Depairing Critical Currents in Superfluid ^3He*

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Critical currents in both superfluid ^3He-A and ^3He-B are calculated within a weak coupling model. The Fermi liquid correction is explicitly included. As a by-product we obtain the nonlinear superfluid densities in ^3He-A and ^3He-B, which depend strongly on the current.

1. INTRODUCTION

The superfluidity[1,2] is certainly one of the most remarkable characteristics of the new phases in liquid ^3He below 3 mK. In the present paper we will study theoretically the current-dependent features of superfluid ^3He when a (large) uniform current (with flow velocity $v_s \approx 10^{-3}$–$10^{-2}\ v_F$) is applied to the system. In particular we will show that superfluidity can be completely destroyed by a large current (i.e., a current larger than the depairing critical current), although to our knowledge experiments in the presence of such large a current have not been done yet. However, we find that even a small current affects significantly the quasiparticle energy gap as well as the superfluid density. Therefore, it is certainly of practical interest to study the effects of a uniform current on the properties of the superfluid phases of ^3He, since even a small temperature gradient introduces an appreciable superflow in liquid ^3He. It may be of importance to distinguish from the outset two kinds of critical currents; the "textural" critical currents[3] and the depairing critical currents. The former is defined as the current that induces a drastic modification in textures associated with the condensate. The associated critical velocity is of the order of Ω_A/p_F in ^3He-A, where Ω_A is the Leggett dipolar shift in NMR frequencies of ^3He-A and p_F is the Fermi momentum. On the other hand, the depairing critical current is of the order of $\Delta(T)/p_F$, where $\Delta(T)$ is the quasiparticle energy gap.

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Therefore the depairing critical current is roughly by a factor 10^2 larger than the textural critical current in ^{3}He-A. In such a large current we expect a variety of nonlinear effects as in the case of superconductivity.[4,5] The analogy is particularly useful for ^{3}He-B. In spite of the difference between the singlet pairing in the BCS superconductor and the triplet pairing in the Balian–Werthamer state,[6] which characterizes the condensate of ^{3}He-B, we have the same expressions for the energy gap and the superfluid density in both cases, except that in superfluid ^{3}He the Fermi liquid correction is significant and has to be included in the theoretical analysis.

An advantage of considering such a large current is that we are now most likely dealing with a uniform texture (see, e.g., Ref. 7); in particular in ^{3}He-A, we may assume that $\mathbf{v}_s \| \hat{l}$, where $\hat{l}$ indicates the direction of the symmetry axis of the energy gap in ^{3}He-A. However, recently Hall and Hook[8] suggested (based on their numerical analysis) that uniform $\hat{l}$ textures are unstable under large superfluid current. We have looked into this possibility. However, contrary to them the uniform texture in the bulk appears to be stable at least in the vicinity of the transition temperature even under large current (see also Bhattacharyya *et al.*[9]). Furthermore, in contrast to the case of a superconductor, a uniform current situation appears quite natural in superfluid ^{3}He, especially when the current is relatively large. In the following we will calculate the quasiparticle energy gap $\Delta(T)$ and the superfluid density in the presence of a uniform current in both ^{3}He-A (with $\mathbf{v}_s \| \hat{l}$) and ^{3}He-B.

Following the formalism developed by Maki and Tsuneto,[5] we introduce the effect of a uniform current as a frequency shift in the single-particle Green's function. The effects of the Fermi liquid correction are then included by renormalizing the quasiparticle mass and the superfluid velocity as done by Leggett.[10] Furthermore, we adopt the weak coupling model for simplicity, since the results can be comparable to experiments at least at semiquantitative levels. In order to have some insight into the qualitative features, the weak coupling model is quite adequate, except possibly for the stability question of the axial state.[11] In particular in the zero current limit ($\mathbf{v}_s = 0$) the superfluid densities obtained agree with the previous results by Combescot.[12]

The calculated superfluid densities in ^{3}He-A and ^{3}He-B are strongly nonlinear in the superfluid velocity $\mathbf{v}_s$ mostly due to the large Fermi liquid correction. The results appear to be readily accessible experimentally. For example, it may be seen as the amplitude dependence of the fourth-sound velocity c_4. Furthermore, although the calculated critical currents vary monotonically as functions of temperature, the calculated (depairing) critical velocities v_{sc} exhibit nonmonotonic temperature dependence in both ^{3}He-A and ^{3}He-B, again due to the Fermi liquid correction. (In the absence of the Fermi liquid correction, the critical velocities behave monotonically though!) This could probably be seen in the ion mobility experiments, since we expect that the ion mobility should change dramatically when the ion velocity exceeds the depairing critical velocity. In the experimental data on negative ion mobilities in ^{3}He-A and ^{3}He-B published by Ahonen *et al.*[13] there is no indication of such a nonmonotonic behavior. However, we hope that experiments on the ion mobilities at lower temperatures will soon resolve this question.

2. FORMULATION

The single-particle Green's function for superfluid ^{3}He-A and ^{3}He-B in the absence of currents is given in the Nambu representation by

$$G_0(\mathbf{p}, \omega_n) = [i\omega_n - \xi\rho_3 - \sigma_1\Delta(\hat{p}_1\rho_2 - \hat{p}_2\rho_1)]^{-1} \tag{1}$$

and

$$G_0(\mathbf{p}, \omega_n) = [i\omega_n - \xi\rho_3 - \sigma_2\rho_1\Delta(\hat{\mathbf{p}} \cdot \boldsymbol{\alpha})]^{-1} \tag{2}$$

respectively, where

$$\xi = (1/2m^*)p^2 - \mu, \qquad \hat{\mathbf{p}} = \mathbf{p}/p_\mathrm{F}, \qquad \boldsymbol{\alpha} = (\sigma_1\rho_3, \sigma_2, \sigma_3\rho_3) \tag{3}$$

Δ is the order parameter (which we will refer simply to the energy gap); m^* and μ are the effective mass of the quasiparticle and the chemical potential; and σ_i and ρ_i are Pauli matrices operating on the ordinary spin space and the particle–hole space, respectively.

Following Maki and Tsuneto,[5] the effect of a uniform current is introduced into the above Green's function by replacing $i\omega_n$ by $i\omega_n + \mathbf{v}_s \cdot \mathbf{p}$, where $\mathbf{v}_s$ is the superfluid velocity and $\mathbf{p}$ is the quasiparticle momentum. This replacement gives the correct Green's function in the absence of the Fermi liquid correction and follows from a gaugelike transformation of the condensate order parameter $\Delta(\mathbf{r}) \to e^{i\Phi}\Delta(\mathbf{r})$ with $\Phi = 2m\mathbf{v}_s \cdot \mathbf{r}$, where m is the mass of the ^{3}He atom.

