. Indeed, the concept of event-causation is precisely as objective as the concept of law of nature is. There is nothing else problematic with respect to its objectivity that the objectivity of the concept of event-causation depends upon.

I am not sure whether it would be a catastrophe for the concept of event-causation if it turned out, as is likely, that the concept of law of nature, and therefore the causal necessity involved in event-causation, is not entirely objective. Frequently, indeed, event-causation is spoken of in philosophy with the serious intention of speaking about something entirely objective. That intention of objectivity would have to be abandoned if laws of nature were not entirely objective, but involved in their status as laws of nature (and sometimes also in their content) an essential and non-negligible reference to our epistemic positions, interests, and conventions. But all other things in and around event-causation could remain the same. It seems to me that a subjective component in event-causation would be a catastrophe merely for naturalistically minded metaphysicians, and for nobody else, certainly not for natural scientists.

21 Thus for recognizing an instance of event-causation one need not know, according to the theory of event-causation presented in this paper, the connecting law of nature (which may be difficult to specify), one needn't even correctly surmise that there is a connecting law of nature. One merely needs to be right in the belief that the laws of nature, in their entirety, are such as to enable (set the stage for) the instance of event-causation concerned, via their selection of the nomologically possible worlds. What this is taken to mean precisely will become clear below.
The causal necessity implied in event-causation can be defined in two basic ways by making use of the concept of nomologically possible world. Let \( E \) and \( E' \) be two events (namely, secondary events) such that \( E \) \( Ecauses \) \( E' \). This implies, as we have seen, among other things, that \( E \) \textit{necessitates} \( E' \), and this, in turn, can mean one or the other of two things:

(a) \textit{Unrelativized nomological necessitation}: An element of \( E' \) (that is to say, some element or other of \( E' \)) is a primary subevent of every nomologically possible world of which an element of \( E \) is a primary subevent. (In order to keep the connection to intuition intact and working, it is good to keep in mind the following definitionally equivalent formulations of (a): \( E' \) has a realization in every nomologically possible world in which \( E \) has a realization; \( E' \) happens in every nomologically possible world in which \( E \) happens.)

(b) \textit{Relativized nomological necessitation}: An element of \( E' \) is a primary subevent of every nomologically possible world of which an element of \( E \) is a primary subevent and which \textit{truly coincides} with the actual world before the time of \( E \).

Before proceeding further, three clarificatory comments are in order:

1. We have presupposed above that \( E \) \textit{Ecauses} \( E' \), and therefore \( E \) is of course a real event (a merely possible event cannot cause anything); in other words, \( E \) has a realization in the actual world (an element of it is a primary subevent of the actual world). Hence, the phrase “the time of \( E \)” is well-defined: the time of \( E \) is the time (the domain) of the element of \( E \) which is a real primary event (in other words, it is the time of \( E \)’s realization in the actual world).

2. Worlds \( u \) and \( u' \) \textit{coincide} before a time \( \tau \) iff for all moments of time \( t \) before \( \tau \) (i. e., for every moment of time \( t \) that is before all moments of time in \( \tau \)): \( u(t) = u'(t) \); they \textit{coincide truly} before a time \( \tau \) iff they coincide before \( \tau \) and the set of moments of time before \( \tau \) is not empty.

3. \textit{Relativized} nomological necessitation is called “relativized” because it is in an obvious sense relative to the actual world and to the time of the cause-event.

For the analysis of event-causation, relativized nomological necessitation is preferable to unrelativized nomological necessitation, because unrelativized nomological necessitation makes the relation of event-causation stricter than is necessary or desirable: \textit{Not} in every case all nomologically possible worlds in which a candidate cause-event takes place should be considered \textit{causally relevant} for a candidate instance of event-causation (that is: for an ordered
pair of two real events which might be cause and effect, the candidate cause-event and the candidate effect-event). Otherwise, some completely outlandish nomologically possible world, totally dissimilar to the actual world, in which the candidate cause-event does happen, but the candidate effect-event does not, could easily thwart an instance of event-causation that on all other accounts is intuitively perfect. This problem is avoided if the possible worlds which are causally relevant for a candidate instance of event-causation are taken to be precisely those nomologically possible worlds in which the candidate cause-event happens and which truly coincide with the actual world before the time of the candidate cause-event: they are the worlds which are, in a causally relevant sense, similar — causally similar — to the actual world.

In order to guarantee that for each candidate instance of event-causation there is a non-empty set of worlds that are causally relevant in the sense defined, it is stipulated that for any candidate cause-event there is at least one moment of time before its time. This makes the actual world, as is proper, causally relevant for all candidate instances of event-causation, because the actual world is, trivially, a nomologically possible world, and one in which the candidate cause-event does happen, and because, whatever the time of the candidate cause-event, the actual world will truly coincide with the actual world before the time of the candidate cause-event (since there always is a moment of time before that time). Yet, the stipulation just made, besides having its intended positive effect, also excludes some real events from being causes, what might be considered to be an undesirable side effect of it: (i) all events that have (in the actual world) an infinite duration into the past are excluded by it from being causes — which is not a bad thing, since such events, if there are such, are not normally required to be causes anyway; (ii) all events that (in the actual world) begin with the very first moment of time itself are excluded by the stipulation from being causes — which, again, is not a bad thing, since, again, such events, if there are such, are not normally required to be causes. 22 I conclude that the above stipulation does not lead to results that are undesirable in view of the intended applications of the concept of event-causation; that stipulation may, therefore, be admitted.

---

22 Note that the Big Bang is not automatically excluded by the stipulation from being a cause, since it is debatable whether the Big Bang, although certainly not of infinite duration into the past, has a first moment at all. If it has a first moment and if that moment is the first moment of time itself, then indeed the Big Bang, according to the stipulation, would have to be excluded from being a cause — but not its close relative: Big Bang* which is just like the Big Bang, except that it is not already occurring in the actual world at the very first moment of time.
Causation in a New Old Key

The question remains whether the set of all worlds that are causally relevant for a candidate instance of event-causation should, or should not, be restricted in a general way even further than has been done already. The relation of event-causation, as it now stands, might still be considered to be too strict, and this may lead one to the idea (extractable from a fair amount of everyday causal talk) that the worlds causally relevant for a candidate instance of event-causation are precisely those nomologically possible worlds in which, firstly, the candidate cause-event happens, which, secondly, truly coincide with the actual world before the time of the candidate cause-event, and which, thirdly, are normal possible continuations of the actual world once the candidate cause-event begins to occur. But this view of the worlds causally relevant for a candidate instance of event-causation in the first place abolishes the essentiality of necessitation for event-causation — and hence, indeed, event-causation itself — because an event $E$ does certainly not necessitate an event $E'$ if $E'$ does not happen in all nomologically possible $E$-worlds that truly coincide with the actual world before the time of $E$, but only in such nomologically possible $E$-worlds that truly coincide with the actual world before the time of $E$ which are normal possible continuations of the actual world once $E$ begins to occur. Event-causation based on normality (of worlds) is not event-causation properly speaking, since, being without the essential ingredient of necessitation, event-causation based on normality is not an analog of efficient causation — which, however, event-causation in the proper sense of the word needs to be. In the second place, the invocation of normality for the analysis of event-causation very likely implies that all hopes for an entirely objective concept of event-causation have definitively to be abandoned, because the concept of normality — if distinguished from nomological possibility — very much appears to be irredeemably subjective (whereas the concept of nomological possibility may still be considered to hold out some hope for complete objectivity).

Therefore, the set of the worlds that are causally relevant for a candidate instance of event-causation should not be restricted in a general way further than has been done already. (There is a briefly mentioned afterthought to this in the next section, part (III).)

VII. A Preliminary Predicate of Event-Causation

The results of the analysis of event-causation so far can be summed up in the following definition of a preliminary predicate of event-causation:

$E Ecauses_p E' := (1st) E$ and $E'$ are [secondary] events, and there is an element $e$ of $E$ and an element $e'$ of $E'$ such that $e$ and $e'$ are primary
subevents of the actual world; (2nd) there is a moment of time antecedent to the time of the element of $E$ which is a primary subevent of the actual world [that is, antecedent to the time of the realization of $E$ in the actual world, i. e., antecedent to the time of $E$]; (3rd) for every nomologically possible world $w$ of which an element $e$ of $E$ is a primary subevent and that truly coincides with the actual world before the time of the element of $E$ which is a primary subevent of the actual world [that is, before the time of the realization of $E$ in the actual world, i. e., before the time of $E$]: an element $e'$ of $E'$ is a primary subevent of $w$, and every moment in the time [domain] of $e$ is before every moment in the time [domain] of $e'$.

