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Abstract

In the framework of the industrial economics approach to banking we extend the analysis of hedging against default on loans to the case of two types of credit risk. Standard results on the optimal hedge volume and the hedging effectivity from the single-risk case are shown to carry over to the portfolio case in a non-trivial but intuitive way.

Keywords: banking, credit risk, loan portfolio, credit derivative, hedging effectivity

JEL classification: G21

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A Note on Hedging a Loan Portfolio

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In the framework of the industrial economics approach to banking we extend the analysis of hedging against default on loans to the case of two types of credit risk. Standard results on the optimal hedge volume and the hedging effectivity from the single-risk case are shown to carry over to the portfolio case in a non-trivial but intuitive way.

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1 Introduction

Recently the industrial economics approach to the microeconomics of banking (cf. Freixas and Rochet 1997, chpt. 3) has been supplemented with aspects of uncertainty and risk aversion (see e.g. Wong 1997). This can be used to analyze credit risk and interest rate risk (see e.g. Wahl and Broll 2000, Broll and Jaenicke 2000). Against the background of an increased importance of markets for credit derivatives (see British Bankers' Association 2002) several authors examined the impact of instruments to hedge against credit risk in such a framework. A typical paper in this literature addresses the question of the optimal hedge volume. If the hedge instrument is perfectly correlated with the credit risk and its price is fair, a full hedge — i.e., a forward sale of all credit risk — is optimal. In the presence of basis risk, i.e., with no perfect correlation between credit risk and credit derivative, a beta-hedge rule performs best. The extent of hedging then depends on the slope of the regression of credit risk on the hedge instrument (see Broll et al. 2002).

The papers dealing with credit risk in the framework of the industrial economics approach can be criticized because they consider only one type of credit risk, i.e., they work with the simplifying assumption that the bank does not hold a loan portfolio or that it hedges the aggregate risk of a given portfolio with only one instrument. In the present note we therefore examine the case of a portfolio consisting of two different credit risks to be hedged with only one credit derivative. We consider this the simplest representation of all real-world situations where there are more types of risk than instruments to hedge against risk. Given this modification, we address

the issues of the optimal hedge volume and the effectivity of hedging. It turns out that the results from the standard case carry over to this more realistic situation in a non-trivial, but still intuitive way. For example, the optimal hedge ratio is a weighted average of the exposure shares of the two types of credit risk.

The plan of the paper is as follows: In section 2 we present the basic model which we use in section 3 to derive our results. Section 4 concludes.

2 The Model

Consider a bank taking deposits and giving loans. Deposit and loan volumes are taken as given. We think of them as having been determined before a hedging decision is to be taken.

There are two types of loans: a type 1 with an interest rate r_1 and a probability of default $\tilde{\theta}_1$, and a type 2 with interest rate r_2 and default risk $\tilde{\theta}_2$. The total loan volume of the bank is fixed and normalized to 1. Given shares α and $1 - \alpha$ of the loan volume are invested in loan type 1 and 2, respectively. The risk averse bank management maximizes a von Neumann-Morgenstern utility function $U(\Pi)$ with $U' > 0$ and $U'' < 0$. There is a hedging market where the bank can exchange a risk $\tilde{\theta}$ against a fixed payment $\bar{\theta}$. We can think of the financial asset traded in this market as a credit derivative being more or less perfectly correlated with the two types of loans in the bank's portfolio. However, our analysis is not limited to credit derivatives. Any tradeable financial asset will do. The effectiveness of the instrument to hedge against credit risk crucially depends on the correlations between $\tilde{\theta}$ and $\tilde{\theta}_i, i = 1, 2$. We assume the following "regression dependence"-type relationships between credit risk i and the hedging instrument:

$$\tilde{\theta}_1 = \beta_1 \tilde{\theta} + \tilde{s}_1 \quad (1)$$

$$\tilde{\theta}_2 = \beta_2 \tilde{\theta} + \tilde{s}_2 \quad (2)$$

where $\beta_i, i = 1, 2$, can take positive or negative values and the \tilde{s}_i 's are zero mean noise terms stochastically independent from $\tilde{\theta}$.