In the presence of the interaction between quasiparticles the effect of the current will be reduced by $(1+\frac{1}{3}F_1\phi)^{-1}$ due to the polarization of the liquid.[10] The above coefficient describes a mean-field screening of the effect of the velocity $\mathbf{v}_s$, where ϕ will be determined later. Therefore in this more general situation the effect of the current is introduced into the Green's function by replacing $i\omega_n$ by $i\omega_n + \mathbf{v}_s^* \cdot \mathbf{p}$, where

$$\mathbf{v}_s^* = (1+\tfrac{1}{3}F_1\phi)^{-1}\mathbf{v}_s \tag{4}$$

Then the Green's functions in the presence of a uniform current are given by

$$G(\mathbf{p}, \omega_n) = G_0(\mathbf{p}, \omega_n - is\cos\theta)$$

with

$$s=(1+\tfrac{1}{3}F_1\phi)^{-1}v_s p_F \tag{5}$$

where $G_0(\mathbf{p}, \omega_n)$ is given by Eqs. (1) and (2) for the axial and the isotropic states, respectively. Here θ is the angle between $\mathbf{v}_s$ and $\mathbf{p}$.

Within the weak coupling limit the gap equation is given by

$$\Delta_i(\hat{\mathbf{p}}) = -3g_1 T \sum \int \frac{d^3p'}{(2\pi)^3}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') \times \tfrac{1}{4}\,\mathrm{Tr}\,[\sigma_2(\rho_1+i\rho_2)\alpha_i G_0(\mathbf{p}', \omega_n - is\cos\theta')] \tag{6}$$

for both ^{3}He-A and ^{3}He-B. The above equation is further simplified to

$$1 = 2\pi\lambda T \sum_{n=0}^{n_0} \int \frac{d\Omega}{4\pi}\left(\frac{3}{2}\sin^2\theta\right) \times \mathrm{Re}\,\{[(\omega_n - is\cos\theta)^2 + \Delta^2\sin^2\theta]^{-1/2}\} \tag{7}$$

and

$$1 = 2\pi\lambda T \sum_{n=0}^{n_0} \int \frac{d\Omega}{4\pi}\{\mathrm{Re}\,[(\omega_n - is\cos\theta)^2+\Delta^2]^{-1/2}\} \tag{8}$$

for the axial and the isotropic states, respectively, where in the axial state we assumed that $\hat{l}$ is parallel to $\mathbf{v}_s$. Here $\lambda = N(0)|g_1|$, $\omega_c = \omega_{n_0} = 2\pi T(n_0+1/2)$ is the cutoff frequency as in the BCS case, and $N(0)$ is the density of states at the Fermi level. In the weak coupling limit the cutoff frequency ω_c as well as the coupling constant λ can be eliminated from Eqs. (7) and (8) by subtracting the corresponding equations at $T=T_c$, the transition temperature, or those at $T=0$. In this case the energy is scaled by T_c or Δ_{00}, the energy gap at $T=0$ and $s=0$.

For example, at $T=0$, Eqs. (7) and (8) are further reduced to

$$-\ln(\Delta/\Delta_{00}) = \tfrac{1}{2}\ln[1+(s/\Delta)^2] - \tfrac{1}{2}s^2/(\Delta^2+s^2) \tag{9}$$

and[4]

$$-\ln(\Delta/\Delta_{00}) = \theta(s-\Delta)[\ln\{(s/\Delta)+[(s/\Delta)^2-1]^{1/2}\} - [1-(\Delta/s)^2]^{1/2}] \tag{10}$$

for the axial and the istropic states, respectively, where $\theta(x)$ is the step function

$$\theta(x) = 1 \quad \text{for } x \geq 0; \qquad = 0 \quad \text{for } x < 0 \tag{11}$$

Here $\Delta_{00} = \tfrac{1}{2}e^{5/6}\Delta_{\mathrm{BCS}}(0)$ and $\Delta_{00} = \Delta_{\mathrm{BCS}}(0)$ for the axial and the isotropic states, respectively. [Note: $\Delta_{\mathrm{BCS}}(0) = (\pi/\gamma)T_c = 1.76T_c$.] In general, Eqs.

(7) and (8) determine the energy gap (or more precisely the order parameter) in the presence of a uniform current.

The superfluid mass current $\mathbf{J}_s$, due to the superfluid velocity $\mathbf{v}_s$ is given by

$$\mathbf{J}_s = mN\mathbf{v}_s - \mathbf{J}_n \tag{12}$$

with

$$\mathbf{J}_n = T\sum_n \int \frac{d^3p}{(2\pi)^3}\mathbf{p}\tfrac{1}{4}\,\mathrm{Tr}\,[G_0(\mathbf{p}, \omega_n - is\cos\theta)] \tag{13}$$

which reduces to

$$|\mathbf{J}_s| = 2\pi TN(0)p_F \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{dz}{2} z\,\mathrm{Re}\frac{i\omega_n + sz}{[(\omega_n - isz)^2 + \Delta^2(1-z^2)]^{1/2}} \tag{14}$$

and

$$|\mathbf{J}_s| = 2\pi TN(0)p_F \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{dz}{2} z\,\mathrm{Re}\frac{i\omega_n + sz}{[(\omega_n - isz)^2 + \Delta^2]^{1/2}} \tag{15}$$

for the axial and the isotropic states, respectively, where $z = \cos\theta$. In particular, at $T = 0$ K, Eqs. (14) and (15) are simplified further to

$$\mathbf{J}_s = \rho_s^0 \mathbf{v}_s^* \tag{16}$$

where

$$\rho_s^0/\rho = \Delta^2/(\Delta^2 + s^2) \tag{17}$$

and

$$\rho_s^0/\rho = 1 - \theta(s-\Delta)[1-(\Delta/s)^2]^{3/2} \tag{18}$$

for the axial and isotropic states, respectively. Here $\rho = mN$ is the (mass) density of liquid ^{3}He.

