No country for old distributions?
On the comparison of implied option parameters between the Brownian motion and variance gamma process

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Abstract

Advanced stochastic approaches are often suggested as a solution to real-world derivative pricing inconsistencies like the non-linearity of the implied volatility smile. Using a novel high-frequency data set with over one million option trades and corresponding order books from the German market, we compare the normal distribution approach with a variance gamma process, which is – as a pure jump process – especially suitable for tick-by-tick data. We are able to report a flattened implied volatility smile with the variance gamma process. Other low-frequency results like time, information, and underlying moment dependencies for both stochastic processes are unchanged. All in-sample residuals of the normal distribution have a smaller variance below the 1% significance level compared to the variance gamma process. Additionally, we reveal a mean-reversion process. We show that the normal distribution is superior to the variance gamma process in an out-of-sample context and conclude that – even in a high-frequency environment – it is still "a country for old distributions".

Keywords: Implied Volatility Smile, Implied Volatility Determinants, Lévy Process, Variance Gamma Process, Buyer-/ Seller-Motivated Trades

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1. Introduction

In the past years a lot of work has been done on new and complex models for stock price modeling and option pricing. These models mostly incorporate real-world phenomena that are not addressed by the classical normal distribution assumption like higher moments and jumps in underlying prices (Kon, 1984). These models achieve superior in-sample fits due to more degrees of freedom. At the same time, there is a long history in the analysis of different determinants that drive option prices. First and foremost, the moneyness smile is such an example, often explained by the influence of higher moments and tail risks being visible in the implied volatility calculated under the normal distribution assumption. Here, the link between the distribution assumptions and the determinants of the implied parameters can be seen, giving rise to a research question: If determinants influence option prices in a classic Black-Scholes setting and modern distribution approaches apply, can we observe a reduced influence from certain determinants or even that the influence vanishes completely?

The phalanx of proposed distributions forms a long line, all with different strengths and weaknesses. Despite the variety of approaches in literature, e.g.: wavelets (Ortiz-Gracia and Oosterlee, 2013), GARCH option pricing with conditional volatility (Byun and Min, 2013), a Cornish–Fisher option pricing model (Aboura and Maillard, 2016), or even neural networks (Cao et al., 2020), the class of Lévy processes have emerged as a common spearhead for option pricing. They have become very popular in Finance in the last few years (e.g., Geman et al., 2001; Carr et al., 2003; Cont and Tankov, 2015). In short, a stochastic process with independent and stationary increments is called a Lévy process (Schoutens, 2005). It has a typical characteristic function and allows for two additional features when compared to the normal distribution, which is essential to our research. Firstly, it is possible to incorporate higher moments like skewness and kurtosis. This means that tail risks can be accounted for. Secondly, this enables modeling price jumps, which are very useful for high-frequency tick data and minimum tick sizes. Therefore, in our particular case we use the well established variance gamma process introduced by Madan and Seneta (1990) to combine ease of use with the application of context-related features. As Lévy processes seem superior in both fitting real movements of the underlyings and pricing real options (e.g., Madan et al., 1998), we ponder the question, is the normal distribution part of an "old-world" and thus outdated and in need of replacing, or do Lévy processes only promise a good in-sample fit, but ultimately lead to estimation problems, instability, computational intensity, and a poor out-of-sample fit, as they introduce many degrees of freedom (Figlewski, 1997).

It is interesting to examine how the influences of individual parameters on the implicit parameters change. Therefore, we briefly review this line of research next. On the front line of the implied volatility determinants, four groups can be categorized: (1) momentum-, (2) time-, (3) liquidity- or information-related, and (4) miscellaneous. Provided one of these interrelation exists, the analysis should show a weakening of the momentum-related effects along with the same results as previously for the other categories. Papers that evaluate deterministic implied volatility functions particularly analyze possible determinants of the implied volatility. Besides the classical first four underlying moments (return, standard deviation, skewness, and kurtosis), we summarize under the first category measurement of historical returns as motivated by Mixon (2002) and Peña et al. (1999) and the smile component. As second category the well-known extension from a simple smile to a volatility surface opens the door to the time-related determinants, including several dummies. For the third category, we follow Bollen and Whaley (2004), who analyze the implication of net buying pressures (NBP) and information flow within a lagged first difference analysis and add other liquidity and risk measures. Lastly, we contribute to the literature by introducing a dummy variable, to control for the order book side (buyer-/seller motivated trade) and is a useful measure for insights in the market microstructure.

