

## THE HUBBARD MODEL WITH LOCAL DISORDER IN $d = \infty$

V. Janiš<sup>1,\*</sup>, M. Ulmke<sup>2</sup>, and D. Vollhardt<sup>1</sup>

<sup>1</sup>Institut für Theoretische Physik C, Technische Hochschule Aachen  
52056 Aachen, Germany

<sup>2</sup>Institut für Festkörperforschung, Forschungszentrum Jülich  
52425 Jülich, Germany

### INTRODUCTION

The scattering of electrons caused by their mutual interaction and by the presence of static disorder, respectively, can lead to very different, and even opposite, effects. For example, on a cubic lattice at half-filling an arbitrarily weak repulsive Hubbard interaction between electrons is sufficient to induce antiferromagnetic long-range order (AFLRO). By contrast, the presence of randomness opposes long-range spatial order. The simultaneous presence of disorder and interactions in electronic systems can hence be expected to lead to fundamentally new phenomena which have no analog in interacting, non-random and non-interacting systems, respectively [1]. To obtain a global picture of the properties of such systems it is desirable to know the solution of a simple, microscopic model which is valid for all input parameters (interaction, disorder, temperature, band filling). Since exact solutions are not available in  $d = 2, 3$  one would like to construct, at least, a thermodynamically consistent mean-field theory that is valid also at strong coupling. Such a (non-perturbative) approximation is provided by the exact solution of a model in  $d = \infty$ . It is now known that even in the limit  $d \rightarrow \infty$  [2,3] the Hubbard interaction remains dynamical [4] and leads to a highly non-trivial single-site problem [5-8] with infinitely many coupled quantum degrees of freedom. This problem is, in fact, equivalent with an Anderson impurity model complemented by a self-consistency condition [6] and is thus amenable to numerical investigations [7] within a finite-temperature quantum Monte-Carlo approach [9]. In the absence of disorder this technique was already used by several groups to investigate the magnetic phase diagram [7,10] the Mott-Hubbard transition [11-13, 10], transport properties [14] and lately also superconductivity in a two-band version [15] of the Hubbard model in  $d = \infty$ . These investigations were also extended to the periodic Anderson model [16] and the Holstein model [17]. The effect of disorder (both diagonal [18] and off-diagonal [19]) on the

---

\*Permanent address: Institute of Physics, Academy of Sciences of the Czech Republic,  
Na Slovance 2, 18040 Praha 8, Czech Republic

low-temperature properties of strongly correlated electrons in this limit also received attention recently.

## MODEL, AVERAGED FREE ENERGY, CORRELATION FUNCTIONS

In this paper we will extend our earlier study [18] of the combined effect of disorder and interactions in terms of the Hubbard model with diagonal disorder (random local potentials) in  $d = \infty$

$$\hat{H} = -\frac{t^*}{\sqrt{Z}} \sum_{\langle ij \rangle, \sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i\sigma} (\epsilon_i - \mu) \hat{n}_{i\sigma}. \quad (1)$$

Here  $\mu$  is the chemical potential,  $\epsilon_i$  are random atomic energies and  $Z$  is the number of nearest neighbors. In particular, we will investigate the importance of the disorder distribution. To this end two qualitatively different distributions are employed in the Monte-Carlo evaluation of the averaged staggered susceptibility and averaged compressibility of the model. The ground state of (1) was studied earlier in  $d = 1, 3$  by use of a real-space renormalization group [20], and in the strong-coupling limit through a slave boson mean-field formulation of the effective  $t - J$  model [21].