The predicate thus defined is a preliminary predicate of event-causation, since some modifications of its definiens seem to be required in order to obtain the definiens of the final predicate of event-causation:

(I) Suppose an event $E'$ happens in every possible world, or in every nomologically possible world, or at least in every (and some) nomologically possible world that truly coincides with the actual world before the time of some real event $E$. Suppose, in addition, that $E$ happens in $w$ — if it happens in $w$ — strictly before $E'$ happens in $w$, for each nomologically possible world $w$ which truly coincides with the actual world before the time of $E$. Is such an event $E'$ caused by the event $E$? Some would deny this on the grounds that $E$ is irrelevant for $E'$: $E$ cannot cause $E'$, because $E'$ will happen anyway, no matter whether $E$ happens or not. Others would not deny that $E'$ is caused by $E$ (as is indeed required by the above definition), but will call the causation of $E'$ by $E$ an instance of trivial event-causation.

What stance is adopted seems to be of no great importance. Yet, if we take into consideration that event-causation is to be an analog of efficient causation (properly speaking), then it is clear that $E$ does not cause $E'$, for $E$ certainly does not make $E'$ actual or real, not even in the purely analogical sense of making actual that alone is possible for it.

Therefore, since event-causation is to be an analog of efficient causation, the above definiens needs modifying, in so doing moving from the already defined preliminary predicate of event-causation towards the final one. We need to add to the definiens:

---

23 There are such events. The sentence "It is in between three and four o'clock on Sept. 4, 1964" expresses an eventlike state of affairs that corresponds to the secondary event $E^*$ which is the set of all maximal primary events $e$ such that $d(e)$ is the set of all moments of time in between three and four o'clock on Sept. 4, 1964. $E^*$ has a realization in each possible world, and therefore it happens in every possible world.
(4th) $E'$ does not have a realization in every nomologically possible world which truly coincides with the actual world before the time of $E$.\textsuperscript{24}

(II) The above preliminary predicate of event-causation is an atemporal one. This is far from being inadequate. Event-causation has, indeed, a lot to do with time and its order, but all of this is a matter of the causal relata, of the events that are causally related. There seems to be no time-relatedness of event-causation that is extrinsic to the causal relata and their position in time. Only if there were such an extrinsic time-relatedness, it would be required that an extra place is added to the predicate of event-causation and that the above definiens of its preliminary version is accordingly modified.

Yet, we do speak of the time of causation. What can be meant by this if there is indeed no time-relatedness of event-causation that is extrinsic to the causal relata and their position in time? I suggest that the time of causation (for an instance of event-causation) is some part of the time of the cause-event, that is, some part of the time of its realization in the actual world. One can take the entire time of the cause-event as being the time of causation, but it seems just as appropriate to merely consider a prominent proper part of that time to be the time of causation. Indeed, when using the words “the time of causation” we usually have in mind some specific instant of time. I therefore suggest that that moment of time in the time of the cause-event is the time of causation (for the instance of event-causation concerned) at which the occurrence of the effect-event is for the first time inevitable in case the cause-event is completely realized (hence the time of causation is the time of the onset of causation). But that moment of time can only be the first moment of the time of the cause-event, if it has a first moment; if not, then there simply is no moment of time at which the occurrence of the effect-event is for the first time inevitable in case the cause-event is completely realized, and therefore there is, then, no time of causation, at least no momentary one.

In order to guarantee a momentary time of causation for every instance of event-causation, I stipulate that the time of every candidate cause-event has a first moment; that moment, or rather its singleton set, will be the momentary time of causation for the candidate instance of event-causation.

\textsuperscript{24} Note that this condition already guarantees that $E'$ does not happen entirely before $E$ in the actual world. For if it did, then an element of $E'$ (namely, precisely the realization of $E'$ in the actual world) would be a primary subevent of each nomologically possible world that truly coincides with the actual world before the time of $E$, and thus $E'$ would have a realization in every nomologically possible world which truly coincides with the actual world before the time of $E$. — Note also that, given the definition of true coincidence, the 4th condition implies the 2nd.
Of course, this stipulation does have a drawback: all events that do not have a first moment are excluded from being causes. But, all things considered, not much seems to be lost by this.\footnote{Except perhaps: if the Big Bang does indeed not have a first moment (cf. footnote 22), then according to this further stipulation it cannot be a cause. But a very close relative of it that has a first moment can be, and it may well be able to do most or all of the causal work the Big Bang is taken to do.} Therefore, the stipulation may be admitted, and for arriving at the final predicate of event-causation, we need to add to the above definiens of the preliminary predicate the following condition:

(5th) the time of $E$ has a first moment (i.e., there is a moment in the time of $E$ such that there is no moment in the time of $E$ that is prior to it).

(III) There are other things one could consider to be of relevance for moving from the defined preliminary predicate of event-causation to the final one. Should we consider only such worlds to be causally relevant in which the cause-event is not already going on when it begins in the actual world? Should the cause-event have a first moment in every causally relevant world? Should the effect-event be temporally contiguous to the cause-event in every causally relevant world? Should it be temporally contiguous to the cause-event at least in the actual world? Should the $w$-time of an event, or at least the $w$-time of an event that is a cause or an effect, always be a time-interval in every world $w$ in which the event occurs?

All these questions are more or less important, an answer to them more or less controversial or arbitrary. I leave it at that, and at the two specified additions to the definiens of the above defined preliminary predicate of event-causation, which turn that definiens into the definiens of the final predicate of event-causation.

VIII. The Final Predicate of Event-Causation

I am now in the position to offer the following definition of event-causation (in which for brevity's sake the preliminary predicate of event-causation is used that was defined in the previous section).

$E$ Ecauses $E'$ := $E$ Ecauses, $E'$ [defined in Section VII by the (1st) - (3rd) conditions]; (4th) $E'$ does not have a realization in every nomologically possible world that truly coincides with the actual world before the time of $E$; (5th) the time of $E$ has a first moment.

This given, a secondary, temporalized predicate of event-causation can be obtained from the final, atemporal one in a way that obviously fits the
motivation for adding the specified 5th condition to the definitions of the preliminary predicate of event-causation:

$E$ Ecauses $E'$ at $t := E$ Ecauses $E'$, and $t$ is the first moment of the time of $E$.

The following statement is an obvious theorem: $\forall E \forall E' (E$ Ecauses $E' \equiv \exists^{-1} t(E$ Ecauses $E'$ at $t))$. Another obvious theorm is $\forall E \forall E' (E$ Ecauses $E'$ $\supset \neg E'$ Ecauses $E$), of which $\forall E (\neg E$ Ecauses $E)$ is an obvious corollary. Thus, event-causation is an asymmetrical, and therefore irreflexive, relation.

It is not, however, a provably transitive relation. Suppose (a) that $E$ Ecauses $E'$, and that $E'$, in its turn, Ecauses $E''$; suppose (b), contrary to transitivity, that $E$ does not Ecause $E''$, and precisely for the reason that there is some nomologically possible world in which $E$ happens [i. e., of which some element of $E$ is a primary subevent] that truly coincides with the actual world before the time of $E$ [i.e., before the time of the element of $E$ that is a primary subevent of the actual world], but in which $E''$ does not happen. This situation is not precluded by assumption (a). Therefore, event-causation, as defined in this paper, is not provably transitive.

This is no evil. If one absolutely requires transitivity for event-causation, then the above definition of event-causation can be taken not to define the final predicate of event-causation after all, but only the basic predicate for the definition of a relational product, and that relational product, then, would indeed be the final predicate of causation: $E$ Ecauses* $E' := E$ Ecauses $E'$, or for some natural number $n \geq 1 : \exists E_1 ... \exists E_n (E$ Ecauses $E_1$, and $\ldots$, and $E_n$ Ecauses $E'$). The predicate "$E$ Ecauses* $E'$" is guaranteed to be transitive.

But should we absolutely require transitivity for event-causation? The paradigm of truly efficient causation that has guided the analysis of event-causation in this paper is indifferent on this issue. Agent-causation, indeed, is trivially transitive since, due to the categorial difference of cause and effect in agent-causation, "$x$ Acauses $y$, and $y$ Acauses $z$" cannot be true for any $x$, $y$ and $z$. The utter triviality of the transitivity of agent-causation — because of the impossibility of fulfilling the antecedent condition of agent-causation's transitivity — makes it a feature of agent-causation that does not seem to be of great importance, in sharp contrast to agent-causation's asymmetry. From the point of view of truly efficient causation, event-causation — its analog — may, therefore, as well be not transitive.

---

26 At this point, the contrast between my views on event-causation and the views of the late David Lewis can be highlighted. Lewis insists that "cause must always be transitive" ("Causation", p. 167), whereas he allows symmetrical and reflexive instances of
IX. The Causal Sentence Connective $C(A, B)$

Suppose we have a regimented, semi-formalized fragment of natural language, or indeed some simple artificial language. In any case, the sentences we are referring to are referred to by the variables: $A, B, A', B', A'', B'' \ldots$, and the language we are talking about is called “L”.