To perform a comprehensive study on the influence of the determinants of the implied parameters under different distribution assumptions, we use a novel high-frequency option dataset with trades and corresponding order book data. Existing literature is mostly limited to low-frequency, end of day data or narrow time windows. From the contributions to the strand of implied volatility mentioned above only Wallmeier and Hafner (2001), Wallmeier (2015) and to some extent, Corrado and Su (1997) use a sub-5-minute frequency for their central models. Even if some others also have reported a high-frequency dataset like bid-ask intra-day prices, they are not mentioned as high-frequency literature, because after transforming and filtering the data to e.g., a daily frequency, they use low-frequency for their analysis. This state of affairs does not reflect the rising importance of high-frequency trading and data. The choice of the variance gamma process, which is a pure jump process, is particularly suited for high-frequency tick-by-tick data. Due to minimum tick requirements, such data consists only of jumps and should not be assumed to be continuously distributed. With this dataset, can we additionally open our paper to the possibility that we may end up with superior knowledge, thus contradicting the market efficiency hypothesis? This is especially relevant in the context of high-frequency traders, who often base their strategy on this kind of information. We are interested in this topic as it is one of the most fundamental questions in finance. The relevance is made clear, not least because of the 2013 Nobel Prize for Eugene Fama, Lars Hansen, and Robert Shiller. With this, it is noteworthy to mention that non-normally distributed returns are not incompatible to efficient markets (Samuelson, 2016). Much work has been done on the question of how efficient markets are. The topic is still discussed with some evidence for (among others Fama et al. (1969); Muntermann and Guettler (2007); Welch and Goyal (2008); Fama and French (2010)) and against efficient markets (e.g., Chan (1992); Foster and Viswanathan (1993); Hong et al. (2000); Kirilenko et al. (2017)). So far, the literature analyzes mainly long-term effects within days, weeks, or years. However, for markets to be truly efficient, the hypothesis must also hold with high-frequency data. Besides addressing the market efficiency question in the context of high-frequency trading, our dataset – covering over one million DAX equity option trades of the year 2012 on a microsecond base and corresponding order book data – is used to construct further determinants, like a dummy variable for buyer-/seller motivated trades.

We contribute to the literature by showing that novel distributions in a high-frequency context cover momentum-related effects quite well at the cost of being more prone to changes in other determinants and leaving a ragged image when it comes to stability and out-of-sample errors. Our results are in line with the previous low-frequency literature. The new order book based parameters like clustering and controlling for buyer-/seller-motivated trades support the novel approach and give new insights. We show that the implied volatility follows a mean-reversion process, even if the order book side is controlled for. Although we have a very good fit, we find no sign of violation of the market efficiency hypothesis and no limits to arbitrage in an algorithmic trading scenario.

With this novel high-frequency data, the existing literature – in particular in the market microstructure field – can be challenged further in subsequent studies. This paper is an essential step in the context of high-frequency data and Lévy processes and is structured as follows. In section 2, we present the research design, such as the regression-based moneyness and time-dependent foundation model, adoptions with further incorporated determinants (normal model), and a delta approach. Section 3 follows with the novel dataset, consisting of XETRA and EUREX tick data with corresponding order books and further information. We analyze the results for the different regression models (delta, normal) separately and present the out-of-sample results in section 4. In sector 5, we discuss the results in the context of the previous literature. The last sector provides a conclusion and an outlook for further research.