In terms of ρ_s^0/ρ, we can now define ϕ introduced in Eq. (4) as

$$\rho_s^0/\rho \equiv 1 - \phi(s) \tag{19}$$

Therefore Eq. (16) may be rewritten as

$$\mathbf{J}_s = \rho_s \mathbf{v}_s$$

with

$$\rho_s/\rho = (1-\phi)(1+\tfrac{1}{3}F_1\phi)^{-1} \tag{20}$$

which is a natural generalization of a similar expression in the linear regime.[12] At $T = 0$ K, Eq. (19) indicates that ϕ is given by

$$\phi = s^2/(\Delta^2 + s^2) \tag{17'}$$

and[4]

$$\phi = \theta(s - \Delta)[1 - (\Delta/s)^2]^{3/2} \tag{18'}$$

for the axial and the isotropic states, respectively.

3. LIMITING CASES

In general Eq. (7) [or Eq. (8)] has to be solved for given s. Then, making use of Δ thus determined, we can extract $\phi(s)$ from Eq. (14) [or Eq. (15)], which enables us to calculate $\mathbf{v}_s$ as well as $\mathbf{J}_s$ (or ρ_s/ρ). Besides the zero-temperature case described above, there are a few instances where we have relatively simple expressions, although in general the above program can be excuted only numerically. We will discuss briefly these simple cases.

3.1. $T \ll T_c$

First let us consider the case of ^{3}He-A. Since the quasiparticle spectrum is gapless in ^{3}He-A, we expect that both $\Delta(T)$ and $\phi(s, T)$ can be expanded in some powers of T, the temperature. The gap equation (7) can be transformed as

$$\begin{aligned} 1 &= 2\pi\lambda T \sum_{n=0}^{n_0} \int_{-1}^{1} dz\, \tfrac{3}{4}(1 - z^2) \\ &\quad \times \mathrm{Re}\, \{[(\omega_n - isz)^2 + \Delta^2(1 - z^2)]^{-1/2}\} \\ &= 2\pi\lambda T \sum_{n=0}^{n_0} I_A(\omega_n) \end{aligned} \tag{21}$$

where

$$\begin{aligned} I_A(\omega_n) = \frac{3}{2} \frac{1}{(\Delta^2 + s^2)^{1/2}} \Bigg\{ \left[1 - \frac{1}{2} \frac{\Delta^2}{\Delta^2 + s^2} + \frac{1}{2} \frac{(2s^2 - \Delta^2)\omega_n^2}{(\Delta^2 + s^2)^2} \right] \cot^{-1} \frac{\omega_n}{(\Delta^2 + s^2)^{1/2}} \\ + \frac{1}{2} \frac{\Delta^2 - 2s^2}{(\Delta^2 + s^2)^{3/2}} \omega_n \Bigg\} \end{aligned} \tag{22}$$

Then, making use of the Euler–Maclaurin formula, we can evaluate the right-hand side of Eq. (21) as

$$1 = \lambda \left\{ \ln \frac{\omega_c}{(\Delta^2 + s^2)^{1/2}} + \frac{5}{6} + \frac{1}{2} \frac{1}{\Delta^2 + s^2} \left[s^2 - s^2\eta_+ - \frac{7}{30}(\Delta^2 - s^2)\eta_+^2 + 0\eta_+^3 \right] \right\}$$

or

$$\ln\frac{(\Delta^2+s^2)^{1/2}}{\Delta_{00}}=\frac{1}{2}\frac{1}{\Delta^2+s^2}\left[s^2-s^2\eta_+-\frac{7}{30}(\Delta^2-s^2)\eta_+^2+0\eta_+^3\right]$$

with

$$\eta_\pm=(\pi T)^2/(s^2\pm\Delta^2) \tag{23}$$

[η_- appears in Eqs. (33) and (34)].

Similarly, the expression for ϕ can be transformed as

$$1-\phi(s)=\frac{2\pi T}{s}\sum_{n=0}^{\infty}J_A(\omega_n) \tag{24}$$

with

$$J_A(\omega_n)=\frac{3}{2}\frac{s\Delta^2}{(\Delta^2+s^2)^{5/2}}\left[(\Delta^2+s^2+3\omega_n^2)\right.$$
$$\left.\times\cot^{-1}\left(\frac{\omega_n}{(\Delta^2+s^2)^{1/2}}\right)-3\omega_n(\Delta^2+s^2)^{1/2}\right] \tag{25}$$

The summation over the Matsubara frequency ω_n yields

$$\phi(s)=\frac{s^2}{\Delta^2+s^2}+\frac{\Delta^2}{\Delta^2+s^2}\left(\eta_+-\frac{7}{15}\eta_+^2-\frac{31}{105}\eta_+^3-0\eta_+^4\right) \tag{26}$$

In ^{3}He-B, on the other hand, we have to consider two regions separately at $T\simeq 0$ K. For $\Delta-s>0$, the quasiparticle spectrum has a nonvanishing energy gap and the temperature-dependent corrections are exponentially small; we have for the order parameter and ϕ

$$\Delta\simeq\Delta_{00}\left[1-\left(\frac{2\pi T}{\Delta}\right)^{1/2}\frac{T}{s}\sinh\left(\frac{s}{T}\right)e^{-\Delta/T}\right] \tag{27}$$

and

$$\phi=3\left(\frac{2\pi T}{\Delta}\right)^{1/2}\frac{\Delta T}{s^2}\left[\cosh\left(\frac{s}{T}\right)-\frac{T}{s}\sinh\left(\frac{s}{T}\right)\right]e^{-\Delta/T} \tag{28}$$

In the second regime, $\Delta-s<0$, the energy spectrum is gapless. Then we can exploit a similar method used for deriving Δ and ϕ in ^{3}He-A; Eqs. (8) and (15) reduce to

$$1=2\pi\lambda T\sum_{n=0}^{n_0}I_B(\omega_n) \tag{29}$$

and

$$1-\phi=(2\pi T/s)\sum_{n=0}^{\infty} J_B(\omega_n) \tag{30}$$

with

$$I_B(\omega_n)=\frac{1}{s}\,\mathrm{Im}\left(\sinh^{-1}\frac{is-\omega_n}{\Delta}\right) \tag{31}$$

and

$$J_B(\omega_n)=-\frac{3}{2}\left(\frac{\Delta}{s}\right)^2 \mathrm{Im}\left\{\frac{\omega_n+is}{\Delta}\left[\left(\frac{\omega_n-is}{\Delta}\right)^2+1\right]^{1/2}-\sinh^{-1}\frac{is-\omega_n}{\Delta}\right\} \tag{32}$$

respectively.

