Flow Dependence of Spin Susceptibilities in Superfluid ^{3}He*

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The flow dependence of spin susceptibilities in superfluid ^{3}He-A and -B is studied theoretically. It is shown that in the A phase $\chi_{A\parallel}$, the component of the static spin susceptibility parallel to $\hat{\mathbf{d}}$, increases rapidly in the presence of mass current. A similar behavior is found for all components of χ_B, the susceptibility tensor of the B phase.

1. INTRODUCTION

In a previous paper[1] we studied the superfluid density and the order parameter of superfluid ^{3}He in the presence of a uniform current. It was shown that the effect of the mass current is rather significant, primarily due to the large Fermi liquid correction. The object of this paper is to study the effect of the current on other physical quantities. The effect appears to be most accessible in the case of the spin (or magnetic) susceptibilities. In superfluid ^{3}He-A in order to measure the flow-dependent susceptibility a special device is required, as under normal conditions only $\chi_{A\perp}$ (the component perpendicular to $\hat{\mathbf{d}}$) can be determined,[2] which is practically the same as the normal susceptibility χ_n. (Note that in the absence of external constraint the $\hat{\mathbf{d}}$ vector is perpendicular to the static magnetic field in the equilibrium configuration.) However, in the presence of a current parallel to a weak external magnetic field (i.e., the anisotropic magnetic energy has to be smaller than the dipolar interaction energy, which forces the $\hat{\mathbf{d}}$ vector parallel to the $\hat{\boldsymbol{l}}$ vector—the flow direction), it is in principle possible to measure $\chi_{A\parallel}$. The latter susceptibility depends very sensitively on the flow. In superfluid ^{3}He-B, on the other hand, it is shown that the spin susceptibility becomes anisotropic in the presence of a mass current. The considerable current dependence as well as the anisotropy in the spin susceptibility appear to be readily accessible.

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2. FORMULATION

In order to calculate the spin susceptibility, it is most convenient to express it in terms of Green's functions. The effects of a uniform current are then easily incorporated by replacing the Green's functions by those in the presence of a uniform current. For the moment let us neglect the Fermi liquid corrections. Then the static spin susceptibility is in general a tensor and is given by

$$(\boldsymbol{\chi}^0)_{ij} = -\langle [s_i, s_j] \rangle = -\frac{T}{4}\sum_n \int \frac{d^3p}{(2\pi)^3} \operatorname{Tr}[\alpha_i G(\mathbf{p}, \omega_n)\alpha_j G(\mathbf{p}, \omega_n)] \tag{1}$$

where $\boldsymbol{\alpha} = (\sigma_1\rho_3, \sigma_2, \sigma_3\rho_3)$ are the spin operators and $G(\mathbf{p}, \omega_n)$ is the single-particle Green's function in the Nambu representation. For a general spin triplet p-wave condensate the Green's function in the absence of current is given by

$$G^{-1}(\mathbf{p}, \omega_n) = i\omega_n - \xi\rho_3 - \sigma_2(\rho_1\boldsymbol{\Delta}_1 + \rho_1\boldsymbol{\Delta}_2)\cdot\boldsymbol{\alpha} \tag{2}$$

where

$$\xi = \frac{p^2}{2m} - \mu, \qquad (\boldsymbol{\Delta}_1 + i\boldsymbol{\Delta}_2)_i = A_{ij}\hat{\mathbf{p}}_j, \qquad \hat{\mathbf{p}}_j = \mathbf{p}_j/p_{\mathrm{F}} \tag{3}$$

and A_{ij} is the nine-component complex order parameter describing the triplet p-wave condensate.

The expression (1) is further simplified. First, replacing $d^3p/(2\pi)^3$ by $N(0)(d\Omega/4\pi)\, d\xi$, where $N(0)$ is the density of states at the Fermi level (for one spin component), we can integrate over ξ. The trace sum is then easily worked out to give

$$(\boldsymbol{\chi}^0)_{ij} = N(0)\left\{\delta_{ij} - \int \frac{d\Omega}{4\pi} \operatorname{Re} \hat{\boldsymbol{\Delta}}_i(\hat{\mathbf{p}})\hat{\boldsymbol{\Delta}}_j^*(\hat{\mathbf{p}})[1 - \tilde{Y}(T, \hat{\mathbf{p}})]\right\} \tag{4}$$

where

$$\tilde{Y}(T, \hat{\mathbf{p}}) = \frac{\beta}{2}\int_0^\infty d\xi\, \operatorname{sech}^2\left[\frac{\beta}{2}E(\hat{\mathbf{p}})\right] = 1 - 2\pi T \sum_{n=0}^{\infty} \frac{\Delta^2(\hat{\mathbf{p}})}{[\omega_n^2 + \Delta^2(\hat{\mathbf{p}})]^{3/2}} \tag{5}$$

is the Yosida function and

$$\beta = 1/T, \qquad E(\hat{\mathbf{p}}) = [\xi^2 + \Delta^2(\hat{\mathbf{p}})]^{1/2}, \qquad \hat{\boldsymbol{\Delta}}_i(\hat{\mathbf{p}}) = \boldsymbol{\Delta}_i(\hat{\mathbf{p}})/\Delta(\hat{\mathbf{p}}) \tag{6}$$

So far we have neglected the effects of a uniform current. As already shown elsewhere, the effects of the current (still without Fermi liquid corrections) are incorporated by shifting $i\omega_n$ by $i\omega_n + v_s p_{\mathrm{F}} \cos\theta$, where θ is the angle between $\mathbf{v}_s$, the superfluid velocity, and the quasiparticle momentum and p_{F} is the Fermi momentum. (Here we assume that $\mathbf{v}_s$ lies in the z direction).