The following truth rules obtain for the five basic sentence connectives that belong to L:

For all possible worlds $w$ (i.e., for all $w \in W$) and all moments of time $t$ (i.e., for all $t \in T$), and for all sentences $A$ and $B$ of $L$:

$\neg A$ is true in $w$ at $t$ iff $A$ is not true in $w$ at $t$;

$(A \supset B)$ is true in $w$ at $t$ iff $A$ is not true in $w$ at $t$ or $B$ is true in $w$ at $t$;

$HA$ is true in $w$ at $t$ iff $\forall t'(t' < t : A$ is true in $w$ at $t')$;

$GA$ is true in $w$ at $t$ iff $\forall t'(t < t' : A$ is true in $w$ at $t')$;

$NA$ is true in $w$ at $t$ iff $w \in NW$ and $\forall w'(w' \in NW$ and $\forall t'(t' < t : w'(t') = w(t')) : A$ is true in $w'$ at $t$).

Note that the truth rule for the operator “$N$” of historical nomological necessity could conceivably be formulated in two ways different from the way just presented — ways that both lead to a somewhat less complex logic for “$N$”.

(1) The language $L$ could be nomologically relativized, that is, it could be interpreted exclusively with respect to nomologically possible worlds. This would mean replacing the reference to $W$ — the set of all possible worlds — at the beginning of the above statement of the truth rules of $L$ by the event-causation (see “Causation”, p. 213). Seen from the point of view of the paradigm of truly efficient causation, however, event-causation can very well be not transitive, whereas its not being asymmetrical or not irreflexive is as absurd as can be. Even quite independent from the categorial difference between cause and effect in truly efficient causation, we have: nothing can make itself actual, nothing that makes something actual is made actual by that which it makes actual. And event-causation, being the analog of truly efficient causation, had better yield the analogs of these principles. (I add that the present approach to causation avoids the problems of causal overdetermination and preemption that beset Lewis’ approach.)

27 For simplicity’s sake, the symbol “$\supset$” will regularly be also used as the operator of material implication of the metalanguage. No confusion can result from this. Note that the bracket-saving convention that “$\supset$” (“if, then”) binds less strongly than “and” (“$\land$”) and “or” (“$\lor$”) is in force here. Note also that the technical expression “$A$ is true in $w$ at $t$” should not be taken to suggest that the sentence $A$ is actually uttered in world $w$, but should be taken to suggest that $w$ is a truthtalker for $A$. It’s only for reasons of brevity and style that “$A$ is true in $w$ at $t$” is here preferred to “$A$ is true at $w$ at $t$” and to “$A$ is true with respect to $w$ at $t$."

—— end of page ——
reference to NW — the set of all nomologically possible worlds. Then the first conjunct on the right-hand side of the "iff" in the truth rule for "N" could be omitted.

The semantical relativization of L to nomologically possible worlds does not appear to have serious drawbacks as long as one does not want to speak in L about possibilities that are not nomologically possible: to the extent that fairy tales or science fiction stories are meant to be about such possibilities, they cannot be formulated in nomologically relativized L. But a fairly decisive reason for abstaining from the nomological relativization of L is that it would obliterate the distinction between nomological truths of L (sentences that are true at all times in all nomologically possible worlds) and analytical truths of L (sentences that are true at all time in all possible worlds). Every language that could, in principle, display the distinction between analytical and nomological truths should display that distinction (and not be interpreted in such a way that it cannot display it). However, there is some pressure — entirely unsuspected at this point — that may motivate a rescinding of this policy (see Section X).

(2) The concept of nomologically possible world could be made relative to worlds. This would mean that "NW" (:= “the set of nomologically possible worlds”) in the above truth rule for “N” is replaced by “NW(w)” (:= “the set of nomologically possible worlds relative to w”). The conjunct “w ∈ NW(w)”, obtained from the conjunct “w ∈ NW” in that truth rule, can then be omitted (because, of course, w ∈ NW(w) for every possible world w).

But this maneuver, if it is to be more than merely technical, requires us to make clear (and preferably objective) sense of the following notion: laws of nature of a world which is not nomologically possible relative to the actual world, because which worlds are nomologically possible with respect to a world w that is not nomologically possible with respect to the actual world depends on the laws of nature of w, which surely are not those of the actual world. The concept of law of nature is a difficult one, even if we are merely talking about the laws of nature of the actual world (the laws of nature); we should not also have to worry about the laws of nature of other possible worlds, of worlds, in particular, which are not nomologically possible with respect to the actual world.

---

28 Stories about miracles, however, can be formulated in nomologically relativized L; for such stories are, as a matter of fact, not meant to be about possibilities that are not nomologically possible (otherwise the occurrence of miracles would be excluded by the very definition of what is a miracle, everything that occurs being nomologically possible), but only about possibilities that are generally thought to be not nomologically possible.
On the whole, it seems best to stick with the truth rule for "\(\neg N\)" already stated, and, in consequence, with an unrestricted interpretation of \(L\) and an absolute concept of nomological possibility. That truth rule contains a stipulation about the truth value of sentences \(\neg N\) in worlds that are not nomologically possible: they are false in those worlds (at all moments of time). Thus we can express the property of being a nomologically possible world \(w\) by \(N(A \supset A)\) (being true in \(w\) at all moments of time), and the property of being a world \(w\) that is not nomologically possible by \(\neg N(A \supset A)\) (being true in \(w\) at all moments of time), since no matter whether the world \(w\) is nomologically possible or not, we always, for all moments of time \(t\), have: \(\forall w'(w' \in W \text{ and } \forall t'(t' < t : w'(t') = w(t')) : A \supset A\) is true in \(w'\) at \(t\)). But this nice feature of the stated interpretation of "\(N\)" immediately demonstrates that "\(N\)" does not function in the way which is customary for necessity operators: "If \(A\) is provable, then \(\neg N\) \(A\) is provable" cannot be a sound rule for "\(N\)" (leading from logical truths to other logical truths).

The state of affairs that is expressed by a sentence \(A\) is designated by "\([A]\)"; this semantical term, which will be much needed below, is defined as follows: \([A]\) := \(\{(w, t) : w \in W \text{ and } t \in T\text{ and } A\) is true in \(w\) at \(t)\).

In addition to the basic sentence connectives, the following sentence connectives are introduced into \(L\) by definition (unnecessary brackets are omitted):

\[
A \lor B := \neg A \supset B,
\]
\[
A \land B := \neg(A \supset \neg B),\quad \tag{29}
\]
\[
P A := \neg H \neg A,
\]
\[
F A := \neg G \neg A,
\]
\[
SA := P A \lor A \lor FA,
\]
\[
IA := HA \land A \land GA.
\]

And finally there is the following definition of a dyadic sentence connective that is important for the theory of causation:

\[
C(A, B) := S(A \land H \neg A \land P \neg A \land NI(A \supset \neg B \land H \neg B) \land N(SA \supset SB) \land \neg NSB).
\]

Making use of this definition, we can prove the Translation Theorem:

Let \(w^*\) be the actual world and \(t\) any moment of time, and let the sentences \(A\) and \(B\) of \(L\) express eventlike states of affairs (this introductory sentence constitutes the presupposition of the Translation Theorem):

\[
C(A, B) \text{ is true in } w^* \text{ at } t \text{ iff } [A] \text{ Scauses } [B].
\]

\(^{29}\) For brevity's sake, the symbol "\(\land\)" will occasionally be also used as the conjunction operator of the metalanguage. No confusion can result from this.
The first thing to remember in proving this is that “x Scauses y” has already been defined in Section III, (B): x Scauses y := x and y are eventlike states of affairs, and $\Phi^*_2(x)$ Ecauses $\Phi^*_2(y)$. Thus, by making use of the definition just mentioned, the biconditional to be proven is turned into

$$C(A, B) \text{ is true in } w^* \text{ at } t \text{ iff } [A] \text{ and } [B] \text{ are eventlike states of affairs, }$$
$$\Phi^*_2([A]) \text{ Ecauses } \Phi^*_2([B]),$$

which can be simplified, because the phrase “[A] and [B] are eventlike states of affairs” is already stated in the presupposition (see above) of the Translation Theorem. Hence the biconditional to be proven becomes

$$C(A, B) \text{ is true in } w^* \text{ at } t \text{ iff } \Phi^*_2([A]) \text{ Ecauses } \Phi^*_2([B]).$$

$\Phi^*_2$ is the mapping inverse to $\Phi_2$, which one-to-one mapping of secondary events onto eventlike states of affairs was defined above (in Section III, Representation results, (2)) as follows:

$$\Phi_2(E) := \{(w, t) : w \in W \text{ and } t \in T \text{ and } \exists e(e \in E \text{ and } e \text{ is a primary subevent of } w \text{ and } t \in d(e))\}, \text{ for every secondary event } E.$$ 

The inverse $\Phi^*_2$ of $\Phi_2$ is, in turn, definable as follows:

$$\Phi^*_2(p) := \{e : \text{Fct}(e) \text{ and } \exists w(\exists t((w, t) \in p) \text{ and } d(e) = \{t : (w, t) \in p\} \text{ and } \forall t(t \in d(e) : e(t) = w(t)))\}, \text{ for every eventlike state of affairs } p.$$ 

Since $\Phi^*_2$ is the inverse of $\Phi_2$, and since the latter mapping is a one-to-one mapping of secondary events onto eventlike states of affairs, $\Phi^*_2$ is a one-to-one mapping of eventlike states of affairs onto secondary events such that for all secondary events $E : \Phi^*_2(\Phi_2(E)) = E$.