Then, making use of the Euler–Maclaurin formula again, we obtain for $T \ll T_c$ (and for $s > \Delta$)

$$-\ln\frac{\Delta_0}{\Delta_{00}}\simeq\cosh^{-1}\left(\frac{s}{\Delta}\right)-\left[1-\left(\frac{\Delta}{s}\right)^2\right]^{1/2}\left[1-\frac{1}{6}\eta_-+\frac{7}{360}\frac{2s^2+\Delta^2}{s^2-\Delta^2}\eta_-^2+0\eta_-^3\right] \tag{33}$$

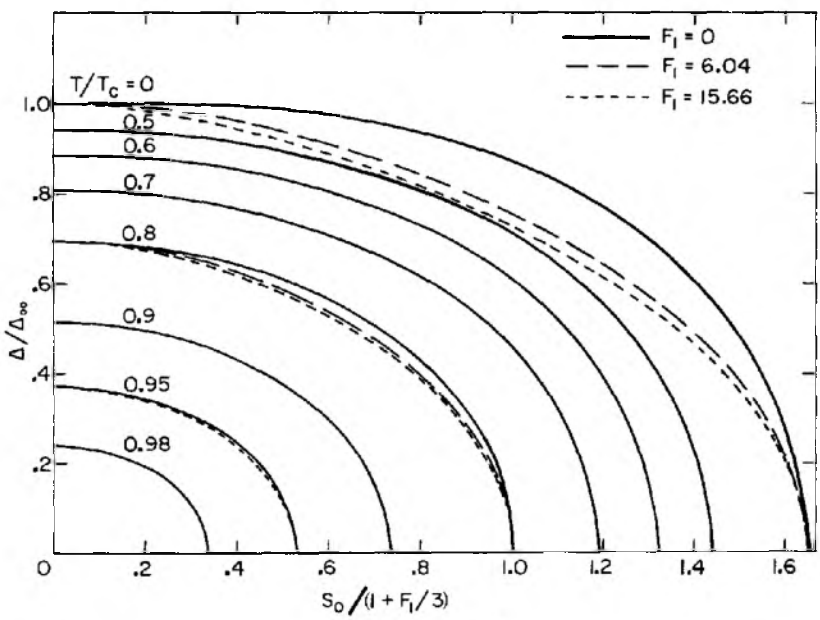


Fig. 1. The order parameter Δ of ^{3}He-A v. $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures. The solid curves are for $F_1=0$ (no Fermi liquid correction). The dashed curves are for $F_1=6.04$ and $F_1=15.66$ with $T/T_c=0$, 0.8, and 0.95.

and

$$\phi=\left[1-\left(\frac{\Delta}{s}\right)^2\right]^{3/2}+\left[1-\left(\frac{\Delta}{s}\right)^2\right]^{1/2}\eta_-\left[1+\frac{7}{60}\frac{4s^2-\Delta^2}{s^2-\Delta^2}\eta_-+0\eta_-^2\right] \quad (34)$$

respectively, where η_- has already been defined in Eq. (23).

In both ^{3}He-A and ^{3}He-B, the temperature-dependent corrections start with T^2, as it should be in the gapless regime.

3.2. $T \simeq T_c$

In the vicinity of the transition temperature (i.e., the Ginzburg–Landau regime), we can expand Eqs. (7), (8), (14), and (15) in powers of Δ, the order parameter. In ^{3}He-A we have then

$$\Delta^2 = \Delta_0^2(T)-\tfrac{1}{2}s^2 \quad (35)$$

and

$$1-\phi=\frac{14}{5}\zeta(3)\left(\frac{\Delta}{2\pi T}\right)^2=\left(1-\frac{T}{T_c}\right)-\frac{7}{5}\zeta(3)\left(\frac{s}{2\pi T_c}\right)^2 \quad (36)$$

where

$$\Delta_0(T)=\left[\frac{10}{7\zeta(3)}\right]^{1/2}\left(1-\frac{T}{T_c}\right)^{1/2}(\pi T_c) \quad (37)$$

is the energy gap in the absence of the current.

Similarly, in ^{3}He-B we have

$$\Delta^2 = \Delta_0^2(T)-\tfrac{2}{3}s^2 \quad (38)$$

and

$$1-\phi=7\zeta(3)\left(\frac{\Delta}{2\pi T}\right)^2=2\left(1-\frac{T}{T_c}\right)-\frac{14}{3}\zeta(3)\left(\frac{s}{2\pi T_c}\right)^2 \quad (39)$$

where

$$\Delta_0(T)=\left[\frac{8}{7\zeta(3)}\right]^{1/2}\left(1-\frac{T}{T_c}\right)^{1/2}(\pi T_c) \quad (40)$$

In a more general situation Eqs. (7), (8), (14), and (15) are evaluated numerically. The results are shown in Figs. 1 and 2 for ^{3}He-A and Figs. 4 and 5 for ^{3}He-B. In Fig. 1 the energy gap Δ (or more precisely the order parameter) is plotted as a function of $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures T/T_c, where $s_0 = v_s p_F/\Delta_{00}$. The solid curves are calculated for $F_1 = 0$ (no Fermi liquid correction) while the results for $F_1 = 6.04$ and