By selling a volume H of risk the bank makes a deterministic payment $\bar{\theta}H$ in exchange for a stochastic claim $\tilde{\theta}H$ at the end of the period.¹ This stochastic claim partially or fully offsets losses due to credit default to an extent which is controlled by the decision variable H . The hedge operation described then contributes $H(\tilde{\theta} - \bar{\theta})$ to the bank's profit. Since we do not want to consider the decisions on deposit volume, loan volume and portfolio structure, the profit function of the bank can be written

¹To keep the model as simple as possible we assume that the premium is paid at the end of the period, so there is not need for discounting.

as

$$\tilde{\Pi} = (1 - \tilde{\theta}_1)\alpha r_1 + (1 - \tilde{\theta}_2)(1 - \alpha)r_2 + H(\tilde{\theta} - \bar{\theta}), \quad (3)$$

where we define credit default as default on interest payment alone and ignore operational costs. The decision problem of the bank management is then given by the expected utility maximization problem

$$\max_H \mathbb{E} [U(\tilde{\Pi})], \quad (4)$$

leading to the first-order condition

$$\mathbb{E} [U'(\tilde{\Pi})(\tilde{\theta} - \bar{\theta})] = 0. \quad (5)$$

In the following section we examine (5) to derive properties of optimally hedging a risky loan portfolio in this framework.

3 Results

Assume the hedging instrument to be unbiased, i.e., $\mathbb{E}(\tilde{\theta}) = \bar{\theta}$. From this we can derive Proposition 1 on the optimal hedge H^* :

Proposition 1 *In the presence of basis risk (a) the optimal hedge for a loan portfolio consisting of two types of credit risk is $H^* = H_1^* + H_2^* = \alpha\beta_1 r_1 + (1 - \alpha)\beta_2 r_2$ (b) the optimal hedge ratio for each type of credit risk is $h_i^* = \beta_i$. (c) The hedge ratio h^* for the loan portfolio is a weighted average $\beta_1\gamma_1/(\gamma_1 + \gamma_2) + \beta_2\gamma_2/(\gamma_1 + \gamma_2)$ of the exposure shares of the two types of credit risk.*

Proof (a) Using the assumption of unbiasedness, (5) implies $\text{Cov}(U'(\tilde{\Pi}), \tilde{\theta}) = 0$, i.e.,

$$\begin{aligned} & \text{Cov} [\tilde{\theta} (\alpha\beta_1 r_1 + (1 - \alpha)\beta_2 r_2 - H^*) \\ & + \alpha\tilde{s}_1 r_1 + (1 - \alpha)\tilde{s}_2 r_2 + \theta H^* + \text{const.}, \tilde{\theta}] = 0. \end{aligned}$$

Due to stochastic independence of \tilde{s}_i and $\tilde{\theta}$ this can only be true, if $H^* = \alpha\beta_1 r_1 + (1 - \alpha)\beta_2 r_2$.

(b) Denote by $\gamma_1 = \alpha r_1$ and $\gamma_2 = (1 - \alpha)r_2$ the bank's exposure to and by $H_1^* = \alpha\beta_1 r_1$ and $H_2^* = (1 - \alpha)\beta_2 r_2$ the optimal hedge volume for credit risk of type 1 and 2, respectively. Therefore, the optimal hedge ratios are $h_i^* = \beta_i$.

(c) The result immediately follows from (a) and (b). q.e.d.

A generalization of Proposition 1 to more than two types of credit risk in the loan portfolio is straightforward. Notice that hedging decisions can be de-centralized in the bank. As long as each decision unit calculates its correct values H_i^* and h_i^* ,

the optimal hedge ratio h^* for the whole loan portfolio will hold. Our result can be considered a generalization of the well-known beta-hedge rule to the case of a portfolio (for a single credit risk see Broll et al., 2002). The optimal hedge strategy for each risk type is a beta-hedge. For the portfolio these strategies lead to an exposure weighted average of betas. Adjustments of the optimal hedge volume H^* to changes in the parameters — beta values β_i , interest rates r_i and risk shares $\alpha, (1 - \alpha)$ — can be easily derived from the solution presented in Proposition 1.

Markets for credit derivatives have grown considerably during recent years (cf. British Bankers' Association 2002). From market data we know that banks are major players in these financial markets both as sellers and buyers of credit risk. Our approach provides one possible explanation of why a bank may want to use a negative hedging position H . In this case the bank receives a deterministic payment in exchange for a stochastic liability. Inspection of H^* shows that this will happen for β_1 and β_2 less than zero and can happen, if one of β_1 or β_2 is negative and dominates the optimal hedge volume. Take as an example a situation where the derivative used for hedging purposes is highly correlated with the business cycle and the bank under consideration has a loan portfolio heavily biased towards borrowers from an industry where business conditions move counter-cyclically.