The thermodynamics of (1) is determined by the averaged free energy

$$\Omega_{av} = -\beta^{-1} \langle \ln \text{tr} \exp(-\beta \hat{H}) \rangle_{av} \quad (2)$$

where  $\langle \dots \rangle_{av}$  is the configurational average over the random energies  $\epsilon_i$  defined by

$$\langle \mathcal{O} \rangle_{av} = \prod_{i=1}^L \left[ \int_{-\infty}^{\infty} d\epsilon_i P(\epsilon_i) \right] \mathcal{O}(\epsilon_1, \dots, \epsilon_L) \quad (3)$$

with  $P(\epsilon_i)$  as the distribution function of the (uncorrelated) random potentials  $\epsilon_i$ . It was recently shown by two of us [22] that in  $d = \infty$  the averaging involves only a single site  $\mathbf{R}_i$  where the electrons encounter both a Hubbard interaction and the random energy  $\epsilon_i$ . This site is surrounded by a *homogeneous* effective medium, described by a dynamical potential  $\Sigma(\omega)$ , which contains the full information about the physical processes taking place at all other sites. These processes may, for example, lead to AFLRO in the system as in the non-random model. Hence we must allow for breaking of translational symmetry in  $\Sigma(\omega)$ . For simplicity we only consider bi-partite lattices, in which case the symmetry breaking is caused by a staggered field  $\alpha h$  with  $\alpha = \pm 1$  on A- and B-sites, respectively, with  $h$  as a magnetic field. In  $d = \infty$ ,  $\Omega_{av}$  then takes the form [5,22]

$$\begin{aligned} 2\beta\Omega_{av}/L &= - \sum_{\sigma,n} \int dE N(E) \ln [ (i\omega_n + \mu_A - \Sigma_{A\sigma n})(i\omega_n + \mu_B - \Sigma_{B\sigma n}) - E^2 ] \\ &+ \sum_{\alpha,\sigma,n} \ln G_{\alpha\sigma n}^{-1} - \sum_{\alpha} \langle \ln \mathcal{Z}_{\alpha} \{ G, \Sigma, \epsilon_i \} \rangle_{av} \end{aligned} \quad (4)$$

where  $L$  is the number of lattice sites,  $N(E)$  is the density of states of the non-interacting electrons,  $\mu_{\alpha} = \mu + \alpha h$ , and  $\omega_n = (2n+1)\pi T$  are Matsubara frequencies. The quantities  $\Sigma_{\alpha\sigma n} \equiv \Sigma_{\alpha\sigma}(i\omega_n)$  and  $G_{\alpha\sigma n}$  specify the potential of the medium and the local part of the averaged propagator of the (non-interacting!) electrons moving in this potential, respectively. Here

$$\begin{aligned} \mathcal{Z}_\alpha\{G, \Sigma, \epsilon_i\} &= \int \mathcal{D}\psi \mathcal{D}\psi^* \exp \left[ \sum_{\sigma, n} \tilde{\psi}_{\alpha\sigma n}^* (G_{\alpha\sigma n}^{-1} + \Sigma_{\alpha\sigma n} - \epsilon_i) \tilde{\psi}_{\alpha\sigma n} \right. \\ &\quad \left. - U \int_0^\beta d\tau \psi_{\alpha\uparrow}^*(\tau) \psi_{\alpha\uparrow}(\tau) \psi_{\alpha\downarrow}^*(\tau) \psi_{\alpha\downarrow}(\tau) \right] \end{aligned} \quad (5)$$

is the partition function for electrons from the medium which encounter a Hubbard interaction  $U$  and a random energy  $\epsilon_i$  at  $\mathbf{R}_i$  [5-7]. The fields  $\psi_{\alpha\sigma}(\tau)$  and  $\tilde{\psi}_{\alpha\sigma n}$  are Grassmann variables in imaginary times and frequency, respectively. From the stationarity conditions  $\delta\Omega_{av}/\delta\Sigma_{\alpha\sigma n} = 0$ ,  $\delta\Omega_{av}/\delta G_{\alpha\sigma n} = 0$  [5,22] one obtains two self-consistency equations determining the physical values of  $\Sigma_{\alpha\sigma n}$  and  $G_{\alpha\sigma n}$ , respectively, i.e.

$$G_{\alpha\sigma n} = \int_{-\infty}^{\infty} dE N(E) \left[ i\omega_n + \mu_\alpha - \Sigma_{\alpha\sigma n} - E^2 / (i\omega_n + \mu_{-\alpha} - \Sigma_{-\alpha\sigma n}) \right]^{-1} \quad (6a)$$

$$G_{\alpha\sigma n} = - \int_0^\beta d\tau e^{i\omega_n \tau} \langle \langle \psi_{\alpha\sigma}(\tau) \psi_{\alpha\sigma}^*(\tau^+) \rangle \rangle_T \rangle_{av} \quad (6b)$$

where  $\langle \dots \rangle_T$  denotes the thermal average with the local partition function, (5).