2. Theoretical background and research design

Our empirical analysis is based on the comparison of underlying prices fitted by a straight forward normal distribution and an advanced jump process, which can cope with real world high-frequency non-continuous tick data. Correspondingly the implied volatility under the normal distribution assumption of dividend-paying American style options is calculated using an adapted version of Barone-Adesi and Whaley (1987) (BAW) (see appendix C.4). For the variance gamma process (VG), a Lévy-process, the parameters (σ, θ, and ν) are obtained using a fast Fourier transformation and convolution properties following the work of Lord et al. (2008) and Kienitz and Wetterau (2012) (see appendix C.5).
To analyze how different features like moneyness influence these implied volatilities and other higher implied moments, we make use of deterministic volatility models, which have already been used to describe the implied volatility as a function. Based on the literature, we use two approaches to analyze and discuss the impact of different distribution assumptions in detail in a high-frequency setting. Hereby, we follow the idea of Peña et al. (1999), who explain the implied parameter directly (normal model), as well as Bollen and Whaley (2004), who use a first difference approach (delta model). As a foundation for our panel regressions, we use a volatility surface, as described in section C.1 in the appendix. In this case, we compare the smile approaches of Peña et al. (1999), Wallmeier (2015), and Wallmeier and Hafner (2001) with an additional time-to-maturity parameter. While adding flexibility increases the fitting quality of the in-sample model, the important out-of-sample performance suffers. Therefore, and in line with Dumas et al. (1998), we restrict our foundation model to be less complex with at most a single time parameter. To choose which specification fits best, the AIC, AICc, and BIC are analyzed. All tested specifications with corresponding AIC values can be found in chapter C.1 in the appendix. Peña et al. (1999), and Tanha and Dempsey (2015) review the parameters of the daily estimated individual smile components. This approach does not make use of high-frequency data and decouples any intra-day relationships, which is why we do not elaborate this further. However, for the sake of completeness, we also analyzed this procedure without any remarkable gains in knowledge. It is only worth mentioning that the parameters show a cyclical pattern relative to the rolling or expiration days, as seen in figure B.2 in the appendix. Noteworthy at this point is the approach of Chen et al. (2016), who model the implied volatility surface with a nonlinear Kalman filter using moneyness and time as parameters. However, this results in a far more complex dependency structure, which does not allow easy comparison of the influence of the individual determinants within the BAW and VG setting – not to mention the additional computational complexity of real tick-data, which we use. We estimate the models separately for puts and calls, and cluster in buyer- and seller-motivated trades as well, as for example, puts and calls could have a different dependency structure on certain determinants (Dash, 2019). All in all, as we are interested in the potential direct shift of determinant influences on implied volatility, we use the following regression models to explain the implied volatility or change in implied volatility directly.

2.1. Normal model

Inspired by the approach of Peña et al. (1999), we first develop a model to explain the implied volatility directly, thus allowing review of determinants using trade-by-trade data. As a foundation for the models we also determine if adding an autocorrelation process is suitable. The optimal lag length is analyzed using information criteria (see table A.4 in the appendix) and a one-sided F-test (see table A.5). Thereby, we find that an AR(1)-process fits best. Based on these insights, we apply a so-called 'normal model'

$$\sigma(t) = \beta_0 + \beta_1 \cdot \sigma(t-1) + \beta \cdot Determinants + \varepsilon, \tag{1}$$

where $\sigma(t)$ is an individual implied parameter (volatility, skewness and kurtosis, respectively), and $\sigma(t-1)$ is the preceding implied parameter from the same option and the same day. All determinants are derived from literature and explained in detail in section 2.3. For our main results, we conduct a panel regression with fixed-effects (FE) for every individual underlying, whereby we challenged the FE-panel against a random-effects model. As a result of this, firm-clustered errors are used. For robustness checks, we develop additional normal model variations (see the appendix C.2, equations 20, 21, and 22)