In order to include the Fermi liquid corrections, which have been neglected up to now, we proceed in two steps as follows:

(1) v_s in the Green's functions has to be replaced by $v_s^* = v_s(1+\frac{1}{3}F_1\phi)^{-1} \equiv s/p_F$. Here F_1 is the usual Landau parameter and ϕ is defined by $\rho_s^0/\rho \equiv 1-\phi(s)$, where ρ_s^0 is determined by the expression for the superfluid current $\mathbf{J}_s$, $\mathbf{J}_s = \rho_s^0 \mathbf{v}_s^*$, and ρ is the mass density of liquid ^{3}He.

(2) The susceptibility $(\boldsymbol{\chi}^0)_{ij}$ then has to be replaced by[3]

$$(\boldsymbol{\chi})_{ij} = (1+\tfrac{1}{4}Z_0)\{\boldsymbol{\chi}^0(s)/[1+\tfrac{1}{4}Z_0 N(0)^{-1}\boldsymbol{\chi}^0(s)]\}_{ij} \tag{7}$$

where $\frac{1}{4}Z_0 = F_0^a$ is the s-wave Landau coefficient for the asymmetric component.

$(\boldsymbol{\chi}^0)_{ij}$ is still given by Eq. (4), but $\tilde{Y}(T, \hat{\mathbf{p}})$ has now, because of step 1, been substituted by the (current-dependent) generalized Yosida function:

$$\begin{aligned}\tilde{Y}(s, T, \hat{\mathbf{p}}) &= \tfrac{1}{2}\beta \int_0^\infty d\xi \,\operatorname{sech}^2 \{\tfrac{1}{2}\beta[E(\hat{\mathbf{p}})+s\cos\theta]\} \\ &= 1-2\pi T \sum_{n=0}^{\infty} \operatorname{Re} \frac{\Delta^2(\hat{\mathbf{p}})}{[(\omega_n - is\cos\theta)^2+\Delta^2(\hat{\mathbf{p}})]^{3/2}}\end{aligned} \tag{8}$$

2.1. ^{3}He-A

First let us consider the spin susceptibility in ^{3}He-A. In ^{3}He-A, $\boldsymbol{\Delta}_1 + i\boldsymbol{\Delta}_2$ is given by

$$\boldsymbol{\Delta} = \boldsymbol{\Delta}_1 + i\boldsymbol{\Delta}_2 = \Delta(\hat{\mathbf{p}}_1 + i\hat{\mathbf{p}}_2)\mathbf{e}_3 \tag{9}$$

where we have assumed both the $\hat{\mathbf{d}}$ and $\hat{\boldsymbol{l}}$ vectors to point to the z direction, parallel to the current. Here $\hat{\mathbf{d}}$ and $\hat{\boldsymbol{l}}$ are unit vectors designating the spin component and the symmetry axis of the energy gap of the condensate of ^{3}He-A, respectively. Since the condensate in ^{3}He-A has only the one spin component parallel to $\hat{\mathbf{d}}$, Eq. (4) in the presence of current reduces to

$$(\boldsymbol{\chi}_A^0)_{ij} = N(0)[\delta_{ij} - \hat{\mathbf{d}}_i\hat{\mathbf{d}}_j(1-Y_A)] \tag{10}$$

where

$$Y_A = \int \frac{d\Omega}{4\pi} \tilde{Y}_A(s, T, \hat{\mathbf{p}})$$

and

$$\tilde{Y}_A = \tfrac{1}{2}\beta \int_0^\infty d\xi \,\operatorname{sech}^2\{\tfrac{1}{2}\beta[(\xi^2+\Delta^2\sin^2\theta)^{1/2}+s\cos\theta]\} \tag{11a}$$

$$= 1-2\pi T \sum_{n=0}^{\infty} \operatorname{Re} \frac{\Delta^2 \sin^2\theta}{[(\omega_n - is\cos\theta)^2+\Delta^2\sin^2\theta]^{3/2}} \tag{11b}$$