To determine the thermodynamic stability of various phases we calculate the susceptibilities  $X_{av} = -L^{-1} \partial^2 \Omega_{av} / \partial x^2$  corresponding to the averaged compressibility  $\kappa_{av}$  and staggered susceptibility  $\chi_{AF}$  for  $x = \mu, h$ , respectively,

$$\kappa_{av} = -\frac{1}{L} \frac{\partial \Omega_{av}}{\partial \mu^2}, \quad \chi_{AF} \equiv \chi_{av}^{stag} = -\frac{1}{L} \frac{\partial \Omega_{av}}{\partial h^2} \quad (7)$$

Details of the calculation are given elsewhere [23]. By restricting the investigation to the paramagnetic and antiferromagnetic phases (excluding, for example, *ferrimagnetism*) the number of independent parameters can be reduced by setting  $\Sigma_{A_n} \equiv \Sigma_{A\uparrow n} = \Sigma_{B\downarrow n}$ ,  $\Sigma_{B_n} \equiv \Sigma_{B\uparrow n} = \Sigma_{A\downarrow n}$  and similarly for  $G_{\alpha\sigma n}$  and  $\psi_{\alpha\sigma}$ . Using (4)  $X_{av}$  may be expressed in terms of a two-particle irreducible vertex function  $\Gamma$  as

$$X_{av} = \beta^{-2} \sum_{\alpha\alpha'} \sum_{nn'} f_\alpha^x \Gamma_{nn',n'n}^{\alpha\alpha'} \gamma_{\alpha'n'}^x \quad (8a)$$

where  $f_\alpha^\mu = 1$ ,  $f_\alpha^h = \alpha$ ,  $\gamma_{\alpha n}^x = (\partial/\partial x)(G_{\alpha n}^{-1} + \Sigma_{\alpha n})$  and

$$\begin{aligned} \Gamma_{n_1 n_1', n_2' n_2}^{\alpha\alpha'} &= \int_0^\beta d\tau_1 d\tau_1' d\tau_2 d\tau_2' \exp [i (\omega_{n_1} \tau_1 + \omega_{n_1'} \tau_1' - \omega_{n_2'} \tau_2' - \omega_{n_2} \tau_2)] \\ &\quad \times \langle \langle \psi_\alpha(\tau_1) \psi_\alpha^*(\tau_2) \psi_{\alpha'}(\tau_1') \psi_{\alpha'}^*(\tau_2') \rangle \rangle_T \rangle_{av} \end{aligned} \quad (8b)$$

In the paramagnetic phase the dynamical response function  $\gamma_{\alpha n}^x$  obeys the integral equation

$$\beta^{-1} \sum_{\alpha'} \sum_{n'} (\beta \delta_{\alpha\alpha'} \delta_{nn'} R_{\alpha n}^x + \Gamma_{nn',n'n}^{\alpha\alpha'}) \gamma_{\alpha'n'}^x = f_\alpha^x R_{\alpha n}^x \quad (9)$$

where

$$R_{\alpha n}^\mu = [G_{\alpha n}^{-2} - \langle G_{\alpha n}^2 \rangle^{-1}]^{-1} \quad (10a)$$

$$R_{\alpha n}^h = [G_{\alpha n}^{-2} - (i\omega_n + \mu - \Sigma_{\alpha n}) G_{\alpha n}^{-1}]^{-1} \quad (10b)$$

with  $\langle G_{\alpha n}^2 \rangle \equiv \int dE N(E) [i\omega_n + \mu - \Sigma_{\alpha n} - E]^{-2}$ . Since  $\Gamma$  is a purely local quantity the information about long-range correlations in the system is contained in  $\gamma$ . Physically

speaking  $\gamma$  measures the response of the effective medium to an infinitesimal change of the field  $x$ . It is this response function, which decides about the (in)stability of a given phase. In the antiferromagnetic phase (8) does not change, but (9) acquires a more complicated dependence on the index  $\alpha$  [23].