2.2. Delta model

Besides a general potential decrease of determinant dependency using a VG process measured with the normal model, a decreased influence on variations in the determinants could also be seen. In the prominent case of moneyness (which is one determinant of many), we could ask if a moneyness shift has as severe an effect on implied volatility calculated using the VG-process as on one calculated based on BAW. To measure the change within the implied volatility, we use the change of the implied volatility between two observations. This approach can also be found in the literature. Therefore, we introduce a delta approach based on Bollen and Whaley (2004), whereby we use our model foundation as a starting point again and add an autocorrelation term. The optimal lag length is determined in line with the normal model with information criteria and a one-sided F-test (see table A.6 and A.7 in the appendix). The delta model is defined by

$$\Delta\sigma(t) = \beta_0 + \beta_1 \cdot \Delta\sigma(t-1) + \beta \cdot Determinants + \varepsilon, \tag{2}$$

where $\Delta\sigma(t) = \sigma(t) - \sigma(t-1)$. Again, we also utilize additional specifications of the delta model for robustness (see the appendix C.3, equations 23 and 24).

Consequently, in contrast to other approaches that model the volatility surface dynamic and evolution (Chen et al., 2016), we analyze the dependency on variations of the determinants (e.g.: moneyness) with the delta model. This allows us to compare a potential change of these dependencies between implied volatilities calculated based on the normal distribution and the VG process.

2.3. Determinants

The reviewed parameters are derived from literature and based on economic reasoning, whereby we use our model foundation with the volatility surface and the AR(1)-process with the addition of the specification by Bollen and Whaley (2004) as a starting point. We contribute to the literature by additionally introducing new determinants, which cover the unique high-frequency order book data (e.g., buyer-/seller-motivated trades) as well as dummies for days relative to the time to maturity.

For reasons of reader-friendliness, we group the determinants into (1) momentum-, (2) time-, (3) liquidity- or information-related and (4) miscellaneous. Ergo, a change in the distribution assumption should only affect the momentum-related determinants.

Within the momentum-category (1) fall the classical first four underlying moments (return, standard deviation, skewness and kurtosis), the history of returns measure of Peña et al. (1999), and the smile component. Thereby, we compute the underlying return as the log-return between the closing price of the previous day and the underlying price during the option trade. Peña et al. (1999) define a variable to capture the information of return history as

$$\text{momentum effects / history of returns} = \log \frac{\frac{1}{60}\sum_{\tau=t-1}^{t-60} Price_\tau}{Price_t}, \tag{3}$$

where the option price of day t is given as $Price_t$. We estimate the underlying standard deviation, skewness, and kurtosis within a 60-minute window on underlying minute-by-minute return data, to fulfill the high-frequency approach and simultaneously avoid issues due to unequal time intervals. Additionally, we test a 60-day window of end-of-day returns for higher underlying moments. Wherever possible, trade-by-trade based parameters are used. Bollerslev (2006) determines a negative effect of underlying returns on implied volatility. This is explained by a more greatly leveraged and, therefore, riskier firm if the prices go down (leverage effect) or by a price reduction in the market if volatility goes up (volatility feedback hypothesis). Especially for deep-in and deep-out-of-the-money options the skewness is negatively and the kurtosis positively correlated with the implied volatility (Corrado and Su, 1997). The smile shape over the moneyness is covered by Peña et al. (1999), who use a linear smirk and quadratic smile component and review the role of different determinants on daily smiles. Their smile model is backed theoretically by Zhang and Xiang (2008), who highlight that the implied volatility depends on a general level, on a linear (slope), and on a squared (curvature) moneyness. Furthermore, Wallmeier (2015) fits smiles and expands the smile function with a spline to the power of three.

When moving forward from the volatility smile to the volatility surface, we dive into the second group of time-related determinants. Dumas et al. (1998) report different models with time parameters and conclude that while time is essential simpler models should be favored, especially regarding out-of-sample performance. They use index options and estimate the volatility function once every week. Moreover, Ncube (1996) analyzes the time to maturity by using daily data and additionally points out that volatility is higher for put options in general. Based on this, we use the (square root of) time to maturity in days in our normal model and the (square root of) time between two trades in seconds for the delta approach. A weekly pattern is pointed out by Jones and Shemesh (2018) and Peña et al. (1999). In the style of these approaches and selection criteria of e.g., Day and Lewis (1992), we extend the analysis and add dummy measures for the last trading day and shortest-term options.