Equation (10) tells us that $(\boldsymbol{\chi}_{\rm A}^0)_{ij}$ has two diagonal components

$$\chi_{\rm A\perp}^0 = N(0) \qquad \text{and} \qquad \chi_{\rm A\parallel}^0 = N(0) Y_{\rm A} \tag{12}$$

where $\chi_{\rm A\perp}^0$ is equal to the normal liquid value. $\chi_{\rm A\perp}^0$ is the susceptibility perpendicular to the $\hat{\mathbf{d}}$ vector, whereas $\chi_{\rm A\parallel}^0$ is that parallel to $\hat{\mathbf{d}}$. In the absence of any external constraint, $\hat{\mathbf{d}}$ is perpendicular to the external field $\mathbf{H}$, thus minimizing the magnetic anisotropic energy. Therefore under ordinary conditions only $\chi_{\rm A\perp}^0$ is accessible. However, in the presence of a large current, $\hat{\boldsymbol{l}}$ is aligned parallel to the flow direction $\mathbf{v}_s$. Then, due to the nuclear dipole interaction, $\hat{\mathbf{d}}$ is forced into the direction parallel to $\hat{\boldsymbol{l}}$. In this situation, if a weak magnetic field parallel to $\hat{\mathbf{d}}$ is used, it is in principle possible to measure $\chi_{\rm A\parallel}^0$. Making use of Eq. (7), we obtain

$$\chi_{\rm A\perp} = \chi_n \qquad \text{and} \qquad \chi_{\rm A\parallel}G = \chi_n \frac{(1+\frac{1}{4}Z_0)Y_{\rm A}}{1+\frac{1}{4}Z_0 Y_{\rm A}} \tag{13}$$

The parallel susceptibility $\chi_{\rm A\parallel}$ decreases rapidly as the temperature decreases.

2.2. ^{3}He-B

In ^{3}He-B the static susceptibility tensor $(\boldsymbol{\chi}_{\rm B}^0)_{ij}$ is again given by Eq. (4), where the order parameter is now[4]

$$A_{ij} = \frac{\Delta}{\sqrt{3}} R_{ij} = \frac{\Delta}{\sqrt{3}}[(\cos\Theta)\delta_{ij} + (1-\cos\Theta)n_i n_j + \varepsilon_{ijk} n_k \sin\Theta] \tag{14}$$

and $\mathbf{n}$ and Θ are the axis of the rotation and the rotation angle of the spin components from the orbital components in the Balian–Werthamer condensate.[5] Under normal conditions the condensate is at the minimum of the dipolar interaction energy, which implies $\Theta = \Theta_0[=\cos^{-1}(-\frac{1}{4})]$. With this general A_{ij}, Eq. (4) reduces to

$$(\boldsymbol{\chi}_{\rm B}^0)_{ij} = N(0)\left\{\delta_{ij} - R_{ik}R_{jl}\int \frac{d\Omega}{4\pi}\hat{\mathbf{p}}_k\hat{\mathbf{p}}_l[1-\tilde{Y}_{\rm B}(s,T,\hat{\mathbf{p}})]\right\} \tag{15}$$

with

$$\tilde{Y}_{\rm B}(s,T,\hat{\mathbf{p}}) = \tfrac{1}{2}\beta\int_0^\infty d\xi\,\text{sech}^2\{\tfrac{1}{2}\beta[(\xi^2+\Delta^2)^{1/2}+s\cos\theta]\} \tag{16a}$$

$$= 1 - 2\pi T\sum_{n=0}^{\infty}\text{Re}\frac{\Delta^2}{[(\omega_n - is\cos\theta)^2+\Delta^2]^{3/2}} \tag{16b}$$

In the presence of a relatively large current, it is likely that $\mathbf{n}$ is aligned parallel to $\mathbf{v}_s$ in an open geometry. Then, since $\tilde{Y}_{\rm B}$ has azimuthal symmetry, Eq. (15) can be decomposed into two components:

$$\chi^0_{B\perp} = N(0)\left[\frac{2}{3} + \int \frac{d\Omega}{4\pi}\,\frac{1}{2}(\sin^2\theta)\,\tilde{Y}_B(s, T, \hat{\mathbf{p}})\right] \tag{17}$$

and

$$\chi^0_{B\|} = N(0)\left\{1 - \int \frac{d\Omega}{4\pi}(\cos^2\theta)[1 - \tilde{Y}_B(s, T, \hat{\mathbf{p}})]\right\}$$

$$= N(0)\left[\frac{2}{3} + \int \frac{d\Omega}{4\pi}(\cos^2\theta)\,\tilde{Y}_B(s, T, \hat{\mathbf{p}})\right] \tag{18}$$

respectively. Here $\chi^0_{B\|}$ is the component parallel to **n** and $\chi^0_{B\perp}$ is that perpendicular to **n**.

By means of Eq. (7) we obtain

$$\chi_{B\perp} = (1 + \tfrac{1}{4}Z_0)\{\chi^0_{B\perp}/[1 + \tfrac{1}{4}Z_0(\chi^0_{B\perp}/\chi_n)]\} \tag{19}$$

and

$$\chi_{B\|} = (1 + \tfrac{1}{4}Z_0)\{\chi^0_{B\|}/[1 + \tfrac{1}{4}Z_0(\chi^0_{B\|}/\chi_n)]\} \tag{20}$$

3. LIMITING BEHAVIORS

If we confine ourselves to limiting cases, the integrals reduce to simple expressions; however, to obtain results for the whole temperature range, numerical methods have to be employed.