Accordingly, the biconditional to be proven finally turns into

---

30 This definition is what is relevant now, since by identifying events with secondary events (in the defined sense) I have opted for distinguishing events from states of affairs, and against identifying events with eventlike states of affairs. (See Section III, (B).)

31 That $\Phi^*_2$, as defined above, is the inverse of $\Phi_2$ can be seen as follows. In the proof of Theorem 2 in the Appendix, using the above definition of $\Phi^*_2$, the proposition is shown that for all eventlike states of affairs $p$: $\Phi_2(\Phi^*_2(p)) = p$. Since $\Phi_2$ is provably a one-to-one mapping assigning eventlike states of affairs to all secondary events, and $\Phi^*_2$ provably a one-to-one mapping assigning secondary events to all eventlike states of affairs, the mentioned result establishes that $\Phi_2$ is the inverse of $\Phi^*_2$, and consequently we also have: $\Phi^*_2$ is the inverse of $\Phi_2$. 

---
\[
C(A, B) \text{ is true in } w^* \text{ at } t
\]

iff
\[
\{ e : \text{Fct}(e) \land \exists w (\exists t ((w, t) \in [A]) \land d(e) = \{ t : (w, t) \in [A] \}) \\
\land \forall t (t \in d(e) : e(t) = w(t))) \} \\
\land \forall t (t \in d(e) : e(t) = w(t))) \}.
\]

Ecauses
\[
\{ e : \text{Fct}(e) \land \exists w (\exists t ((w, t) \in [B]) \land d(e) = \{ t : (w, t) \in [B] \}) \\
\land \forall t (t \in d(e) : e(t) = w(t))) \}.
\]

The proof of this can be found in the Appendix.

The Translation Theorem clearly shows (presupposing that A and B express eventlike states of affairs) that \(C(A, B)\) is a causal sentence connective which mirrors, in an obvious sense, the predicate of causation "x Scauses y" for states of affairs, which, in its turn, mirrors the predicate of causation "x Ecauses y" for events (that is, for secondary events). Sentence connective and predicates can be taken to be dedicated to the expression of one and the same conception of causation, a conception that takes causation between events (or states of affairs) to be an analog of efficient causation properly speaking (or of truly efficient causation), that is, of agent-causation. Agent-causation, therefore, is conceptually central for event-causation, but, as I hope to have made very clear, in a sense that is totally different from the sense in which agent-causation is usually considered by philosophers to be conceptually relevant for event-causation.\(^{32}\)

X. Towards a Complete Logic of Event-Causation

Since the causal sentence connective defined in the previous section is defined exclusively with the help of temporal and modal operators (and the truth-functional connectives \(\land\) and \(\lor\), of course), the logic of event-causation becomes a part of temporal modal logic. Prima facie, there exist axiomatic systems for the operators that have here been used for the definition of \(C(A, B)\), which have been proven to be sound and complete with respect to logical validity defined, on the basis of a certain class of models, for the formulas of

\(^{32}\) If agent-causation is regarded to be conceptually relevant for event-causation, then, usually, in the sense of an agent-causal, interventionistic or actionistic element in event-causation itself. This idea was put forward, notably, by Georg Henrik von Wright. See his paper "On the Logic and Epistemology of the Causal Relation".
an appropriate artificial language. Thus, prima facie, we seem to be in the possession of a provably sound and complete logic of causation. However, a closer look reveals that the completeness results have been reached on the basis of a framework that is not the framework that has here been employed, and by assigning to the operator “N” a meaning that is not the meaning that has here been assigned to it.

A user of the alternative framework simply assumes a non-empty set of possible worlds as basic (in the framework here employed, the possible worlds have been defined), and a non-empty set of moments of time, which is taken to be linearly ordered. In addition, a relation of momentary coincidence of worlds at a moment of time is also assumed (in the framework here employed, such a relation is definable), which is postulated to be an equivalence relation at each moment of time (in the present framework, this could be proven) and which moreover is postulated to satisfy the following stricture: If worlds coincide at some moment of time, then they also coincide at every moment of time previous to that moment of time (in the present framework, this postulate has no parallel). The alternative truth rule for “N” looks like this: NA is true in a world w at a moment of time t if A is true at t in all worlds w’ that coincide with w at t.

If we want to represent the entire alternative approach, on which the completeness results are based, within the approach mainly presented in this paper, that is: within the first approach, then this can be done as follows: (1) The possible worlds of the alternative framework are identified with the nomologically possible worlds (the worlds in NW), also imposing the following condition: \( \forall w \forall w' \forall t [w \in NW \land w' \in NW \land t \in T \land w(t) = w'(t), \text{then } \forall t' (t' < t : w(t') = w'(t'))] \). (2) The relevant artificial language is interpreted with respect to the worlds in NW only (and not with respect to all the worlds in W). (3) The truth rule for NA becomes: NA is true in a world w (in NW) at a moment of time t if \( \forall w' (w' \in NW \land w'(t) = w(t)) \land A \text{ is true in } w' \text{ at } t \).

On the face of it, the alternative approach, even if transposed into the first approach, looks rather different from the latter. Yet, \( \forall w \forall w' \forall t [w \in NW \land w' \in NW \land t \in T \land w(t) = w'(t), \text{then } \forall t' (t' < t : w(t') = w'(t'))] \) is a plausible condition on NW, which we could add without any detriment at all to the first approach. And if we decide, in spite of the objection brought

---

33 See for these results: Franz von Kutschera, “T-W-Completeness”, and Stefan Wöß, *Kombinierte Zeit- und Modallogik. Vollständigkeitsresultate für prädikatenlogische Sprachen*, and, by the same author, “Combinations of Tense and Modality for Predicate Logic”. An axiomatic system of propositional temporal modal logic for which completeness has been proven is presented in chapter XIX of the first part of my *Theorie der Kausalität*. 
forward against this (see the previous section), to nomologically relativize the above interpretation of L (taken to be the relevant artificial language), then the truth rule for “N” in the first approach turns into: NA is true in w at t iff \( \forall w'(w' \subseteq \text{NW} \land \forall t'(t' < t: w'(t') = w(t')) \): A is true in w’ at t), while at the same time the following clause is already an equivalent formulation of the truth rule for “N” in the (transposed) alternative approach: NA is true in w at t iff \( \forall w'(w' \subseteq \text{NW} \land \forall t'(t' \leq t: w'(t') = w(t')) \): A is true in w’ at t). The two truth rules are still not identical, but conceivably the difference between them is not such as to block the completeness results that have been reached for the alternative approach from holding also for the (now modified) first approach.

What should not be done is to simply abandon the truth rule for “N” in the first approach in favor of the truth rule for “N” in the alternative approach, since obviously the former, not the latter, is suitable for the operator-representation of event-causation (by a sentence connective), given the definition of the causal predicate (the predicate-representation) of event-causation which has here been proposed. This situation could be changed by replacing in the third condition of the definiens of “E Ecauses E’” (see Sections VIII and VII above) the phrase “that truly coincides with the actual world before the time of the element of E which is a primary subevent of the actual world” by the phrase “that coincides with the actual world up to and including the first moment of the element of E which is a primary subevent of the actual world”. But, it seems to me, the historico-nomological background or basis of an instance of event-causation and the cause-event itself should be kept distinct, and certainly should not be systematically run into each other by having them, in every instance, overlap in the actual world at one moment of time.

Nevertheless, a provably complete logic of causation based on the first approach does not seem to be far away, once that approach and the alternative approach have been, in the way described, assimilated to each other. The nomological relativization of the object-language L is, however, as serious step, and this also for the following reason: If all the worlds that are drawn into consideration for the semantical evaluation of the sentences of L are nomologically possible, then the (secondary) events and eventlike states of affairs that are (directly, respectively indirectly) expressed by the sentences of that language need to be also nomologically relativized. But can this be done in such a manner as to leave the one-to-one correlation between events and eventlike states of affairs — one of the mainstays of the theory of causation presented in this paper — untouched?
It can be done if the nomological relativization of events and eventlike states of affairs is effected in the following manner:

A **nomological (secondary) event** is a non-empty set of maximal primary events that each occur in at least one nomologically possible world, and which is such that there are no two elements in it that are primary subevents of the same nomologically possible world.

A **nomological state of affairs** is a set of ordered pairs in which the second element is a moment of time, and the first a nomologically possible world.

A **nomological eventlike state of affairs** \( p \) is a non-empty ("non-selfcontradictory") nomological state of affairs such that for all nomologically possible worlds \( w \) and \( w' \) and times \( \tau \): if \( \forall t(\langle w, t \rangle \in p \iff t \in \tau) \) and \( \forall t \in \tau: w'(t) = w(t) \), then \( \forall t(\langle w', t \rangle \in p \iff t \in \tau) \).

Given the above nomological relativization of events and eventlike states of affairs, it can be proven: Nomological eventlike states of affairs and nomological (secondary) events are one-to-one correlated to each other.