Besides the parameters for the implied volatility surface, we review important liquidity and information flow measures in the third category. Thence, we subsume trade and volume-based variables for the underlying and options. Bollen and Whaley (2004) analyze the implication of net buying pressure (NBP) on different moneyness categories for daily closing prices using a first difference regression. The daily calculated information parameter NBP is defined for a single underlying, option style, and moneyness category (see table A.3 in the appendix) as

$$\begin{aligned} \text{NBP}_{cat}^{put/call} = \sum_{Trades} & \frac{\Delta_{opt} \cdot Vol_{opt} \cdot D_{cat} \cdot D_{put/call} \cdot D_{buy}}{\Delta_{opt} \cdot Vol_{opt} \cdot D_{put/call}} - \\ & \frac{\Delta_{opt} \cdot Vol_{opt} \cdot D_{cat} \cdot D_{put/call} \cdot (1 - D_{buy})}{\Delta_{opt} \cdot Vol_{opt} \cdot D_{put/call}}, \end{aligned} \tag{4}$$

where Δ_{opt} is the option delta and Vol_{opt} the corresponding volume. The dummy variables are D_{cat}, which equals one if the trade matches the specified moneyness category, $D_{put/call}$, which equals one if the traded option matches the specified contract type, and D_{buy} as defined below. For the NBP, all moneyness categories (separated and combined), as well as option styles (put and call), are checked. As a control variable for information flow, Bollen and Whaley (2004) apply daily trade volume of the underlying. Besides trade volume, Longstaff (1995) notes that transaction costs and liquidity play a crucial role. He reviews these determinants with a regression model using data of option prices and bid-ask quotations. With this, we subsume trade and volume-based variables for the underlying and options. We calculate another information criterion – the relative bid-ask spread – as the difference between the respective top book entries divided by the mean bid-ask price, whereby the results are similar to the normal bid-ask spread and therefore not reported separately. The last information parameter (round trip costs) is computed as relative (compared to the hypothetical investment amount) costs to buy and sell an (delta adjusted) amount of the underlying considering the order book depth and volume of each entry. We review different investment amounts (1,000, 10,000, and 100,000 euros) with similar results for all.

Closely linked to plain volume and liquidity is the flow toxicity measure of Easley et al. (2012). They introduce an estimated volume-synchronized probability of informed trading (VPIN), to measure flow toxicity as a potential risk management tool for market makers. Volume-based, it is specifically designed for a high-frequency tick-by-tick context. Compared to regularly traded index and crude oil futures (Easley et al., 2012) or stock prices (Pöppe et al., 2016; Abad et al., 2018) our data consists of sparsely traded options, with several option series per underlying, which is why we need to adjust the proposed buy and sell volume estimates. Nonetheless, we strive to calculate a VPIN for each underlying. The basic idea is that each respective volume is allocated based on the likelihood that it is a buy or sell order. Thereby, the likelihood is measured as a trade price change from the previous traded volume bucket to the current. This is exactly why we cannot merge different option series with different option premiums easily. A possible workaround would be to calculate implied spot prices; however, for this procedure we would need an estimate of the implied volatility itself. This is not applicable in our case; therefore, we apply a different methodology. First, we compute for every trade i within the individual option series the likelihood of a buy (L_i^B) and the delta adjusted volume (V_i). The buy and sell likelihood (L_i^B and L_i^S) for a call is computed as

$$L_i^B = 1 - L_i^S = Z\left(\frac{P_i - P_{i-1}}{\sigma_{\Delta P}}\right), \tag{5}$$

where P_i is the trade price, and $\sigma_{\Delta P}^2 = \frac{1}{I-1}\sum(P_i - P_{i-1})^2$ with I as the total number of trades. For puts the buy and sell likelihoods reverse. Please note that the first trade for every option series cannot be included as we need a first P_{i-1}. After this procedure, the data is now comparable across each underlying and sorted by time. The trading day is divided into n equal volume buckets V_Υ, to calculate the buy and sell volume (V_Υ^B and V_Υ^S) for every bucket as

$$V_\Upsilon^{B/S} = \sum_{i \in \Upsilon} L_i^{B/S} \cdot V_i. \tag{6}$$

The daily VPIN is then simply the absolute difference of the buy and sell volume divided by the total volume:

$$VPIN = \frac{\sum_{\Upsilon=1}^{n} |V_\Upsilon^S - V_\Upsilon^B|}{nV_\Upsilon}. \tag{7}$$