3.1. $T = 0$ K

At $T = 0$ K we can replace the frequency summation by an integral over n. Then Eqs. (12), (17), and (18), by means of Eqs. (11b) and (16b), reduce to

$$\frac{\chi^0_{A\|}}{\chi_n} = \mathrm{Re}\int \frac{d\Omega}{4\pi}\,\frac{s|\cos\theta|}{[(s^2+\Delta^2)\cos^2\theta - \Delta^2]^{1/2}} = \frac{s^2}{s^2+\Delta^2} \tag{21}$$

$$\frac{\chi^0_{B\perp}}{\chi_n} = \frac{2}{3} + \int \frac{d\Omega}{4\pi}\,\frac{1}{2}(\sin^2\theta)\,\mathrm{Re}\frac{s|\cos\theta|}{(s^2\cos^2\theta - \Delta^2)^{1/2}}$$

$$= \frac{2}{3} + \frac{1}{3}\theta(s-\Delta)\left[1 - \left(\frac{\Delta}{s}\right)^2\right]^{3/2} \tag{22}$$

$$\frac{\chi^0_{B\|}}{\chi_n} = \frac{2}{3} + \frac{1}{3}\theta(s-\Delta)\left[1 + 2\left(\frac{\Delta}{s}\right)^2\right]\left[1 - \left(\frac{\Delta}{s}\right)^2\right]^{1/2} \tag{23}$$

where $\theta(x)$ is the step function

$$\theta(x)=1 \qquad \text{for} \quad x\geq 0$$
$$=0 \qquad \text{for} \quad x<0$$

From these integrals we see immediately[1] that

$$\chi^0_{A\parallel}/\chi_n = 1-\rho^0_s/\rho \qquad \text{in} \quad {}^3\text{He-A}$$

and

$$\chi^0_{B\perp}/\chi_n = 1-\tfrac{1}{3}\rho^0_s/\rho \qquad \text{in} \quad {}^3\text{He-B}$$

at least at $T=0$ K, where ρ^0_s/ρ is again defined by the superfluid current $\mathbf{J}_s$, $\mathbf{J}_s=\rho^0_s\mathbf{v}^*_s$, ρ^0_s/ρ being the reduced superfluid density in the case of $F_1=0$.

3.2. $T \ll T_c$

At low temperatures (compared to T_c), we can carry out the angular integral of Eq. (4) first; then we have

$$\chi^0_{A\parallel}/\chi_n = 1-2\pi T\sum_{n=0}^{\infty} J_A(\omega_n) \tag{24}$$

with

$$J_A(\omega_n)=\frac{\Delta^2}{(\Delta^2+s^2)^{3/2}}\left[\cot^{-1}\left(\frac{\omega_n}{(\Delta^2+s^2)^{1/2}}\right)-\frac{\omega_n(\Delta^2+s^2)^{1/2}}{(\Delta^2+s^2+\omega_n^2)}\right] \tag{25}$$

Then the frequency summation is carried out by making use of the Euler–Maclaurin expansion, so that

$$\frac{\chi^0_{A\parallel}}{\chi_n}=\frac{s^2}{\Delta^2+s^2}+\frac{1}{3}\frac{\Delta^2}{\Delta^2+s^2}\eta_+\left(1+\frac{7}{15}\eta_+ +\frac{31}{35}\eta_+^2+0\eta_+^3\right) \tag{26}$$

with $\eta_+=(\pi T)^2/(s^2+\Delta^2)$.

For ^{3}He-B in most of the region of interest the energy gap is nonvanishing. We make use of Eq. (16a) and find the temperature-dependent corrections to be exponentially small:

$$\frac{\chi^0_{B\perp}}{\chi_n}=\frac{2}{3}+\frac{\beta}{2}\int_{-1}^{1}\frac{dz}{2}\,\frac{1-z^2}{2}\int_0^{\infty}d\xi\,\operatorname{sech}^2\left[\frac{\beta}{2}(E+sz)\right]$$

$$\approx\frac{2}{3}+3\left(\frac{2\pi T}{\Delta}\right)^{1/2}\frac{\Delta T}{s^2}[\cosh(\beta s)-(\beta s)^{-1}\sinh(\beta s)]\,e^{-\beta\Delta} \tag{27}$$

and

$$\frac{\chi^0_{B\parallel}}{\chi_n} = \frac{2}{3} + \frac{\beta}{2}\int_{-1}^{1}\frac{dz}{2}z^2\int_0^\infty d\xi\,\text{sech}^2\left[\frac{\beta}{2}(E+sz)\right]$$
$$\approx \frac{2}{3} + 3\left(\frac{2\pi T}{\Delta}\right)^{1/2}\frac{\Delta T}{s^2}\{[2(\beta s)^{-1}+\beta s]\sinh(\beta s) - 2\cosh(\beta s)\}\,e^{-\beta\Delta} \quad (28)$$

with $\beta = T^{-1}$. In the above derivation it is assumed that $s \leq \Delta$.