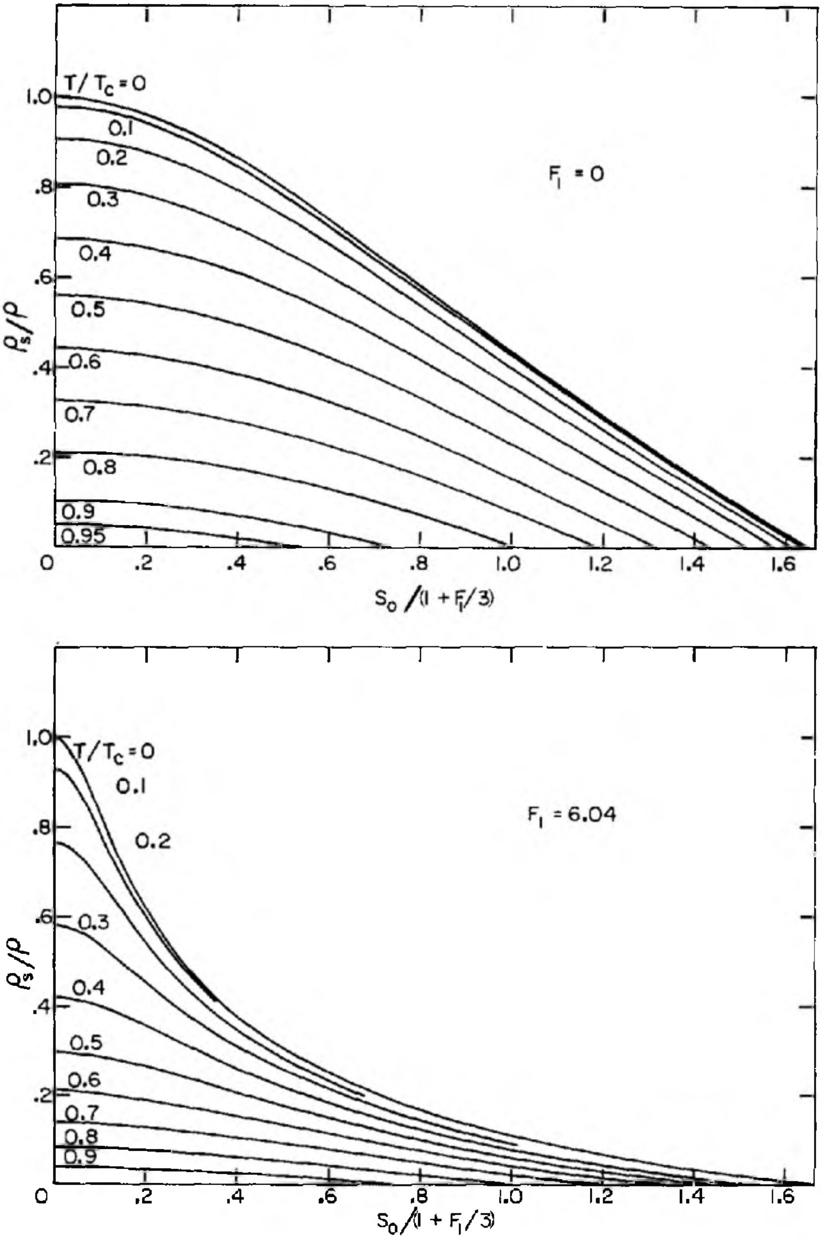


Fig. 2. The superfluid density ρ_s of ^{3}He-A v. $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures and for $F_1=0$ (no Fermi liquid correction), $F_1=6.04$ (corresponding to zero pressure), and $F_1=15.66$ (corresponding to melting pressure).

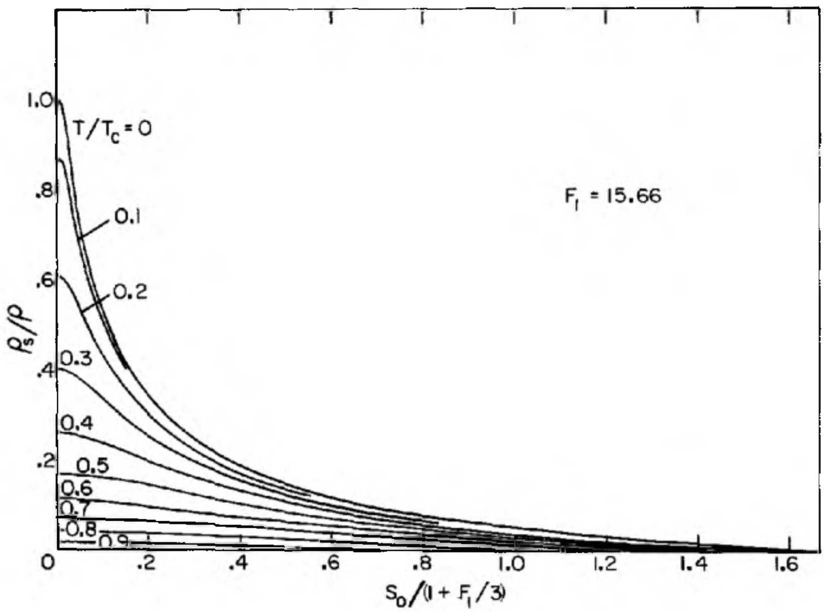


Fig. 2. Continued.

$F_1 = 15.66$ are shown only for $T/T_c = 0$, 0.8, and 0.95 for comparison. The above F_1 correspond to the case $P = 0$ and the case at the melting curve, respectively.[14]

If Δ is plotted against $s_0/(1+\frac{1}{3}F_1)$, the effect of the Fermi liquid correction is significant only at low temperatures. In the high-temperature regime (i.e., $T/T_c > 0.9$) the effect of the Fermi liquid correction can be accounted for by simply replacing $\mathbf{v}_s$ by $\mathbf{v}_s/(1+\frac{1}{3}F_1)$ (i.e., the Fermi liquid correction with $\phi = 1$). In Fig. 2 the superfluid density ρ_s is plotted again as

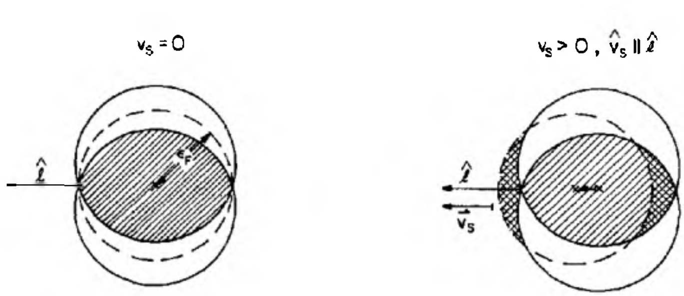


Fig. 3. Anisotropic energy gap of ^{3}He-A in the presence of a current for $v_s = 0$ and $v_s > 0$, respectively.

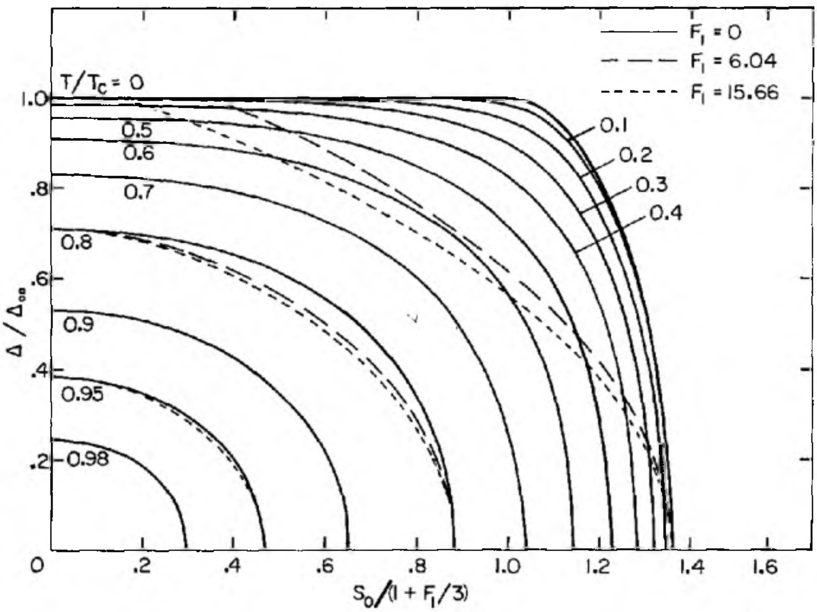


Fig. 4. The order parameter Δ of ^{3}He-B v. $s_0/(1+\frac{1}{3}F_1)$. The solid curves are for $F_1=0$, while the dashed curves are for $F_1=6.04$ and 15.66.