## NUMERICAL PROCEDURE

Eqs. (8), (9) form the basis for the numerical evaluation of  $\kappa_{av}$  and  $\chi_{AF}$ . The numerical calculations were performed with a semi-elliptic density of states (DOS) with total width  $2w$ , i.e.

$$N(E) = \frac{2}{\pi w^2} (w^2 - E^2)^{1/2} \quad (11)$$

This DOS is chosen because of its sharp algebraic band edges (resembling those typical for  $d = 3$ ) and its simple analytic form; it is exact for a Bethe lattice in the limit  $Z \rightarrow \infty$ .

To study the influence of the disorder we investigate, and compare, two qualitatively different distributions of random potentials:

a) Discrete, binary-random-alloy distribution

$$P_{binary}(\epsilon_i) = \frac{1}{2} \delta\left(\epsilon_i - \frac{\Delta}{2}\right) + \frac{1}{2} \delta\left(\epsilon_i + \frac{\Delta}{2}\right), \quad (12a)$$

where the atomic potentials  $\epsilon_i = \pm\Delta/2$  occur with equal probability. This distribution is important since it leads to a disorder-induced MIT due to band-splitting in the noninteracting system (an exact result in  $d = \infty$  [24,22]) which may compete with the interaction-induced Mott-Hubbard MIT.

b) Continuous, semi-elliptic distribution

$$P_{semi}(\epsilon_i) = \frac{8}{\pi \Delta^2} \left[ \left(\frac{\Delta}{2}\right)^2 - \epsilon_i^2 \right]^{1/2} \quad (12b)$$

This is a much softer type of disorder. The two distributions allow us to test the universality of the magnetic behavior of the disordered model obtained for different types of disorder. – It should be noted that in  $d = \infty$  electrons are delocalized for arbitrary strength of the disorder, i.e. Anderson localization does not occur in this limit. This is a consequence of the single-site character of the theory, which implies that vertex corrections to the conductivity vanish identically [24,25].

To be able to study the competition between magnetic order caused by the electronic interactions and the disordering effects caused by the random potential, respectively, we work with an average band filling  $n = 1$ . Due to the symmetry of the distributions (12a,b) we can fix the chemical potential at  $\mu = U/2$ .

For the numerical evaluation of the functional integral (5) we employ the algorithm of Hirsch and Fye [9]. We discretize the time variable, i. e.  $\beta = \Lambda \delta\tau$ , with  $0.25 \leq \delta\tau \leq 1$ , and then extrapolate the quantities under investigation to  $\delta\tau \rightarrow 0$ . Exact summations over spin variables in the discrete Hubbard-Stratonovich transformation were used whenever possible, i. e. for  $\Lambda \leq 22$ . For  $\Lambda > 22$  we used Monte-Carlo sampling. After 4-8 iterations an accuracy of  $10^{-5}$  and  $10^{-3}$  was reached in the exact summations and in the Monte-Carlo sampling with  $10^4$  sweeps per iteration, respectively.

Setting  $\hbar = k_B = 1$  the only remaining physical dimension is that of an energy ( $U, \Sigma, \Delta, T$ , etc.) or inverse energy ( $G, \kappa_{av}, \chi_{AF}, \beta, \delta\tau$ ). Departing from our earlier