3.3. $T \approx T_c$

Finally, in the vicinity of the transition temperature we can expand Eqs. (11b) and (16b) in powers of Δ^2. Keeping only the lower order terms, we obtain

$$\frac{\chi^0_{A\parallel}}{\chi_n} = 1 - \frac{14}{3}\zeta(3)\left(\frac{\Delta}{2\pi T}\right)^2 + \frac{124}{5}\zeta(5)\left(\frac{\Delta}{2\pi T}\right)^2\left[\left(\frac{s}{2\pi T}\right)^2 + \left(\frac{\Delta}{2\pi T}\right)^2\right] \quad (29)$$

and

$$\frac{\chi^0_{B\perp}}{\chi_n} = 1 - \frac{7}{3}\zeta(3)\left(\frac{\Delta}{2\pi T}\right)^2 + 31\zeta(5)\left(\frac{\Delta}{2\pi T}\right)^2\left[\frac{2}{5}\left(\frac{s}{2\pi T}\right)^2 + \frac{1}{2}\left(\frac{\Delta}{2\pi T}\right)^2\right] \quad (30)$$

$$\frac{\chi^0_{B\parallel}}{\chi_n} = 1 - \frac{7}{3}\zeta(3)\left(\frac{\Delta}{2\pi T}\right)^2 + 31\zeta(5)\left(\frac{\Delta}{2\pi T}\right)^2\left[\frac{6}{5}\left(\frac{s}{2\pi T}\right)^2 + \frac{1}{2}\left(\frac{\Delta}{2\pi T}\right)^2\right] \quad (31)$$

where, to lowest order in s, the order parameters are given by $\Delta^2 = \Delta_0^2(T) - \frac{1}{2}s^2$ and $\Delta^2 = \Delta_0^2(T) - \frac{2}{3}s^2$ for the axial and the isotropic states, respectively.

We can see from the foregoing results that $\chi^0_{B\perp} \leq \chi^0_{B\parallel}$, which in fact is true for all temperatures $0 \leq T \leq T_c$.

4. DISCUSSION

The static susceptibilities $\chi_{A\parallel}$, $\chi_{B\perp}$, $\chi_{B\parallel}$ are evaluated numerically and shown in Figs. 1–3. In Fig. 1 the static spin susceptibility $\chi_{A\parallel}/\chi_n$ of ^{3}He-A is plotted as a function of $s_0/(1+\frac{1}{3}F_1)$ for various reduced temperatures T/T_c, where $s_0 = v_s p_F/\Delta(s=0, T=0)$. Figure 1a shows the results for $F_1 = 0$, $Z_0 = 0$ (no Fermi liquid corrections), while in Figs. 1b and 1c Fermi liquid corrections have been included: in Fig. 1b, $F_1 = 6.04$, $Z_0 = -2.69$ and in Fig. 1c, $F_1 = 15.66$, $Z_0 = -2.95$. The above F_1 and Z_0 correspond to $P = 0$ (no pressure) and melting pressure, respectively.[6] The dashed curves represent the critical velocities. In Figs. 2 and 3 the static spin susceptibilities $\chi_{B\perp}/\chi_n$ and $\chi_{B\parallel}/\chi_n$ are shown, plotted in the same way as $\chi_{A\parallel}/\chi_n$. We only represent results for flow velocities smaller than the critical velocity since for a higher

