A tetrahedron-shaped inverted pendulum with torque-actuation

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Extended Abstract

In this contribution, the tetrahedron-shaped inverted pendulum (hereinafter: *TSIP*) is investigated. This pendulum (see Fig. 1) consists of a tetrahedron-shaped rigid body housing. During the process of balancing, the housing is standing on one of its corners with three rotational degrees of freedom. To prevent the pendulum from moving too far out of the considered equilibrium position (and thus from irreversible falling), some actuation is required. Therefore, the *TSIP* comes with four motor-actuated reaction wheels.
Probably the most popular representation of the superordinate class of pendulums and source of inspiration is the *Cubli*-Demonstrator from ETH Zurich, see [1]. This demonstrator's housing has the shape of a cube and stands on one of its corners. The *Cubli*-system is actuated by three reaction wheels with orthogonal rotation axes, therefore its inverse dynamics are well-defined. In [2], the pendulum was extended by a braking system, which allows the cube to switch from a stable into an instable equilibrium without external impact.
Like the tetrahedron, the cube is a platonic solid. This class of polytopes consists of regular, equal faces [3, p. 4-5], which results in high symmetries and is therefore interesting for modeling and control. The current state of research covers a generalization of the modeling concept of the *Cubli* to convex, polyhedron-shaped housings with an arbitrary number of reaction wheels. In this elaboration, this generalization is used for modeling a tetrahedron-shaped system, which exists as s real demonstrator[1].

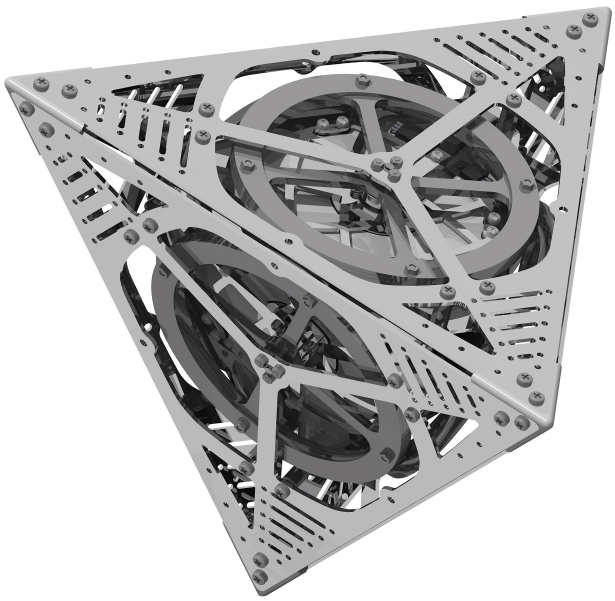

Fig. 1 Rendered image (CAD) of the *TSIP*-Demonstrator.

The *TSIP*-Demonstrator consists of a housing out of an aluminium-alloy with four motorized reaction wheels manufactured from stainless steel. It has an edge length of $0.3\,m$ and a total weight of $3.2\,kg$.
The complete demonstrator consists, in analogy to the four faces of a tetrahedron, of four almost identical assembly groups. A big advantage of this subdivision is, that as long as all four assembly groups are built up identical and oriented correctly, the center of mass (*CoM*) of the resulting tetrahedron-like shape is at its center of geometry, independent of the position of the assembly groups' *CoM*. Therefore, the *CoM* of the *TSIP*-Demonstrator is almost at its geometrical center.
As in Fig. 2, each assembly group subdivides (from bottom to top) into the outer housing, the reaction wheel, the inner housing, an electronic mainboard and an electric motor for actuation. The actuator's rotation axis points in direction of the corresponding surface normal. The demonstrator's translatory acceleration and angular velocity is measured using four *IMU*s (*Inertial Measurement Unit*), each one located in a corner of the tetrahedron.