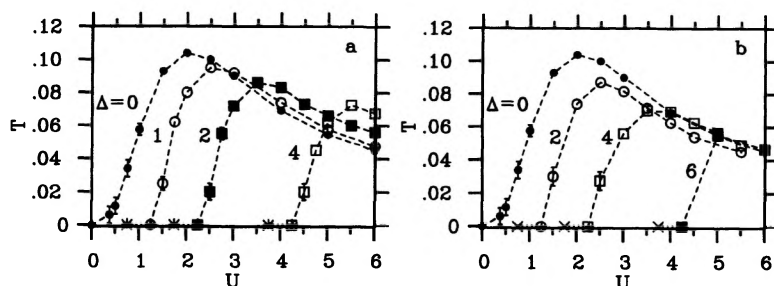
convention we now choose the *half-band width*  $w$  as our energy unit since it does not depend on the limiting process  $Z \rightarrow \infty$ . (This is in contrast to the scaling of the hopping amplitude  $t = \alpha t^* / \sqrt{Z}$  where  $\alpha$  may be chosen at will). This convention agrees with that used by Kotliar and collaborators [6,11,13]. To be able to compare the results presented in this paper with our earlier ones [18], all numerical values of quantities with dimension of energy (inverse energy) obtained in Ref. [18] must be divided (multiplied) by a factor of 2. To compare with the results of Jarrell's group [7,14] and that of Georges and Krauth [10,12], their numbers have to be divided (multiplied) by a factor of  $\sqrt{2}$ .

## MAGNETIC PHASE DIAGRAM

We first study the influence of disorder on the formation of AFLRO. The latter arises from correlations between local magnetic moments whose static average,  $m_{av}$ , is defined by  $m_{av}^2 \equiv L^{-1} \sum_i \langle (\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow})^2 \rangle_T \rangle_{av} = 1 - 2d_{av}$ . Here  $d_{av} = L^{-1} d\Omega_{av}/dU$  is the average double occupancy of lattice sites. The disorder has two main effects which are independent of the specific disorder distribution[18]: 1) at a fixed value of  $U$  an increase of the disorder *reduces* the moments. 2) For fixed disorder strength  $\Delta$  an increase of  $U$  leads to an increase of the moments, with saturation starting at  $U \gtrsim \Delta$ . Both features are easily explained in terms of the effect the disorder and the on-site repulsion, respectively, have on the average double occupancy  $d_{av}$ . For  $U \ll \Delta$  the local repulsion is weak while the spatial fluctuations of the atomic potentials are strong, such that  $d_{av}$  is at its maximum value ( $d_{av} \simeq 1/2$ ) and  $m_{av}$  is small. As  $U$  increases the particles are forced to separate and  $d_{av}$  decreases, i.e.  $m_{av}$  increases, too.

We note that in a non-interacting, disordered system at  $T = 0$  with a *discrete* spectrum of random energies, and for lattices with a finite band width, the band will split at some critical value of the disorder,  $\Delta_c$ , i.e. the DOS acquires a gap. This is an exact feature in  $d = \infty$  [22,24]; for a random alloy  $\Delta_c = 1$ . Hence, for  $\Delta > \Delta_c$  the moments disappear completely. By switching on the interaction between the electrons, at fixed  $\Delta$ , energy states will enter the gap. Now, at  $T = 0$  the split-band MIT can be expected to occur even for  $U > 0$  if  $\Delta$  is large enough, i.e.  $\Delta_c(U) \geq \Delta_c(0) \equiv \Delta_c$ . This implies in turn that in the binary alloy case with  $\Delta > 1$  there *always* exists a critical value of the interaction  $U_c^{MI,1}(\Delta) \geq 0$  where the moments vanish and a MIT occurs. (It cannot be ruled out *a priori* that  $U_c^{MI,1} = 0$  for all  $\Delta > 1$ , but due to the many-body effects in the interacting system one expects  $U_c^{MI,1}(\Delta) > 0$ ; see below).