a function of $s_0/(1+\frac{1}{3}F_1)$ for $F_1=0$, 6.04, and 15.66. We can see immediately that the Fermi liquid correction has a significant effect on ρ_s. In the presence of the Fermi liquid correction, ρ_s at lower temperatures decreases rapidly as $s_0/(1+\frac{1}{3}F_1)$ increases. Such a strong nonlinearity in s_0 appears readily accessible experimentally. We note further that Δ and ρ_s vary smoothly as v_s increases. This can be understood by looking at the quasiparticle energy gap of ^{3}He-A in the presence of a uniform current (see Fig. 3). In ^{3}He-A the energy gap is anisotropic and the pair-breaking starts continuously from $v_s=0$, implying continuous reductions of Δ and ρ_s even for very small v_s.

In Fig. 4 the order parameter Δ in ^{3}He-B is shown as a function of $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures. As in Fig. 1, the solid curves are drawn for the case $F_1=0$, while results for $F_1=6.04$ and 15.66 are shown only for $T/T_c=0$, 0.8, and 0.95 for comparison. At low temperatures Δ as a function of s_0 has a sharp break where the system becomes gapless. In such regimes the Fermi liquid correction has a rather significant effect on the v_s dependence of the energy gap. At high temperatures (say $T/T_c>0.9$) the effect of the Fermi liquid correction can be included by reducing $\mathbf{v}_s$ by a factor $(1+\frac{1}{3}F_1)^{-1}$ as in the case of ^{3}He-A. In Fig. 5, the superfluid density ρ_s is shown as a function of $s_0/(1+\frac{1}{3}F_1)$ for $F_1 = 0$, 6.04, and 15.66, respectively.

As in the case of ^{3}He-A, the Fermi liquid correction suppresses ρ_s very rapidly as v_s increases. Compared with the case of ^{3}He-A, Δ and ρ_s stay constant at $T = 0$ until v_s exceeds Δ_{00}/p_F. This is easily seen from Fig. 6. At $T = 0$, as long as $v_s < \Delta_{00}/p_F$, no pair-breaking takes place due to the current. Only when v_s exceeds this critical value are the quasiparticles excited across the energy gap, drastically reducing both Δ and ρ_s.

4. DEPAIRING CRITICAL CURRENT

So far we have considered Δ and ρ_s as functions of v_s. The superflow is given in terms of ρ_s as

$$\mathbf{J}_s = \rho_s \mathbf{v}_s \tag{41}$$

Since ρ_s decreases monotonically as v_s increases, $\mathbf{J}_s$ takes the maximal value for v_s defined by

$$\partial J_s/\partial v_s = 0 \tag{42}$$

where $J_s = |\mathbf{J}_s|$ and $v_s = |\mathbf{v}_s|$. Making use of Eq. (20), we find that Eq. (42) is reduced to

$$d\phi/ds = (1/s)(1-\phi) \tag{43}$$

which determines the critical velocity v_{sc}.

At $T = 0$ K, Eq. (43) is solved explicitly. For ^{3}He-A, Eq. (43) together with Eqs. (9) and (17′) yields

$$\phi = \sqrt{2}-1, \qquad \text{or} \qquad s = (\sqrt{2}-1)^{1/2}\{\exp[\tfrac{1}{2}(\sqrt{2}-1)]\}\Delta_{00} \tag{44}$$

This can be solved for v_{sc} as

$$v_{sc} = [1+\tfrac{1}{3}(\sqrt{2}-1)F_1](\sqrt{2}-1)^{1/2}\{\exp[\tfrac{1}{2}(\sqrt{2}-1)]\}\Delta_{00}/p_F \tag{45}$$

and the critical current

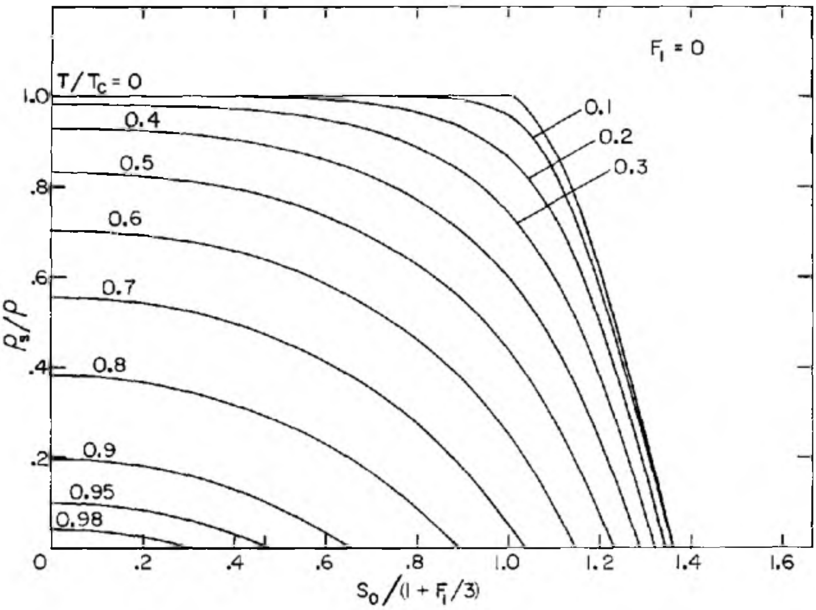
$$J_{sc} = \sqrt{2}(\sqrt{2}-1)^{3/2}\{\exp[\tfrac{1}{2}(\sqrt{2}-1)]\}\Delta_{00}p_F^{-1}\rho \tag{46}$$