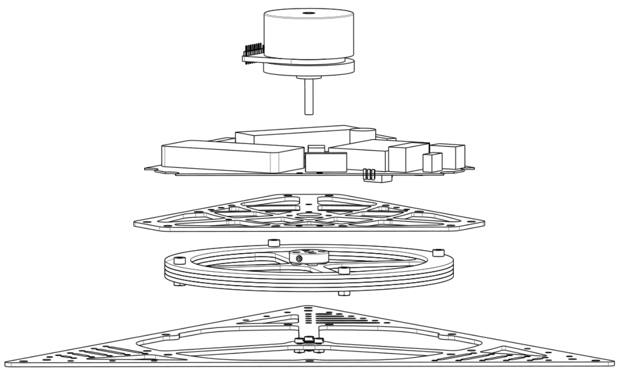

Fig. 2 Face assembly group of the *TSIP*-Demonstrator.

[1] Video available: `https://youtu.be/ILqgUaLrwkY`

An essential difference between the *Cubli* and *TSIP* is, that the inverse dynamics of the *TSIP* is over-determined because of the fourth reaction wheel. As a consequence, the *TSIP* is able to generate a specific resulting torque on the multi-body system with a variety of system inputs. By comparing the two systems, also the arrangement of the actuators leads to very different resulting torques on the housing: If all actuators in the *TSIP* are providing the same torque, the resulting torque on the entire multibody system is zero, whereas it is not in the *Cubli*.

The state of research covers a non-linear state space model for a class of inverted pendulums, based on two simple balances of torque. Therefore, the steps of modeling are not specialized on a cube- or tetrahedron-shaped system, but a superordinate class of systems, consisting of a housing body with a convex, polyhedral shape (not necessarily a platonic solid) and a static number of motor-actuated reaction wheels. The actuators can be located at different positions and orientations inside the housing and need not be of the same type.
In the case of the *TSIP*, the rigid body system counts five bodies. The corner is modeled connected to the plane ground via a spherical joint. The housing has three rotational degrees of freedom and no translational movement is assumed, for this reason no friction model is necessary. Each actuator has exactly one rotational degree of freedom around its motor axis, respectively its reaction wheels's most inert principal axis of inertia. To conclude, the system has 7 degrees of freedom and therefore the state vector counts 14 states.

Before going on with the control design, a simple modal order reduction over the state space model is performed: States and their corresponding differential equations that are not relevant for the controller are removed at first. For control, the absolute angles of the motors are irrelevant and thus omitted in the state vector. The resulting state space representation with 10 states is linearized and further investigated. Despite the linearized system is fully observable, it's controllability matrix does not have full rank, but only rank 9. Therefore it is not fully controllable, which needs to be considered during control design. Thus, *Kalman*-Transformation [4] is used, subdividing the system in an observable-controllable and an observable-non-controllable part. The non-controllable inner process is identified as the conservation of angular momentum around the axis pointing from the spherical constraint in direction of the *CoM* of the multibody system. A more descriptive interpretation of this process is that if an actuator's angular velocity around the considered axis is changed by applying motor torque, the housing's angular velocity is affected in the opposite direction and the uncontrollable state remains unchanged. Although, this is not a problem if the system does start in its equilibrium position.

The linearized, open-loop system can be closed by a LQR-based controller. Because the control design should only happen on the controllable part of the system, the transformed, non-controllable state vector entry is simply ignored in the control concept and the later application. The final controlling concept is based on a linear state feedback, the feedback gain is determined by minimizing the well known cost function of the LQR-design. Note, that there are no further adjustments necessary to handle the over-determined system input. The resulting controlled variables represent one possible torque-configuration and can easily be adapted by adding or subtracting a constant torque on all inputs.

The presented procedure is used to design an appropriate control unit for the *TSIP*-Demonstrator, which is deployed on an ARM-Microcontroller. Running with a stable frequency of 200 Hz, the control unit is able to stabilize the inverted pendulum in its equilibrium position. Even if this contribution focusses on the pendulum balancing on a corner (three *DoF*s), the procedure can be simplified to the balancing on an edge (one *DoF*). Potential applications for the *TSIP* are, for example, in education or as oscillation compensator for the mobile platform of a cable robot, similar to [5] and [6].
The *TSIP*-Demonstrator is still an active topic of research and will be extended by multiple features. One upcoming subject is the planning of trajectory movements outside the system's equilibrium position. Furthermore, a non-linear control design could enhance the balancing precision as well as the sensitivity against disturbance.

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