**Appendix - Proofs of Theorems**

**Theorem 1.** For every secondary event \( E \): \( \Phi_2(E) \) is an eventlike state of affairs.

**Proof.** Let \( E \) be a secondary event. Hence \( \Phi_2(E) \) (as defined) is \( \{ \langle w, t \rangle : w \in W \text{ and } t \in T \text{ and } \exists e \in E \text{ and } e \text{ is a primary subevent of } w \text{ and } t \in d(e) \} \).

(i) \( \Phi_2(E) \) is a non-empty ("non-selfcontradictory") state of affairs: Since \( E \), being a secondary event, is a non-empty set of maximal primary events, there is at least one maximal primary event \( e \in E \); being such an event, \( e \) is a primary subevent of at least one world \( w \) and \( d(e) \) contains at least one moment of time \( t \). Hence: \( \exists w \exists t(\langle w, t \rangle \in W \text{ and } t \in T \text{ and } \exists e \in E \text{ and } e \text{ is a primary subevent of } w \text{ and } t \in d(e)) \), and therefore: \( \Phi_2(E) \) is a non-empty state of affairs. Moreover:

(ii) Let \( w \) and \( w' \) be worlds and \( \tau \) a time (a non-empty set of moments of time) such that \( \forall t(\langle w, t \rangle \in \Phi_2(E) \iff t \in \tau) \) and \( \forall t \in \tau: w(t) = w'(t) \) [assumption]. We need to deduce from this: \( \forall t(\langle w', t \rangle \in \Phi_2(E) \iff t \in \tau) \). Therefore:

(a) Suppose \( t \in \tau \), hence [by assumption] \( \langle w, t \rangle \in \Phi_2(E) \), hence \( \exists e \in E \text{ and } e \text{ is a primary subevent of } w \text{ and } t \in d(e) \).
But $e$ is also a primary subevent of $w'$. For since $e$ is a primary subevent of $w$, we have, one, for all $t' \in d(e) : w(t') \subseteq e(t')$. And we have, two, $d(e) = \tau$. If $t' \in d(e)$, then $\langle w, t' \rangle \in \Phi_2(E)$, hence [making use of the assumption] $t' \in \tau$. If conversely $t' \in \tau$, then [making use of the assumption] $\langle w, t' \rangle \in \Phi_2(E)$, hence $\exists e'(e' \in E$ and $e'$ is a primary subevent of $w$ and $t' \in d(e'))$; but since $E$, as a secondary event, contains at most one primary subevent from each world — hence also at most one from $w$ — and since already $e \in E$, we have: $e' = e$, and consequently: $t' \in d(e)$. Because of one and two, one obtains by making use of $\forall t(t \in \tau : w(t) = w'(t))$ [in the assumption]: for all $t' \in d(e) : w'(t') \subseteq e(t')$. Therefore: $e$ is a primary subevent of $w'$ (as asserted above).

Hence we also have: $\exists e \in E$ and $e$ is a primary subevent of $w'$ and $t \in d(e)$, and therefore we finally obtain: $\langle w', t \rangle \in \Phi_2(E)$.

(b) Suppose now $\langle w', t \rangle \in \Phi_2(E)$, hence $\exists e'(e' \in E$ and $e'$ is a primary subevent of $w'$ and $t \in d(e'))$. Since $\tau$ is a time, i. e., a non-empty set of moments of time [according to assumption], we obtain because of $\forall t((w, t) \in \Phi_2(E) \iff t \in \tau)$ [in the assumption]: $\langle w, t' \rangle \in \Phi_2(E)$, for some moment of time $t' \in \tau$, that is, $\exists e \in E$ and $e$ is a primary subevent of $w$ and $t' \in d(e))$. Just as under (a), we get from this: $d(e) = \tau$. Since $e$ is, moreover, a primary subevent of $w$, we also have: $\forall t(t \in d(e) : w(t) \subseteq e(t))$. Therefore, because of $\forall t(t \in \tau : w(t) = w'(t))$ [in the assumption]: $\forall t(t \in d(e) : w'(t) \subseteq e(t))$, and consequently: $e$ is also a primary subevent of $w'$. But $e'$ (above) is already a primary subevent of $w'$. Therefore, since $E$, as a secondary event, can contain only one primary subevent of the world $w'$, it follows: $e = e'$, and consequently: $e'$ is a primary subevent of $w$ (because $e$ is a primary subevent of $w$). Therefore, since we have $\exists e'(e' \in E$ and $e'$ is a primary subevent of $w$ and $t \in d(e'))$, we can conclude: $\langle w, t \rangle \in \Phi_2(E)$, and hence — because of $\forall t(\langle w, t \rangle \in \Phi_2(E) \iff t \in \tau) — t \in \tau$.

Given (a) and (b), $\forall t(\langle w', t \rangle \in \Phi_2(E) \iff t \in \tau)$ has now been deduced from the assumption.

From the results in (i) and (ii) it follows by employing the relevant definition: $\Phi_2(E)$ is an eventlike state of affairs — which had to be shown.

THEOREM 2. For every eventlike state of affairs $p$ there is a secondary event $E$ such that $\Phi_2(E) = p$. 


PROOF. From every eventlike state of affairs \( p \) a secondary event \( \Phi_2^*(p) \) is obtained by defining \( \Phi_2^*(p) := \{ f : \text{Fct}(f) \text{ and } \exists w \exists t (\langle w, t \rangle \in p) \text{ and } d(f) = \{ t : \langle w, t \rangle \in p \} \text{ and } \forall t \in d(f) : f(t) = w(t) \} \} \).

Let \( p \) be any arbitrary eventlike state of affairs; then \( \Phi_2^*(p) \) is indeed a secondary event:

\( \Phi_2^*(p) \) is obviously a set of maximal primary events (since \( p \) is an eventlike state of affairs, every \( w \) such that \( \exists t (\langle w, t \rangle \in p) \) must be a world, and \( \{ t : \langle w, t \rangle \in p \} \) must be a non-empty set of moments of time).

Moreover, \( \Phi_2^*(p) \) is non-empty, since \( \exists w \exists t (\langle w, t \rangle \in p) \) (\( p \) being an eventlike state of affairs).

Finally, suppose that \( f \in \Phi_2^*(p) \), \( f' \in \Phi_2^*(p) \) and \( w' \in W \), and that \( f \) is a primary subevent of \( w' \) and that \( f' \) is also a primary subevent of \( w' \) [assumption]. Since \( f \) and \( f' \) as elements of \( \Phi_2^*(p) \) are maximal primary events (\( \Phi_2^*(p) \) being a set of maximal primary events), it follows first of all that \( f \) and \( f' \) are maximal primary subevents of \( w' \). It follows also that \( f = f' \).

Using the definition of \( \Phi_2^*(p) \), we obtain from the assumption: \( \text{Fct}(f) \) and \( \exists w_1 \exists t (\langle w_1, t \rangle \in p) \) and \( d(f) = \{ t : \langle w_1, t \rangle \in p \} \) and \( \forall t (t \in d(f) : f(t) = w_1(t)) \), and \( \text{Fct}(f') \) and \( \exists w_2 \exists t (\langle w_2, t \rangle \in p) \) and \( d(f') = \{ t : \langle w_2, t \rangle \in p \} \) and \( \forall t (t \in d(f') : f'(t) = w_2(t)) \). And hence:

(1) \( p \) obtains in \( w_1 \) precisely in the time \( d(f) \), since \( d(f) = \{ t : \langle w_1, t \rangle \in p \} \). So we have: \( \forall t (\langle w_1, t \rangle \in p \iff t \in d(f)) \). Moreover, \( \forall t (t \in d(f) : w'(t) = w_1(t)) \), because \( \forall t (t \in d(f) : f(t) = w_1(t)) \), and \( \forall t (t \in d(f) : f(t) = w'(t)) \), \( f \) being a maximal primary subevent of \( w' \) [as established]. Therefore, since \( p \) is an eventlike state of affairs (and \( w_1 \) and \( w' \) are worlds, \( d(f) \) a time): \( \forall t (\langle w', t \rangle \in p \iff t \in d(f)) \).

(2) \( p \) obtains in \( w_2 \) precisely in the time \( d(f') \), since \( d(f') = \{ t : \langle w_2, t \rangle \in p \} \). So we have: \( \forall t (\langle w_2, t \rangle \in p \iff t \in d(f')) \). Moreover, \( \forall t (t \in d(f') : w'(t) = w_2(t)) \), because \( \forall t (t \in d(f') : f'(t) = w_2(t)) \), and \( \forall t (t \in d(f') : f'(t) = w'(t)) \), \( f' \) being a maximal primary subevent of \( w' \) [as established]. Therefore, since \( p \) is an eventlike state of affairs (and \( w_2 \) and \( w' \) are worlds, \( d(f') \) a time): \( \forall t (\langle w', t \rangle \in p \iff t \in d(f')) \).

From the results in (1) and (2) it follows immediately: \( d(f) = d(f') \). But since \( \forall t (t \in d(f) : f(t) = w'(t)) \) and \( \forall t (t \in d(f') : f'(t) = w'(t)) \), and since \( f \) and \( f' \) are functions, this means: \( f = f' \).