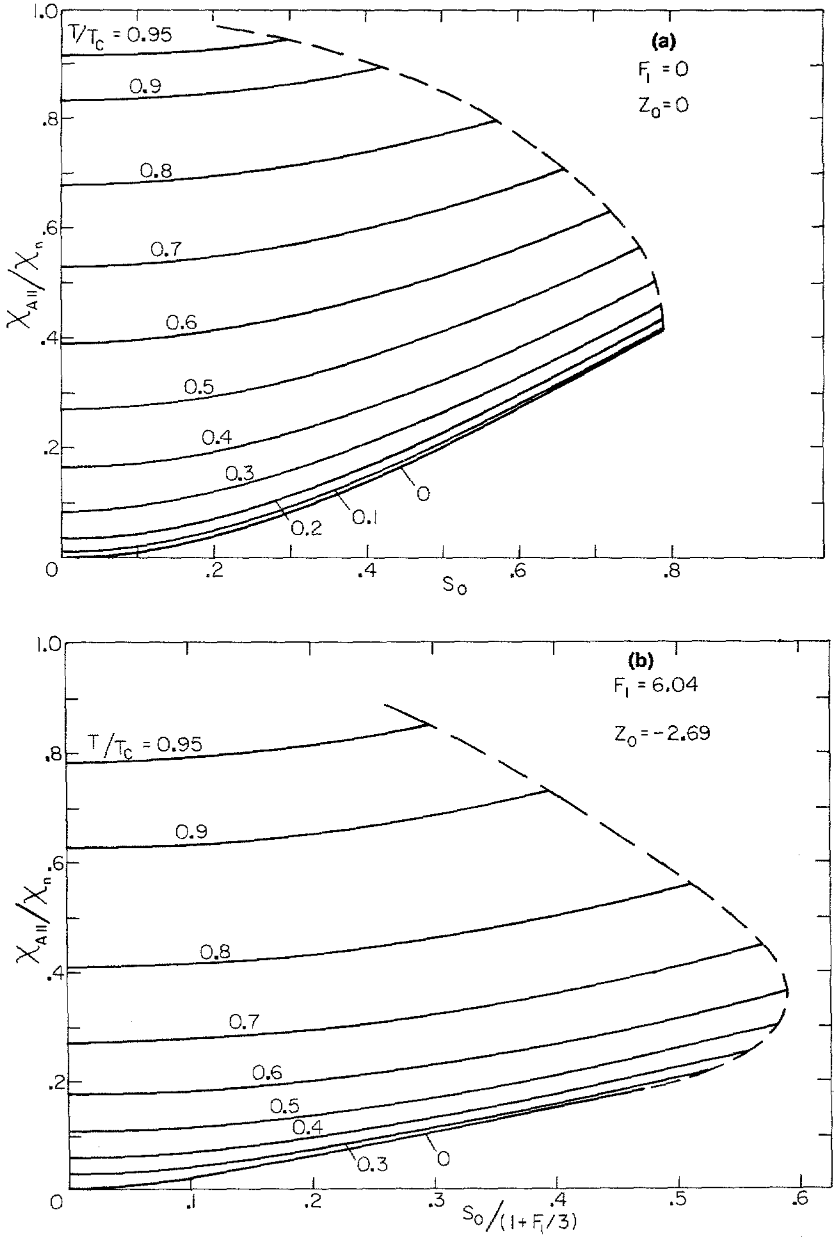

Fig. 1. The static spin susceptibility $\chi_{A\parallel}/\chi_n$ of ^{3}He-A vs. $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures. (a) $F_1=0$, $Z_0=0$ (no Fermi liquid corrections). (b) $F_1=6.04$, $Z_0=-2.69$ (corresponding to zero pressure). (c) $F_1=15.66$, $Z_0=-2.95$ (corresponding to melting pressure). The dashed curves show the critical velocities.

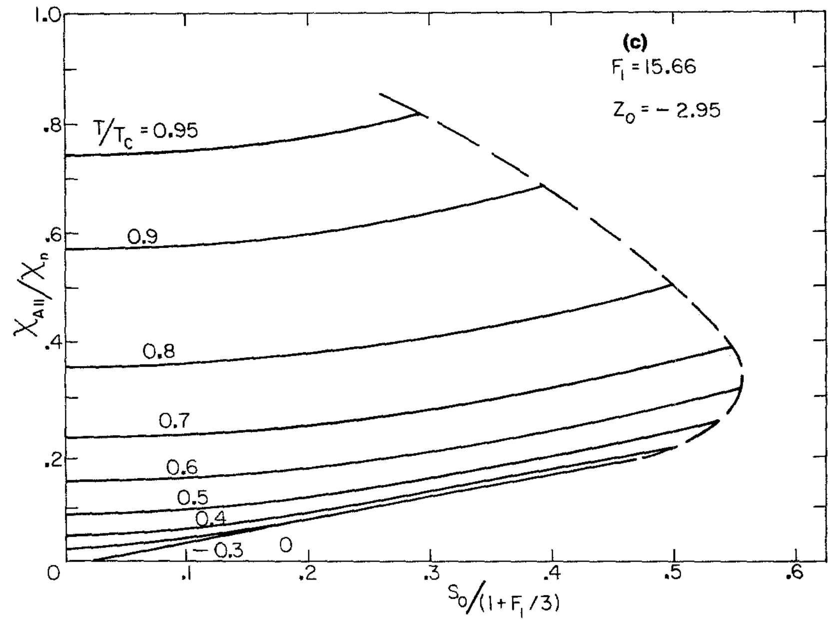

Fig. 1. Continued.

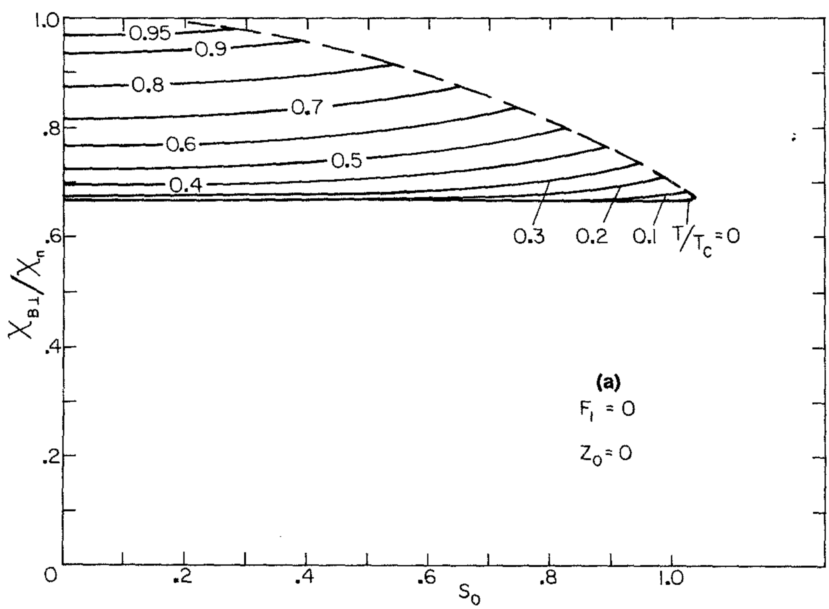

Fig. 2. The static spin susceptibility $\chi_{B\perp}/\chi_n$ of ^{3}He-B vs. $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures. (a) $F_1=0$, $Z_0=0$. (b) $F_1=6.04$, $Z_0=-2.69$. (c) $F_1=15.66$, $Z_0=-2.95$. The dashed curves show the critical velocities.