Next we study the spatial correlations between the local moments. For this we have to evaluate the staggered susceptibility  $\chi_{AF}$ , (7). In particular, to detect the instability of the paramagnetic phase w.r.t. AF fluctuations, we look for a divergence of  $\chi_{AF}$ . To determine  $T_c$  we calculated  $[\chi_{AF}(T; \delta\tau)]^{-1}$  vs.  $T$  for different time slicings  $\delta\tau$ . At sufficiently large  $U$  a Curie law with a mean-field critical exponent  $\nu = 1.0$  is observed to fit the data very well in agreement with ref. 7. For these values of  $U$  one may safely extrapolate  $[\chi_{AF}(T; \delta\tau)]^{-1}$  to zero to determine the critical temperature  $T_c(\delta\tau)$ . However, for values  $U \lesssim \Delta$  the low-temperature behavior of  $\chi_{AF}^{-1}$  no longer follows a Curie law and hence an extrapolation to  $\chi_{AF}^{-1} = 0$  becomes ambiguous [18,23]. For the temperatures investigated here ( $T \geq 1/16$ ) we observe two regimes: (i)  $U \sim \Delta$ : here  $\chi_{AF}^{-1}$  still decreases monotonically for decreasing  $T$ , but an extrapolation would suggest  $T_c(\delta\tau) < 0$ , i.e. a paramagnetic state; (ii)  $U \lesssim \Delta$ : here  $\chi_{AF}^{-1}$  begins to *increase* again. The latter behavior is a direct consequence of the suppression of the local moments due to the disorder as discussed above. For  $U \simeq \Delta$  there is still a possibility that  $\chi_{AF}^{-1}$



**Figure 1.** Magnetic phase diagram,  $T$  vs.  $U$ , for various values of disorder  $\Delta$ ; a) discrete disorder distribution (random alloy), b) continuous disorder distribution. The antiferromagnetic (paramagnetic) phase is stable below (above) the curves. Dotted lines at  $T = 0$  indicate the regimes where the Curie law would give negative  $T_c$ ; below the crosses  $\chi_{AF}^{-1}$  no longer vanishes at all.

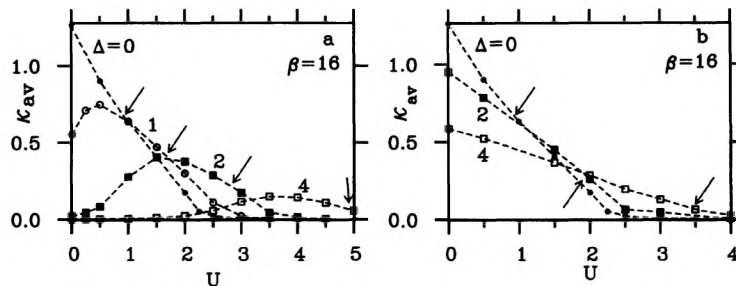
vanishes, e.g. non-analytically. The actual phase boundary between the paramagnetic and the AF phase is determined by extrapolating  $T_c(\delta\tau)$  to  $\delta\tau = 0$ .

The resulting  $T - U$  phase diagram obtained for the two types of disorder is shown in Fig. 1a for binary alloy disorder and in Fig. 1b for continuous disorder. As expected the disorder suppresses the AFLRO for  $U \lesssim \Delta$  and reduces the maximal value of  $T_c$  globally. However,  $T_c$  is not a monotonically decreasing function of the disorder for all  $U$ : for  $U \gtrsim 2.5$  the curves separating the ordered and disordered phase are seen to cross. This implies that weak or medium-strength disorder favors the formation of an ordered phase, i.e. the critical temperature  $T_c(\Delta)$  initially increases with  $\Delta$  and reaches a maximum before it eventually falls to zero [18].

This surprising strong-coupling behavior, whose origin is not clear at present, does not seem to depend on the type of distribution of the random potential. In the case of the continuous disorder distribution it is less pronounced, however, and seems to set in only above a finite value of  $\Delta$ . It will be interesting to investigate whether the above effect survives in the Heisenberg limit,  $U \rightarrow \infty$ , where the interplay between disorder and interactions may well lead to a new phase, e.g. with spin glass properties.