It has now been established that \( \Phi_2^*(p) \) is a secondary event. Moreover, we have: \( \Phi_2(\Phi_2^*(p)) = p. \)
(i) Suppose \( \langle w, t \rangle \in \Phi_2(\Phi_2^*(p)) \).

That is: \( w \in W \) and \( t \in T \) and \( \exists e(e \in \Phi_2^*(p) \) and \( e \) is a primary subevent of \( w \) and \( t \in d(e) \).

That is: \( w \in W \) and \( t \in T \) and \( \exists e(\textrm{Fct}(e) \) and \( \exists w' (\exists t' (\langle w', t' \rangle \in p) \) and \( d(e) = \{ t' : \langle w', t' \rangle \in p \} \) and \( \forall t'(t' \in d(e) : e(t') = w(t')) \)) and \( e \) is a primary subevent of \( w \) and \( t \in d(e) \).

Since \( e \) is a primary subevent of \( w \) and maximal (because of \( \forall t'(t' \in d(e) : e(t') = w(t')), \( w' \) being a world), we have: \( \forall t'(t' \in d(e) : e(t') = w(t')) \). Therefore: \( \forall t'(t' \in d(e) : w'(t') = w(t')) \). Therefore, since \( d(e) \) is a time (a non-empty set of moments of time) and \( \forall t'(\langle w', t' \rangle \in p \) iff \( t' \in d(e) \)), and \( p \) is an eventlike state of affairs: \( \forall t'(\langle w', t' \rangle \in p \) iff \( t' \in d(e) \)). Therefore, since \( t \in d(e) : \langle w, t \rangle \in p \).

(ii) Suppose \( \langle w, t \rangle \in p \). Hence \( w \) is a world and \( t \) a moment of time (\( p \) being a state of affairs).

The function \( e \) is defined by stipulating: \( d(e) = \{ t' : \langle w, t' \rangle \in p \} \), and \( \forall t'(t' \in d(e) : e(t') = w(t')) \); \( e \) is (a) a primary subevent of \( w \), (b) \( t \in d(e) \), and (c) \( \exists t'(\langle w, t' \rangle \in p) \).

Therefore we have: \( w \in W \) and \( t \in T \) and \( \exists e(\textrm{Fct}(e) \) and \( \exists w'(\exists t'(\langle w', t' \rangle \in p) \) and \( d(e) = \{ t' : \langle w', t' \rangle \in p \} \) and \( \forall t'(t' \in d(e) : e(t') = w(t')) \)) and \( e \) is a primary subevent of \( w \) and \( t \in d(e) \). And this means (see above under (i)): \( \langle w, t \rangle \in \Phi_2(\Phi_2^*(p)) \).

What was to be proven has now been established, because \( \Phi_2^*(p) \) has been shown to be a secondary event \( E \) for \( p \) such that \( \Phi_2(E) = p \).

\[ \text{THEOREM 3. Let } w^* \text{ be the actual world and } t \text{ any moment of time, and let the sentences } A \text{ and } B \text{ of } L \text{ express eventlike states of affairs:} \\
C(A, B) \text{ is true in } w^* \text{ at } t \]

\[ \text{iff} \]

\[ \{ e : \textrm{Fct}(e) \land \exists w(\exists t(\langle w, t \rangle \in [A]) \land d(e) = \{ t : \langle w, t \rangle \in [A] \land \forall t(t \in d(e) : e(t) = w(t)) \}) \land \exists w(\exists t(\langle w, t \rangle \in [B]) \land d(e) = \{ t : \langle w, t \rangle \in [B] \land \forall t(t \in d(e) : e(t) = w(t)) \}) \} \]

\[ \text{Ecauses} \]

\[ \{ e : \textrm{Fct}(e) \land \exists w(\exists t(\langle w, t \rangle \in [B]) \land d(e) = \{ t : \langle w, t \rangle \in [B] \land \forall t(t \in d(e) : e(t) = w(t)) \}) \} \]

\[ \text{Proof. Suppose } w^* \text{ is the actual world and } t \text{ a moment of time, and the sentences } A \text{ and } B \text{ of } L \text{ express eventlike states of affairs [assumption]. We} \]
will use the abbreviation $\Phi^*_2([A])$ for $\{e : \text{Fct}(e) \land \exists w(\exists t(\langle w, t \rangle \in [A]) \land d(e) = \{t : \langle w, t \rangle \in [A]\}) \land \forall t(t \in d(e) : e(t) = w(t)))\}$, and the abbreviation $\Phi^*_2([B])$ for $\{e : \text{Fct}(e) \land \exists w(\exists t(\langle w, t \rangle \in [B]) \land d(e) = \{t : \langle w, t \rangle \in [B]\}) \land \forall t(t \in d(e) : e(t) = w(t)))\}.

(1) Suppose: $C(A, B)$ is true in $w^*$ at $t$. That is, according to the definition of $C(A, B)$:
$S(A \land H \land P \land NI(A \supset \neg B \land H \land \neg B) \land N(SA \supset SB) \land \neg NSB)$ is true in $w^*$ at $t$. That is, according to the truth rules for $L$, etc. $[w^* \in NW$, the linearity of time: $\forall t(t' \in T$ and $t' \in T : t < t'$ or $t' < t$ or $t = t')$:
$\exists t^*([w^*, t^*]) \in [A]$ and $\forall t(t', t^*) \in [A] \supset t' \geq t^*)$ and $\exists t'(t' \leq t^*)$ and $\forall w'(w' \in NW$ and $\forall t(t' < t^* : w'(t') = w^*(t')) : I(A \supset \neg B \land H \land \neg B)$ is true in $w'$ at $t^*$ and $w'(w' \in NW$ and $\forall t(t' < t^* : w'(t') = w^*(t')) : (SA \supset SB)$ is true in $w'$ at $t^*$ and $\exists w'(w' \in NW$ and $\forall t(t' < t^* : w'(t') = w^*(t'))$ and $\langle w', t^* \rangle \notin [SB])$.

That is, according to the truth rules for $L$, etc.:

$\exists t^*([w^*, t^*]) \in [A]$ \hspace{10em} [1]
and $\forall t(t', t^*) \in [A] \supset t' \geq t^*)$ \hspace{10em} [2]
and $\exists t'(t' < t^*)$ \hspace{10em} [3]
and $\forall w'(w' \in NW$ and $\forall t(t' < t^* : w'(t') = w^*(t')) :$
$\forall t'(t', t^*) \in [A] \supset (w', t^*) \notin [B]$ and $\forall t '' (t'' < t' \supset (w', t'') \notin [B])]$ \hspace{10em} [4]
and $\forall w'(w' \in NW$ and $\forall t(t' < t^* : w'(t') = w^*(t')) :$
$\exists t'(t', t^*) \in [A] \supset \exists t''(w', t'' \in [B])$ \hspace{10em} [5]
and $\exists w'(w' \in NW$ and $\forall t(t' < t^* : w'(t') = w^*(t'))$
and not $\exists t'(t', t^*) \in [B]$] \hspace{10em} [6].

From this [and the assumption] we need to deduce: \"$\Phi^*_2([A])$ Ecauses $\Phi^*_2([B])\"", employing the following general definition (see Sections VIII and VII):

$E$ Ecauses $E' := (1st)$ $E$ and $E'$ are [secondary] events, and there is an element $e$ of $E$ and an element $e'$ of $E'$ such that $e$ and $e'$ are primary subevents of the actual world; (2nd) there is a moment of time antecedent to the time of the element of $E$ which is a primary subevent of the actual world [that is, antecedent to the time of the realization of $E$ in the actual world, i. e., antecedent to the time of $E$]; (3rd) for every non-nomologically possible world $w$ [every element $w$ of NW] of which an element $e$ of $E$ is a primary subevent and that truly coincides with the actual world before the
time of the element of $E$ which is a primary subevent of the actual world [that is, before the time of the realization of $E$ in the actual world, i.e., before the time of $E$]: an element $e'$ of $E'$ is a primary subevent of $w$, and every moment in the time [domain] of $e$ is before every moment in the time [domain] of $e'$; (4th) $E'$ does not have a realization in every nomologically possible world that truly coincides with the actual world before the time of $E$; (5th) the time of $E$ has a first moment [i.e., there is a moment in the time of $E$ such that there is no moment in the time of $E$ that is prior to it].

The 1st condition for "$\Phi_2([A])$ Ecauses $\Phi_2([B])"$ is fulfilled: Because $[A]$ and $[B]$ are eventlike states of affairs [according to assumption], $\Phi_2([A])$ and $\Phi_2([B])$ are secondary events (see the proof of Theorem 2). Now: $e$ is the function $f$ with $d(f) = \{t': (w^*, t') \in [A]\}$ and $\forall t \in d(f): f(t) = w^*(t)$; $e'$ is the function $f'$ with $d(f') = \{t': (w^*, t') \in [B]\}$ and $\forall t \in d(f'): f'(t) = w^*(t)$. And because of [1], $[5]$ and $w^* \in NW$, we obtain not only $\exists t'(w^*, t') \in [A]$, but also $\exists t'(w^*, t') \in [B]$. This guarantees that $e$ and $e'$ are primary subevents of the actual world, $w^*$. Moreover, we have $e \in \Phi_2([A])$, and $e' \in \Phi_2([B])$.