It may be of interest to note that the critical current is independent of the Fermi liquid correction, while the critical superflow velocity does depend on F_1. For ^{3}He-B, a similar analysis yields

$$\phi = (2^{1/3}-1)^3, \qquad \text{or} \qquad s = 2^{-1/3}[\exp(2^{1/3}-1)]\Delta_{00} \tag{47}$$

This gives

$$v_{sc} = [1+\tfrac{1}{3}(2^{1/3}-1)^3F_1]2^{-1/3}[\exp(2^{1/3}-1)]\Delta_{00}/p_F \tag{48}$$



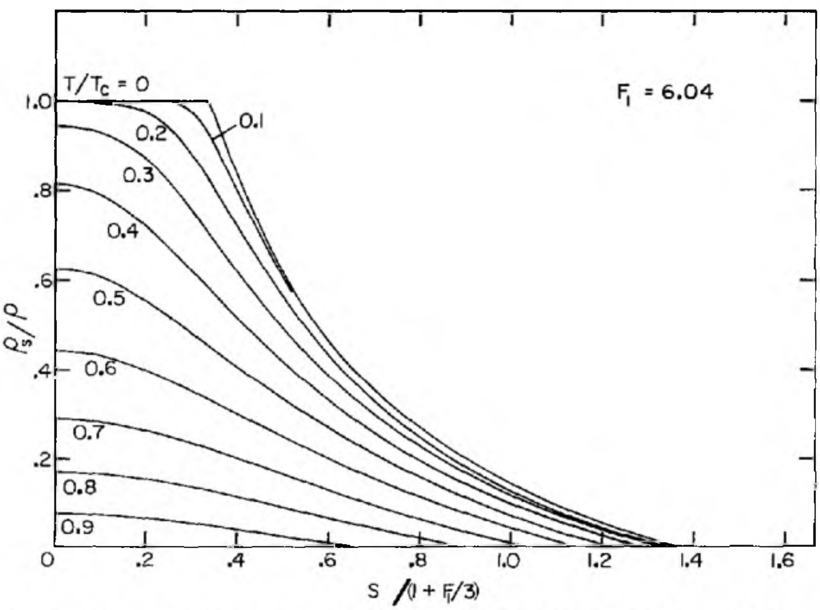


Fig. 5. The superfluid density ρ_s of ^{3}He-B v. $s_0/(1+\frac{1}{3}F_1)$ for $F_1 = 0$, 6.04, and 15.66.

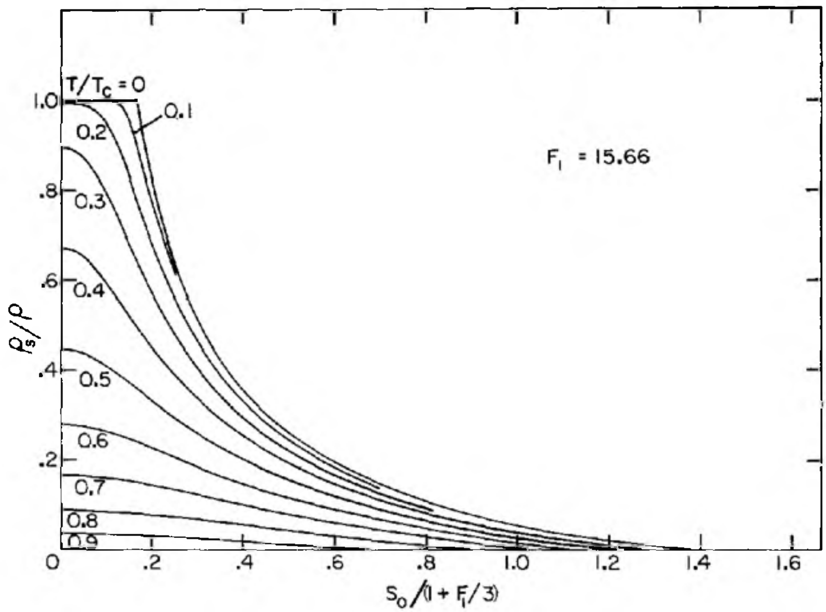


Fig. 5. Continued.

and

$$J_{sc} = [1-(2^{1/3}-1)^3]2^{-1/3}[\exp(2^{1/3}-1)]\Delta_{00}p_F^{-1}\rho \tag{49}$$

Again we find that J_{sc} is independent of the Fermi liquid correction, while v_{sc} depends on F_1.

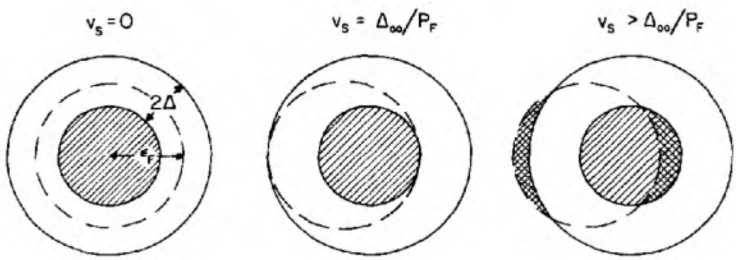


Fig. 6. The energy gap of ^{3}He-B in the presence of current for $v_s = 0$, $v_s = \Delta_{00}/p_F$, and $v_s > \Delta_{00}/p_F$, respectively.

In the vicinity of the transition temperature we can also solve Eq. (43). For ^{3}He-A we have then

$$s = \left(\frac{5}{21\zeta(3)}\right)^{1/2}\left(1-\frac{T}{T_c}\right)^{1/2} 2\pi T_c \tag{50}$$

$$v_{sc} \simeq \left(1+\frac{1}{3}F_1\right)\left(\frac{5}{21\zeta(3)}\right)^{1/2}\left(1-\frac{T}{T_c}\right)^{1/2} \frac{2\pi T_c}{p_F} \tag{51}$$

and

$$J_{sc} = \frac{2}{3}\left(\frac{5}{21\zeta(3)}\right)^{1/2}\left(1-\frac{T}{T_c}\right)^{3/2} \frac{2\pi T_c \rho}{p_F} \tag{52}$$

Similarly, for ^{3}He-B we have

$$s = [7\zeta(3)]^{-1/2}(1-T/T_c)^{1/2} 2\pi T_c \tag{53}$$

$$v_{sc} \simeq (1+\tfrac{1}{3}F_1)[7\zeta(3)]^{-1/2}(1-T/T_c)^{1/2} 2\pi T_c p_F^{-1} \tag{54}$$

and

$$J_{sc} = \tfrac{4}{3}[7\zeta(3)]^{-1/2}(1-T/T_c)^{3/2} 2\pi T_c p_F^{-1} \rho \tag{55}$$