## METALLIC AND INSULATING PHASES

At weak coupling the disorder was found to suppress the AFLRO, irrespective of the disorder distribution. The ground state is then a disordered paramagnet. Whether this phase is metallic or insulating does, however, depend on the type of disorder (discrete or continuous spectrum) and its strength: in the case of the binary alloy disorder [18] there occurs a MIT for  $\Delta_c(U) \geq 1$ , while for the continuous distribution this is not the case. A thermodynamic criterion for an insulating state (both in the paramagnetic and antiferromagnetic phase) is the vanishing of the averaged compressibility:  $\kappa_{av} = 0$  (for a discussion of this criterion see Ref. [23]). In Fig. 2a,b  $\kappa_{av}$  is shown as a function of  $U$  for the two disorder distributions under investigation. They are calculated in the para- and antiferromagnetic phases, the transition points being indicated by arrows. While for  $U \gtrsim \Delta$  the dependence of  $\kappa_{av}$  on  $U$  is qualitatively the same for both distributions, namely a monotonic decrease with increasing  $U$ , the behavior for  $U < \Delta$  is characteristically different. For the random alloy at  $\Delta \gtrsim 2$   $\kappa_n$  is (exponentially) small at small  $U$ , i.e. the system is a *paramagnetic insulator* there. The incompressibility



**Figure 2.** Averaged compressibility  $\kappa_{av}$  vs.  $U$  at  $\beta = 16$  for various values of disorder  $\Delta$ ; a) random alloy disorder, b) continuous disorder distribution. Arrows indicate transition into the antiferromagnetic phase.

is a consequence of the disorder-induced gap in the DOS in this case. As  $U$  increases the gap closes and  $\kappa_{av}$  increases, too. For values of  $U$  larger than the critical value  $U_c^{MI,1}(\Delta)$  discussed above the system is a *paramagnetic metal* (without Fermi liquid properties). Hence, in contrast to an interacting system without disorder the Coulomb interaction may turn an insulator into a metal! The question whether  $U_c^{MI,1} = 0$  or not cannot be decided numerically. As  $U$  is further increased  $\kappa_{av}$  reaches a maximum for  $U \approx \Delta$ , after which it decreases again (Fig. 2a). No such behavior is found for the continuous disorder distribution, where  $\kappa_{av}$  decreases monotonically for all  $U$  (Fig. 2b). At small  $U$  ( $U \ll \Delta$ ) and  $\Delta > 0$  the system is hence a *paramagnetic metal* (without Fermi-liquid properties) in this case. –

In both cases  $\kappa_{av}$  decreases for large enough  $U$  and eventually becomes (exponentially) small. Hence, at some critical value  $U_c^{MI,2}$  a *second* transition, back into an insulating state, occurs. From Fig. 2a,b, obtained at finite temperatures, it appears that this transition occurs well in the antiferromagnetically ordered phase. This would imply the formation of an *antiferromagnetic metal* in a finite region  $U_c^{AF} < U < U_c^{MI,2}$ , at half-filling. The question is then whether, at  $T = 0$ ,  $\kappa_{av}$  vanishes *discontinuously* already at  $U_c^{AF}$ , such that  $U_c^{AF} = U_c^{MI,2}$ . We found that, while for  $\Delta = 0$  the onset of AFLRO suppresses  $\kappa_{av}$  considerably, it has almost no effect on  $\kappa_{av}$  at finite disorder [23]. Apparently the disorder *stabilizes* the metallic state close to  $U_c^{AF}$ . These results suggest that for  $U \gtrsim U_c^{AF}$  the system is indeed an antiferromagnetically ordered metal. A definite answer to this question for  $T = 0$  would, however, require Monte-Carlo calculation at much lower temperatures, i.e.  $\beta \gtrsim 100$ , which cannot at present be achieved by the method used here. Hopefully a new numerical method proposed recently [26], which makes use of an exact diagonalization of an Anderson model with a finite number of sites, will permit a reliable calculation in this limit.

## SUMMARY

We investigated the influence of two different types of disorder, with discrete and continuous spectrum, respectively, on the low-temperature phase diagram of the Hubbard model at half filling. For this we evaluated the staggered susceptibility and compressibility in both the paramagnetic and antiferromagnetic phase in  $d = \infty$  using quantum Monte-Carlo. The competition between the kinetic energy, electron interac-

tion and disorder leads to a sequence of interaction- and disorder-induced transitions. This rich transition scenario, together with the finding of an unexpected strong-coupling effect (the stabilization of the ordered phase due to disorder) clearly shows that the simultaneous presence of disorder and strong electron-electron correlations leads to new, non-perturbative quantum many-body phenomena which deserve further investigations.