The quality of a hedging instrument can be evaluated using the concept of a hedging effectivity as introduced by Ederington (1979). An index of hedging effectivity $HE \in [0, 1]$ is defined as 1 minus the ratio of the variance of profits with hedging over the variance of profits without hedging. If a perfect hedging instrument were available ($\text{Var}(\tilde{s}_i) = 0, i = 1, 2$), HE would be equal to 1. The presence of basis risk — $\text{Var}(\tilde{s}_i) > 0$ for at least one risk type i — leads to a less than perfect hedge. We can prove the following result:

Proposition 2 *For a loan portfolio with two types of credit risk hedging effectivity HE^* under the generalized beta-hedge rule is given by*

$$\frac{(\beta_1\gamma_1 + \beta_2\gamma_2)^2\text{Var}\tilde{\theta}}{\gamma_1\gamma_2\text{Cov}(\tilde{s}_1, \tilde{s}_2) + 2\beta_1\beta_2\gamma_1\gamma_2\text{Var}\tilde{\theta} + \gamma_1^2(\text{Var}\tilde{s}_1 + \beta_1^2\text{Var}\tilde{\theta}) + \gamma_2^2(\text{Var}\tilde{s}_2 + \beta_2^2\text{Var}\tilde{\theta})} \quad (6)$$

Proof Straightforward calculation from the definition of hedging effectivity using the generalized beta-hedge rule yields (6). q.e.d.

For a single risk type under the beta-hedge rule HE can be shown to be equal to the square of the correlation between the future price $\tilde{\theta}$ of the derivative and the credit risk, i.e., $HE_i = \rho_i^2 = \text{Cov}^2(\tilde{\theta}, \tilde{\theta}_i)/(\text{Var}(\tilde{\theta})\text{Var}(\tilde{\theta}_i))$. This can also be written as $HE_i = \beta_i^2\text{Var}(\tilde{\theta})/(\beta_i^2\text{Var}(\tilde{\theta}) + \text{Var}(\tilde{s}_i))$. Notice our generalization of hedging effectivity to the case of a portfolio of risks exhibits a structure similar to the one-risk case.

The hedging effectivity as calculated in (6) can be used to evaluate the risk-reducing effects of alternative financial instruments for hedging purposes. In the case of a portfolio of credit risks all derivatives available will typically only provide an imperfect hedge. Therefore, the bank has to choose from a set of instruments. HE^* offers an operational measure for this selection problem. As we mentioned earlier, not only credit derivatives are candidates for hedging a loan portfolio. Other derivatives, like the new macro derivatives, can also deliver the risk reduction sought by a bank (for the use of macro derivatives to hedge against credit risk see Broll et al. 2003).

4 Conclusion

In this note we extend the analysis of hedging credit risk with a credit derivative to the case of a portfolio with two different risks. We find that the optimal hedge volume is a weighted average of the exposure shares of the two types of credit risk. The hedging effectivity of the optimal hedge turns out to be a straightforward generalization of the hedging effectivity in the single-risk case.

Let us finally emphasize one more time that our results are not limited to the use of a credit derivative. Any financial instrument could replace the credit derivative considered in our model. All we would need to know are the slopes of the regressions of the two credit risks on the risk of the instrument. In particular, a macro derivative could serve as a substitute for the credit derivative. We know from Broll et al. (2003) that such a macro derivative not only carries the advantage of enabling the bank to trade only the systematic part of credit risk, but also has analytic properties analogous to a credit derivative with basis risk of the type examined here.

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Zusammenfassung

Wir erweitern die Analyse der Absicherung gegen Kreditrisiko mit Hilfe eines Kreditderivats im Rahmen des industrieökonomischen Ansatzes der Bank auf den Fall zweier Kreditrisiken. Für ein solches Kreditportfolio untersuchen wir die Frage des optimalen Sicherungsvolumens und der Hedging-Effektivität. Es zeigt sich, dass die Ergebnisse aus dem Fall mit nur einem Kreditrisiko in intuitiv nachvollziehbarer Weise auf den Portfolio-Kontext verallgemeinerbar sind.