The 2nd condition for "$\Phi_2([A])$ Ecauses $\Phi_2([B])"$ is fulfilled: The element of $\Phi_2([A])$ that is a primary subevent of the actual world is precisely the function $e$ which has just been defined. The time, $d(e)$, of that primary event has a first moment: Because of [1]: $t^* \in d(e)$, and there is no $t'$ in $d(e)$ that is prior to $t^*$: suppose $t' \in d(e)$ and $t' < t^*$; hence $(w^*, t') \in [A]$; but this contradicts [2]. Now, according to [3]: $\exists t'(t' < t^*)$. Therefore, since $t^*$ is the first moment of $d(e)$: there is a moment of time antecedent to the time of the element of $\Phi_2([A])$ which is a primary subevent of the actual world.

The 3rd condition for "$\Phi_2([A])$ Ecauses $\Phi_2([B])"$ is fulfilled: Suppose $w \in NW$ and $e'' \in \Phi_2([A])$ and $e''$ is a primary subevent of $w$ and $\exists t'(t' < t^*)$ the time of the element of $\Phi_2([A])$ which is a primary subevent of $w^*$ and $\forall t'(t' < t^*)$ the time of the element of $\Phi_2([A])$ which is a primary subevent of $w^*$: $w(t') = w^*(t')$. As we have seen in proving the fulfillment of the 2nd condition, this supposition can be simplified as follows: $w \in NW$, $e'' \in \Phi_2([A])$, $e''$ is a primary subevent of $w$, $\exists t'(t' < t^*)$, $\forall t'(t' < t^*) w(t') = w^*(t')$. Note also that $e''$, as an element of $\Phi_2([A])$, is a maximal primary event.

(i) Because $e'' \in \Phi_2([A])$, and because $e''$ is a maximal primary subevent of $w$: $\exists w'(t'(w', t') \in [A]) \land d(e'') = \{t': (w', t') \in [A]\} \land \forall t'(t' \in d(e'') : e''(t') = w'(t'))$ and $\forall t'(t' \in d(e'') : e''(t') = w'(t'))$. Hence: $\forall t'(t' \in d(e'') : w(t') = w'(t'))$.

Therefore, since $[A]$ is an eventlike state of affairs, $d(e'')$ a time, $w$ and $w'$ worlds, and $\forall t'(t'(w', t') \in [A]$ iff $t' \in d(e'')$: $\forall t'(t'(w', t') \in [A]$ iff $t' \in d(e'')$). Consequently, because of $\exists w'(w', t') \in [A]$ we have:
\[ \exists t'(t' \in d(\varepsilon')) \], and because of the latter: \[ \exists t'(\langle w, t' \rangle \in \{ A \}). \]

We are now in the position to use condition [5] above, and we obtain: \[ \exists t'(\langle w, t' \rangle \in \{ B \}). \] From this we can conclude: \[ \exists \varepsilon''(\varepsilon'' \in \Phi_2^*([B])) \] and \( \varepsilon'' \) is a primary subevent of \( w \), by defining: \( \varepsilon'' \) is the function \( f'' \) such that \( d(f'') = \{ t' : \langle w, t' \rangle \in \{ B \} \} \) and \( \forall t'(t' \in d(f'') : f''(t') = w(t')) \). It remains to be shown:

(ii) Every moment in the time of \( \varepsilon'' \) is prior to every moment in the time of \( \varepsilon'' \): Suppose \( t' \in d(\varepsilon'') \), hence \( \langle w, t' \rangle \in \{ A \} \) [because of \( \forall t'(\langle w, t' \rangle \in \{ A \} \iff t' \in d(\varepsilon'') \)] which has been established above; therefore, by condition [4]: \( \langle w, t' \rangle \notin \{ A \} \) and \( \forall t''(t'' < t' \Rightarrow \langle w, t'' \rangle \notin \{ B \}) \). Suppose \( \exists t''(t'' \in d(\varepsilon'')) \) and \( (t'' = t' \text{ or } t'' < t') \), hence \( \exists t''(\langle w, t'' \rangle \in \{ B \} \) and \( (t'' = t' \text{ or } t'' < t') \). But this contradicts the result already reached with the help of [4]. Consequently (making use of the linearity of the temporal order): \( \forall t''(t'' \in d(\varepsilon'') \Rightarrow t' < t'') \). This establishes the desired result.

The 4th condition for \( \Phi_2^*([A]) \) Ecauses \( \Phi_2^*([B]) \) is fulfilled: According to condition [6] above: \( \exists \omega'(\omega' \in \Delta W \text{ and } \forall t'(t' < t' : \omega'(t') = \omega'(t')) \) and not \( \exists t'(\langle w, t' \rangle \in \{ B \}) \). Hence: \( \exists \omega'(\omega' \in \Delta W \text{ and } \exists t'(t' < the time of the element of } \Phi_2^*([A]) \) which is a primary subevent of \( w^* \) and \( \forall t'(t' < the time of the element of } \Phi_2^*([A]) \) which is a primary subevent of \( w^* : \omega'(t') = \omega^*(t')) \) and not \( \exists t'(\langle w, t' \rangle \in \{ B \}) \) [because \( t^* \) is the first moment of the time of the element of \( \Phi_2^*([A]) \) which is a primary subevent of \( w^* \), and because of [3]]. Hence: \( \exists \omega'(\omega' \in \Delta W \text{ and } \exists t'(t' < the time of } \Phi_2^*([A]) \) and \( \forall t'(t' < the time of } \Phi_2^*([A]) : \omega'(t') = \omega^*(t')) \) and not \( \exists t'(\langle w', t' \rangle \in \{ B \}) \).

Suppose now that an element \( f \) of \( \Phi_2^*([B]) \) is a primary subevent of \( w' \). Consequently: \( \exists \omega'(\omega'(t', t \in \{ B \}) \land \forall t(\langle w, t \rangle \in \{ B \}) \land \forall t(\langle w, t \rangle \in \{ B \}) = f(t) = w(t)) \) and \( f \) is a primary subevent of \( w' \); hence \( \forall t(\langle w, t \rangle \in \{ B \}) : f(t) = w'(t) \) [\( f \) being a maximal primary event], and therefore \( \forall t(\langle w, t \rangle \in \{ B \}) : f(t) = w'(t) \). Therefore, because \( [B] \) is an eventlike state of affairs, \( d(f) \) a time, \( w \) and \( w' \) worlds, and \( \forall t(\langle w, t \rangle \in \{ B \}) \iff t \in d(f) \) : \( \forall t(\langle w', t \rangle \in \{ B \}) \iff t \in d(f) \), and consequently because of \( \exists t(\langle w, t \rangle \in \{ B \}) \) we have: \( \exists t'(\langle w', t' \rangle \in \{ B \}) \) — in contradiction to what has already been established. Therefore: no element of \( \Phi_2^*([B]) \) is a primary subevent of \( w' \).

The 5th condition for \( \Phi_2^*([A]) \) Ecauses \( \Phi_2^*([B]) \) is fulfilled: This has already been shown in proving that the 2nd condition for \( \Phi_2^*([A]) \) Ecauses \( \Phi_2^*([B]) \) is fulfilled. (Remember that the time of \( \Phi_2^*([A]) \) is the time of the element of \( \Phi_2^*([A]) \) which is a primary subevent of the actual world.)

Since all conditions for \( \Phi_2^*([A]) \) Ecauses \( \Phi_2^*([B]) \) have now been shown to be fulfilled (for \( \Phi_2^*([A]) \) and \( \Phi_2^*([B]) \)), it follows: \( \Phi_2^*([A]) \) Ecauses \( \Phi_2^*([B]) \).
(II) Suppose: $\Phi^*_5([A])$ Ecauses $\Phi^*_5([B])$. This means that all five conditions in the definiens of the definition of "$\Phi^*_5([A])$ Ecauses $\Phi^*_5([B])$" (see above under (I)) are fulfilled for $\Phi^*_5([A])$ and $\Phi^*_5([B])$. We need to show that the following existential quantification statement follows from the supposition just made:

$$\exists t^* \langle w^*, t^* \rangle \in [A]$$
and $\forall t' \langle w^*, t' \rangle \in [A] \supset t' \geq t^*$ \hspace{1cm} [1]$

and $\exists t' \langle w^*, t' \rangle \in [A] \supset t' < t^*$ \hspace{1cm} [2]$

and $\forall w' \langle w', t' \rangle \in [A] \supset \forall t'' \langle w', t' \rangle \in [A] \supset \forall t''' \langle w', t'' \rangle \in [B] [3]$

and $\forall t'' \langle w', t'' \rangle \in [A] \supset \exists t''' \langle w', t''' \rangle \in [B] [4]$\n
and $\exists t'' \langle w', t'' \rangle \in [A] \supset \forall t''' \langle w', t''' \rangle \in [B]$ [5]

and $\forall t' \langle w', t' \rangle \in [A] \supset \forall t'' \langle w', t'' \rangle \in [B]$ [6].