## ACKNOWLEDGEMENTS

We are grateful to Dr. P. van Dongen and Prof. H. Müller-Krumbhaar for discussions. This work was supported in part by the Sonderforschungsbereich 341 of the Deutsche Forschungsgemeinschaft.

## References

- [1] *Critical phenomena, random systems, gauge theories*, part II, Les Houches XLIII, eds. K. Osterwalder and R. Stora (North Holland, Amsterdam, 1986).
- [2] W. Metzner and D. Vollhardt, Phys. Rev. Lett. **62**, 324 (1989).
- [3] D. Vollhardt, in *Correlated Electron Systems*, ed. V. J. Emery (World Scientific, Singapore, 1993), p. 57.
- [4] E. Müller-Hartmann, Z. Physik **B74**, 507 (1989); *ibid* **B76**, 211 (1989).
- [5] V. Janiš, Z. Phys. **B83**, 227 (1991); V. Janiš and D. Vollhardt, Int. J. Mod. Phys. **B6**, 731 (1992).
- [6] A. Georges and G. Kotliar, Phys. Rev. **B45**, 6479 (1992); A. Georges, G. Kotliar and Q. Si, Int. J. Mod. Phys. **B6**, 705 (1992).
- [7] M. Jarrell, Phys. Rev. Lett. **69**, 168 (1992); M. Jarrell and T. Pruschke, Z. Phys. **B90**, 187 (1993).
- [8] See also F. J. Ohkawa, J. Phys. Soc. Jpn. **60**, 3218 (1991), *ibid.* **61**, 1615 (1991).
- [9] J. E. Hirsch and R. M. Fye, Phys. Rev. Lett. **56**, 2521 (1986).
- [10] A. Georges and W. Krauth, Phys. Rev. **48**, 7167 (1993).
- [11] M. J. Rozenberg, X. Y. Zhang and G. Kotliar, Phys. Rev. Lett. **69**, 1236 (1992).
- [12] A. Georges and W. Krauth, Phys. Rev. Lett. **69**, 1240 (1992).
- [13] X. Y. Zhang, M. J. Rozenberg and G. Kotliar, Phys. Rev. Lett. **70**, 1666 (1993); M. J. Rozenberg, G. Kotliar and X.Y. Zhang, preprint; Q. Si, M.J. Rozenberg, G. Kotliar, and A. E. Ruckenstein, preprint.
- [14] T. Pruschke, D. L. Cox and M. Jarrell, Phys. Rev. **47**, 3553 (1993); M. Jarrell and T. Pruschke, Phys. Rev. **B49**, 1458 (1994).
- [15] A. Georges, G. Kotliar and W. Krauth, Z. Physik B **92**, 313 (1993).
- [16] M. Jarrell, H. Akhlaghpour, and T. Pruschke, Phys. Rev. Lett. **70**, 1670 (1993).
- [17] J. K. Freericks, M. Jarrell, and D. J. Scalapino, Phys. Rev. **B48**, 6302 (1993).
- [18] V. Janiš, M. Ulmke, and D. Vollhardt, Europhys. Lett. **24**, 287 (1993).
- [19] V. Dobrosavljević and G. Kotliar, Phys. Rev. Lett. **71**, 3218 (1993), and preprint.
- [20] M. Ma, Phys. Rev. **B26**, 5097 (1982).
- [21] G. T. Zimanyi and E. Abrahams, Phys. Rev. Lett. **64**, 2719 (1990).
- [22] V. Janiš and D. Vollhardt, Phys. Rev. **B46**, 15 712 (1992).
- [23] M. Ulmke, V. Janiš, and D. Vollhardt, preprint RWTH/ITP-C 1/94.
- [24] R. Vlaming and D. Vollhardt, Phys. Rev. **B45**, 4637 (1992).
- [25] A. Khurana, Phys. Rev. Lett. **64**, C 1990 (1990).
- [26] M. Caffarel and W. Krauth, preprint.