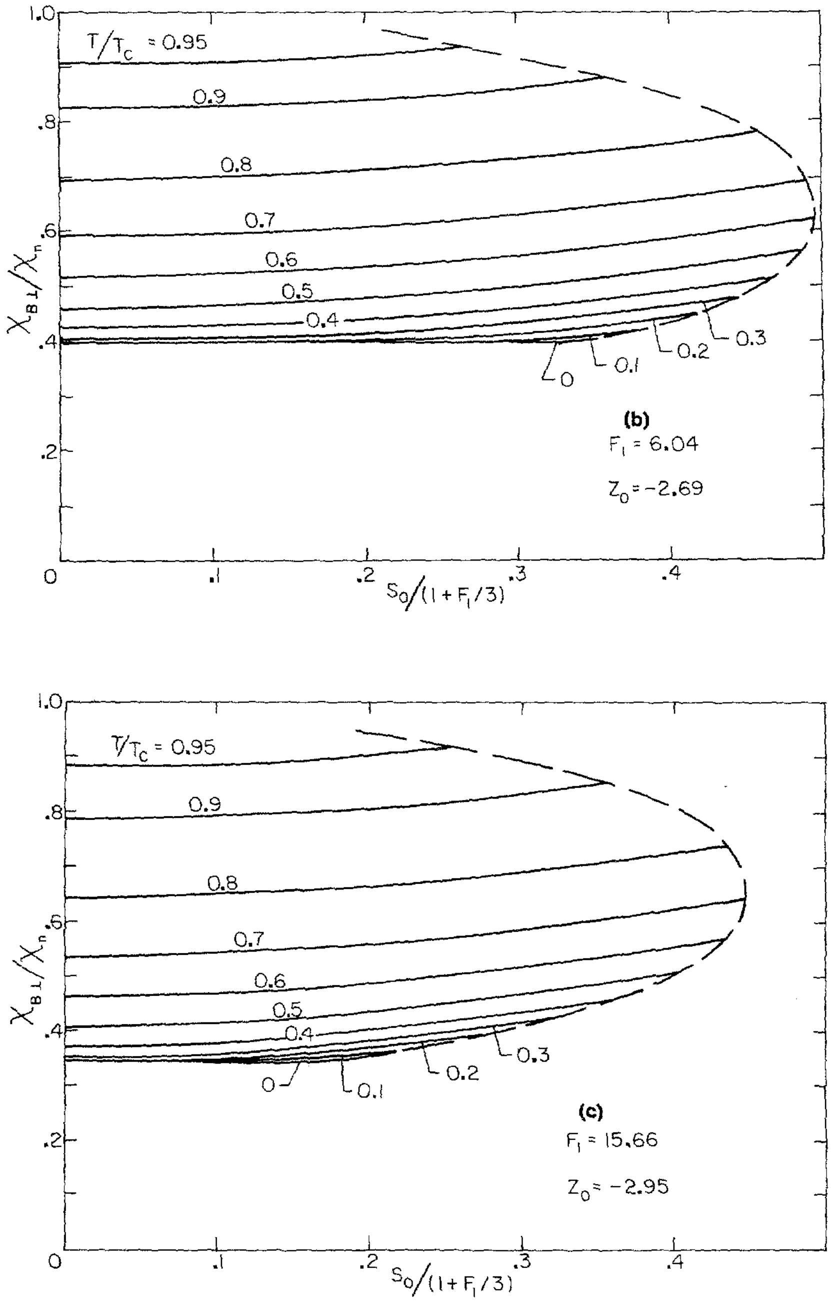

Fig. 2. Continued.