From this existential quantification statement, it follows (by the truth rules of L, etc.) that $S(A \land H \neg A \land P \neg A \land NI(A \supset \neg B \land H \neg B) \land N(SA \supset SB) \land \neg NSB)$ is true in (the actual world) $w^*$ at (the arbitrary moment of time) $t$ (see above under (I)), and hence (according to definition) that $C(A, B)$ is true in $w^*$ at $t$.

As for deducing the existential quantification statement: According to the 1st and 5th condition in the definiens of "$\Phi^*_5([A])$ Ecauses $\Phi^*_5([B])$", the time of the one element $e^*$ of $\Phi^*_5([A])$ which is a primary subevent of the actual world ($w^*$) has a first moment: $t^*$.

[1] $\langle w^*, t^* \rangle \in [A]$.

Since $e^*$ is an element of $\Phi^*_5([A])$, we have: $\exists t(\exists t(\langle w, t \rangle \in [A]) \land d(e^*) = \{t \land (w, t) \in [A] \land \forall t(t \in d(e^*) \land e^*(t))\}$. Since $e^*$ is also a primary subevent of $w^*$, we obtain: $\forall t(t \in d(e^*) \land w^*(t) = w(t))$. Therefore, $e^*$ being a maximal primary event. Therefore, since $[A]$ is an eventlike state of affairs, $d(e^*)$ a time, and $w^*$ worlds, and because $\forall t(\langle w, t \rangle \in [A] \iff t \in d(e^*) \land \forall t(\langle w^*, t \rangle \in [A] \iff t \in d(e^*)\}$, and consequently (because $t^* \in d(e^*)\}$: $\langle w^*, t^* \rangle \in [A]$.

[2] $\forall t(\langle w^*, t' \rangle \in [A] \supset t' \geq t^*)$.

We have already shown: $\forall t(\langle w^*, t \rangle \in [A] \iff t \in d(e^*)$. Since $t^*$ is the first moment of $d(e^*)$, $\forall t(\langle w^*, t' \rangle \in [A] \supset t' \geq t^*)$ is a straightforward consequence.

[3] $\exists t(\langle w^*, t \rangle \in [A])$.

According to the 2nd condition in the definiens of "$\Phi^*_5([A])$ Ecauses $\Phi^*_5([B])$", there is a moment of time antecedent to the time of the element of $\Phi^*_5([A])$
which is a primary subevent of the actual world. Hence there is a moment of time antecedent to \( e^* \), hence prior to \( t^* \).

[4] \( \forall \langle w' \rangle \forall (t' < t^* : w'(t') = w^*(t')) : \forall t''(\langle w', t'' \rangle \in [A] \supset \langle w', t'' \rangle \notin \{B\}) \) and \( \forall t''(t'' < t^* \supset \langle w', t'' \rangle \notin \{B\}) \).

Suppose: \( w' \in [N] \) and \( \forall t'(t' < t^* : w'(t') = w^*(t')) \), and \( \langle w', t' \rangle \in [A] \); suppose also [for reduction] \( \exists t''(t'' = w' \supset \langle w', t'' \rangle \in \{B\}) \). Let \( e_1 \) be the function \( f \) with \( d(f) = \{ t' : \langle w', t' \rangle \in [A] \} \) and \( \forall t'(t \in d(f) : f(t') = w'(t')) \). Let \( e_2 \) be the function \( f' \) with \( d(f') = \{ t' : \langle w', t' \rangle \in [B] \} \) and \( \forall t'(t \in d(f') : f'(t') = w'(t')) \). Obviously, both \( e_1 \) and \( e_2 \) are (maximal) primary subevents of \( w' \), and \( e_1 \in \Phi_2^*[\{A\}] \), and \( e_2 \in \Phi_2^*[\{B\}] \). According to the 3rd condition in the definition of \( \Phi_2^*[\{A\}] \), we can deduce that there is an element \( e' \) of \( \Phi_2^*[\{B\}] \) that is a primary subevent of \( w' \), and every moment in the time of \( e_1 \) is before every moment in the time of \( e' \) [using \( e_1 \in \Phi_2^*[\{A\}] \) and the suppositions, and keeping in mind that “before the time of the element of \( \Phi_2^*[\{A\}] \) which is a primary subevent of the actual world” is equivalent to “before \( t'' \) and keeping in mind that \( \exists t'(t' < t^*) \). But \( \Phi_2^*[\{B\}] \) being a secondary event, and both \( e_2 \) and \( e' \) being elements of it, and both \( e' \) and \( e_2 \) being primary subevents of the world \( w' \), it follows: \( e' = e_2 \). And consequently we have: every moment in the time of \( e_1 \) is before every moment in the time of \( e_2 \). That is, since \( d(e_1) = \{ t' : \langle w', t' \rangle \in [A] \} \) and \( d(e_2) = \{ t' : \langle w', t' \rangle \in [B] \} \) every moment in \( \{ t' : \langle w', t' \rangle \in [A] \} \) is before every moment in \( \{ t' : \langle w', t' \rangle \in [B] \} \). Hence, since \( t'' \) is a moment in \( \{ t' : \langle w', t' \rangle \in [A] \} \) [as we have supposed], \( t'' \) is before every moment in \( \{ t' : \langle w', t' \rangle \in [B] \} \), and hence it cannot be that \( \exists t''(t'' \leq t'^* \supset \langle w', t'' \rangle \in \{B\}) \). Consequently: \( \langle w', t'' \rangle \notin \{B\} \) and \( \forall t''(t'' < t^* \supset \langle w', t'' \rangle \notin \{B\}) \) — which is precisely what had to be deduced.

[5] \( \forall \langle w' \rangle \forall (t' < t^* : w'(t') = w^*(t')) : \exists t''(\langle w', t'' \rangle \in [A] \supset \exists t''(\langle w', t'' \rangle \in [B] \rangle) \).

Suppose: \( w' \in [N] \) and \( \forall t'(t' < t^* : w'(t') = w^*(t')) \), and \( \langle w', t' \rangle \in [A] \). Precisely as in the case of [4], we can deduce from the suppositions, by making use of the 3rd condition in the definition of \( \Phi_2^*[\{A\}] \) we deduce \( \exists t'' \), \( \exists t'' \) of \( \Phi_2^*[\{B\}] \) that is a primary subevent of \( w' \). Hence: \( \exists t'' \langle w(t), t \in \{B\} \rangle \land d(e'') = \{ t : \langle w(t), t \in \{B\} \rangle \}$ and \( \forall t' \in d(e') : e'(t') = w(t)) \). and \( e' \) is a primary subevent of \( w' \). Because \( \forall t' \in d(e') : w(t') = w(t)) \) and because \( e' \) is a primary subevent of \( w' \), it follows: \( \forall t' \in d(e') : w(t') = w(t)) \). [e' being a maximal primary event], and therefore, since \( d(e') \) is a time, \( w' \) and \( w' \) worlds, \( \forall t' \in \{B\} \iff t \in d(e') \), and \( [B] \) an eventlike state of affairs: \( \forall t' \in \{B\} \iff t \in d(e') \). Consequently, because of \( \exists t' \in \{B\} \) we have: \( \exists t''(\langle w', t'' \rangle \in \{B\}) \) — which is what had to be deduced.
[6] \exists w^{*} (w \in NW \text{ and } \forall t^{*} (t^{*} < t^{*} : w^{*}(t^{*}) = w^{*}(t^{*})) \text{ and not } \exists t^{*} (\langle w^{*}, t^{*} \rangle \in [B])): \text{Making use of the 4th condition in the definition of} \Phi^{*}_{2}(\{A\}) \text{Ecauses} \Phi^{*}_{2}(\{B\}) \text{we obtain:} \exists w^{*} (w \in NW \text{ and } \forall t^{*} (t^{*} < t^{*} : w^{*}(t^{*}) = w^{*}(t^{*})) \text{ and there is no element of} \Phi^{*}_{2}(\{B\}) \text{that is a primary subevent of} w^{*} ) \text{. Suppose } \exists t^{*} (\langle w^{*}, t^{*} \rangle \in [B]). \text{But this would mean that there is an element of} \Phi^{*}_{2}(\{B\}) \text{that is a primary subevent of} w^{*} \text{, namely, the function} f \text{for which we have:} \quad d(f) = \{t^{*} : \langle w^{*}, t^{*} \rangle \in [B]\} \text{ and } \forall t^{*} (t^{*} \in d(f) : f(t^{*}) = w^{*}(t^{*})). \text{Hence: not } \exists t^{*} (\langle w^{*}, t^{*} \rangle \in [B]) \text{ — and the deduction of [6] is complete.}\]

The existential quantification statement that had to be deduced has now been deduced. This concludes the second part of the proof of Theorem 3, and the proof of that theorem.

References


UWE MEIXNER
University of Regensburg
uwe.meixner@psk.uni-regensburg.de