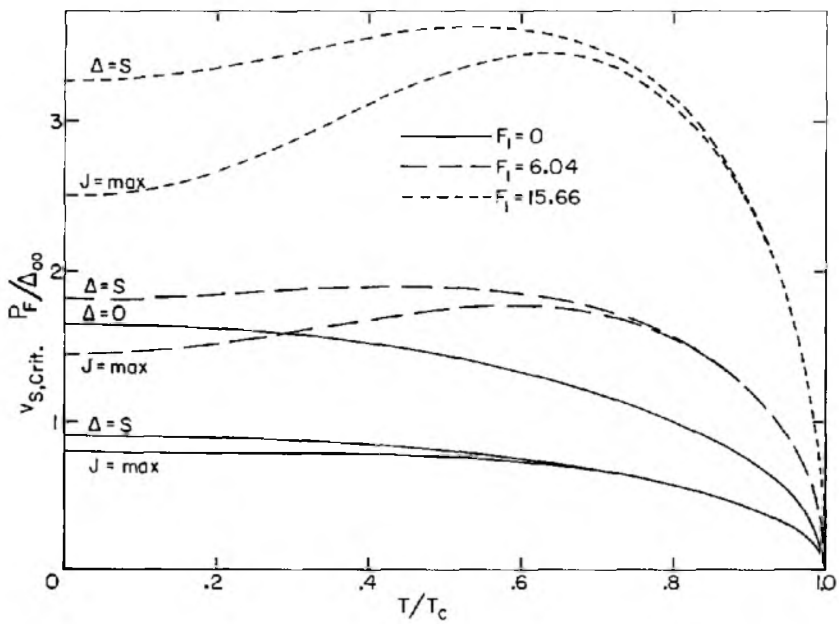


Fig. 7. Critical velocities of ^{3}He-A defined by (1) $J_s = \max$, (2) $v_s = \Delta p_F^{-1}$, and (3) $\Delta = 0$, as functions of the reduced temperature T/T_c. The solid curves are for $F_1 = 0$, while the dashed curves are for $F_1 = 6.04$ and 15.66.

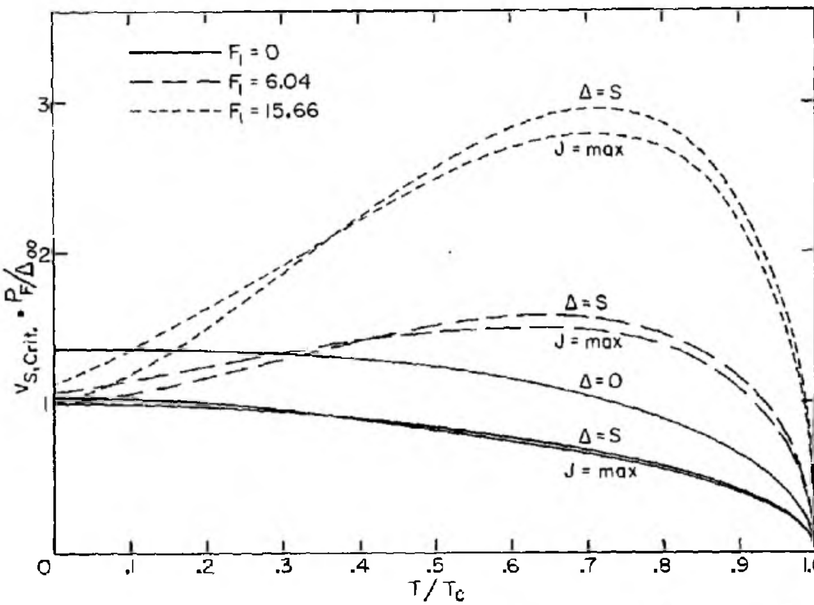


Fig. 8. Critical velocities of ³He-B defined by (1) $J_s = \max$, (2) $v_s = \Delta p_F^{-1}$, and (3) $\Delta = 0$. The solid curves are for $F_1 = 0$, while the dashed curves are for $F_1 = 6.04$ and 15.66.

At intermediate temperatures the critical velocity is evaluated numerically for ³He-A and ³He-B, and is shown in Figs. 7 and 8, respectively. In both figures we have included other critical velocities: the pair-breaking critical velocity, defined as v_s corresponding to $\Delta = s$, and another critical velocity corresponding to $\Delta = 0$. The latter critical velocity is shown only for $F_1 = 0$, as $\Delta = 0$ implies $\phi = 1$ and consequently the corresponding critical velocity for nonvanishing F_1 is obtained by multiplying $(1 + \frac{1}{3}F_1)$ by the one for $F_1 = 0$. The former critical velocity behaves very similarly to the depairing critical velocity (for which J_s has a maximum), although physically this critical velocity (at least in ³He-B) corresponds to the velocity where the pair creation of quasiparticles begins to take place. The latter critical velocity may be completely unaccessible, although the velocity gives the instability limit of the normal liquid against formation of the superfluid condensate. In Figs. 7 and 8 we include the critical depairing velocities in the presence of the Fermi liquid correction. In the presence of a large Fermi liquid correction v_{sc} is no longer monotonic but has a broad peak around $T/T_c = 0.56$ and $T/T_c = 0.7$ for ³He-A and ³He-B, respectively. This unusal behavior of the critical velocity is uniquely due to the Fermi liquid correction, since, as we see from these figures, v_{sc} is monotonic for $F_1 = 0$. We would like to stress here that although the critical velocity behaves nonmonotonically in the presence of the Fermi liquid correction, the critical current is independent of the Fermi liquid correction (i.e., F_1) and is a monotonic function of the temperature.

As already mentioned in the introduction, we think that this effect could probably be seen in the ion mobility experiment, although the published results[13] on the ion mobility do not indicate this unusual temperature dependence.

5. CONCLUDING REMARKS

Making use of the Green's function technique, we have calculated the order parameter Δ and the superfluid density ρ_s in the presence of a large current ($v_s \simeq 10$ cm/sec) of ^{3}He-A and ^{3}He-B within the weak coupling model.

We have shown that ρ_s is strongly nonlinear in $\mathbf{v}_s$ in both ^{3}He-A and ^{3}He-B due to the large Fermi liquid correction. We have also found that the critical velocity is a nonmonotonic function of temperature, which should have a variety of interesting consequences in the behavior of superfluid ^{3}He with a large, uniform current.

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