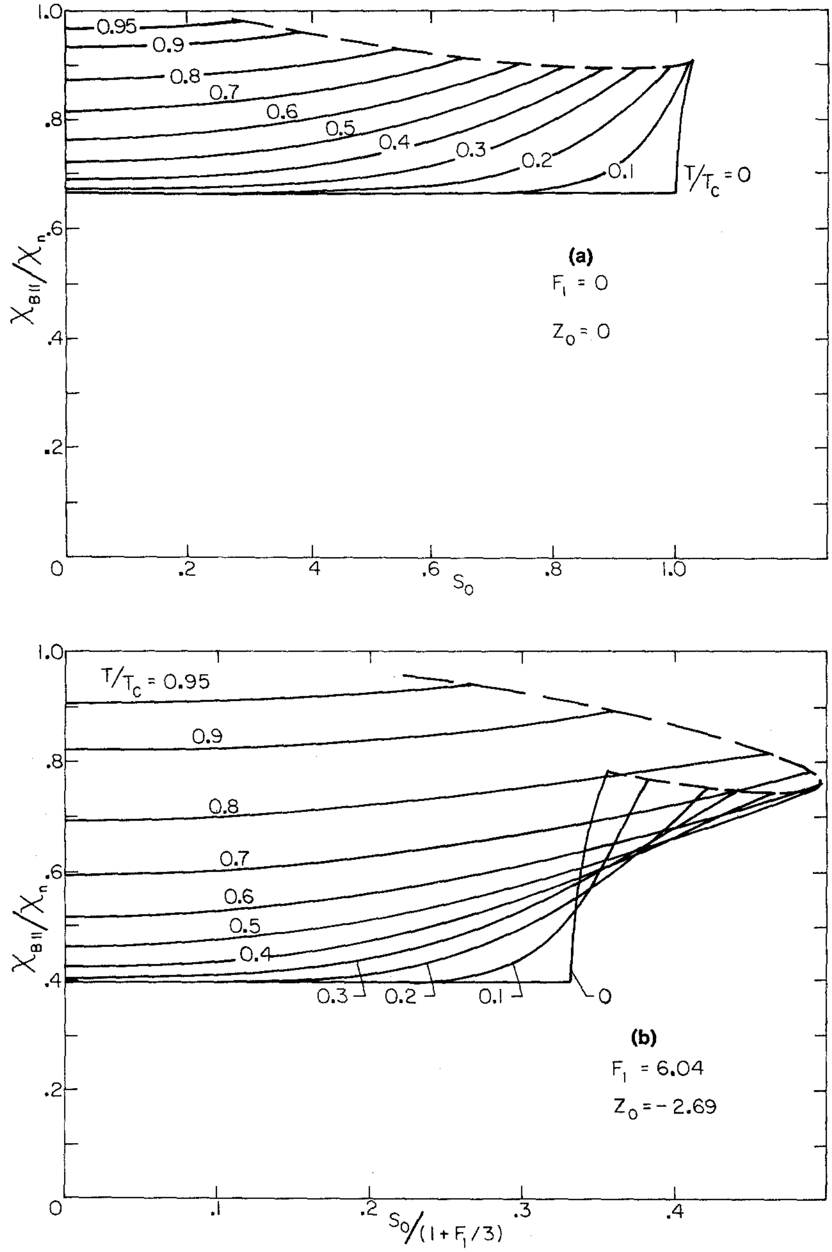

Fig. 3. The static spin susceptibility $\chi_{B\|}/\chi_n$ of ^{3}He-B vs. $s_0/(1+\frac{1}{3}F_1)$ for several reduced temperatures. (a) $F_1=0$, $Z_0=0$. (b) $F_1=6.04$, $Z_0=-2.69$. (c) $F_1=15.66$, $Z_0=-2.95$. The dashed curves show the critical velocities.

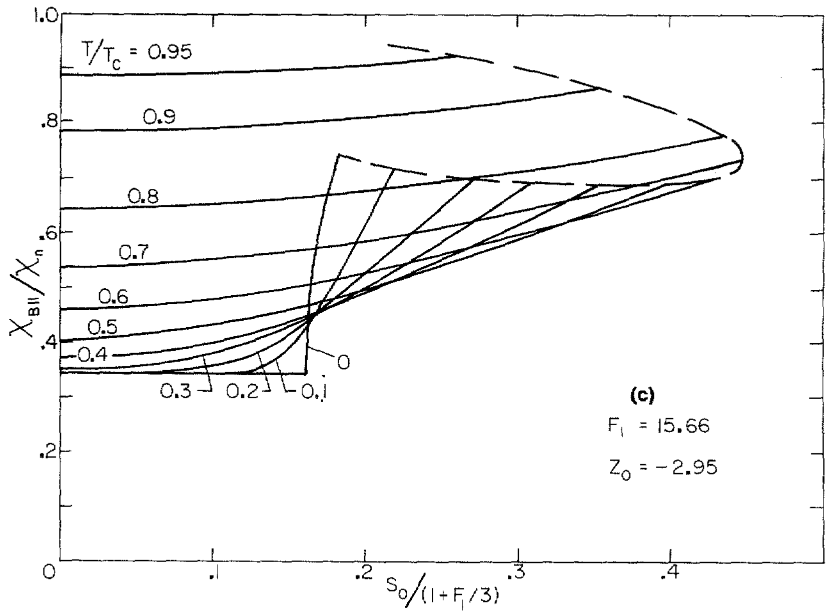

Fig. 3. Continued.

superfluid velocity we do not expect a homogeneous situation. As in the case of the superfluid density, the Fermi liquid corrections are very important.

In superfluid ^{3}He-B, we may think of two typical experimental setups, one where the external magnetic field is parallel and another where it is perpendicular to the flow direction $\mathbf{v}_s$. When the external field is parallel to $\mathbf{v}_s$, the measured susceptibility is $\chi_{B\parallel}$, since in this situation the $\mathbf{n}$ vector is parallel to the external field (the rotation of the spin coordinate by an angle Θ_0 does not change the situation). When the external field is perpendicular to the flow (the field being weak enough so that the anisotropic magnetic energy is smaller than the dipolar interaction energy, which forces $\hat{\mathbf{n}}$ and $\mathbf{H}$ to be parallel), then the $\mathbf{n}$ vector is perpendicular to the external field, and the rotation of the spin coordinate again mixes the perpendicular components only. Therefore in this case the measured susceptibility should be $\chi_{B\perp}$.

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