We utilize each trade (which might be split by the volume bucket) to make use of the high-frequency context. We choose for each underlying ten equal volume buckets for each day, taking into consideration the option volume relative to the underlying volume as well as a potential intraday information shift. However, we also have tested different bucket sizes without change in the general results. As the VPIN measure and the indirect classification in buyer- and seller-motivated trades is very much disputed in the literature (Andersen and Bondarenko, 2014b; Easley et al., 2014; Andersen and Bondarenko, 2014a) we use both NPB and VPIN.

Within the fourth category, we add three measures. First, a dummy to control for the order book side (buyer-/seller motivated trade) is a useful measure for insights in the market microstructure. The buyer-/seller-motivated trade dummy (D_{buy}) equals one if the trade price is above the mean bid-ask price (buyer-motivated) and zero otherwise (seller-motivated). The volatility itself, and consequently the implied volatilities are viewed as risk measures (e.g., Slim et al., 2020); thus, it is of no surprise that Dennis and Mayhew (2000) find a clear positive relationship between the daily beta factor of the CAPM and the implied volatility. To calculate the CAPM beta, we use the DAX as the market benchmark. Lastly, the volatility is highly autocorrelated (Wallmeier and Hafner, 2001). Therefore, we use lagged implied volatility, analogous to Bollen and Whaley (2004).

For all daily measures (e.g., beta of CAPM), we apply a time window of 60 days following Peña et al. (1999). Other time frame specifications are tested but provide similar results. As mentioned above, the determinants to be used are selected based on economic reasoning and information criteria. The picked determinants are benchmarked against factors selected by applying the lasso method following Tibshirani (1996). The F-test results show that the manually-picked factors perform significantly better than the lasso picked, both out and in-sample and in panel as well as in company-individual regressions.

Table 1 gives an overview of the finally reviewed determinants, their expected influence ($+$ denotes a positive relation and $-$ a negative, respectively) and literature references.

To conclude the research design, it should be noted that we apply Cook's distance for all regressions to exclude outliers and use the $\frac{4}{n}$ recommendation of Bollen and Jackman (1985) as cut off criteria, whereby n is the number of observations. Furthermore, we exclude all second moments larger than 1 as well as relative pricing errors larger than 1%, as this could hint at bad convergence of our pricing models. To evaluate for the pricing error, we first fit the implied parameters (e.g., implied volatility for the normal distribution) based on the traded option prices and then use the implied parameters directly without any further transformation to calculate the option price of the model. If the implied parameters fit well, the traded price and the option price obtained with the implied parameters should not deviate by much.

3. Data and descriptive statistics

For our analysis of the determinants of the implied volatility smile, we use American DAX equity options traded at the EUREX. The observation horizon covers 252 trading days of the sample period of the year 2012 and contains 1,044,976 trades from all 30 DAX members as of the end of the year 2012. Use of this period has the advantage that it covers bullish as well as bearish times with different volatility conditions. The option and underlying DEUTSCHE BOERSE trade and order book data is obtained via the European Financial data Institue (EUROFIDAI) and provided by the BEDOFIH (Base Européenne de Données Financières à Haute Fréquence) database. Trade-by-trade option data covers for every trade at least the series specification, the price, the time, and the quantity (volume). After disregarding trades that breach the arbitrage boundaries of Merton (1973) during extreme market situations, trades for which it

Table 1: Determinants and expected influence on implied volatility

Source (developed from)	Determinants	Expected influence on implied volatility
	Momentum and Smile	
Wallmeier (2015)	Moneyness	+ / − (different for smirk, smile and spline)
Bollerslev (2006)	Underlying return	−
Mixon (2002)	Momentum effects/history of returns	+
Black and Scholes (1973)	Underlying volatility	+
Corrado and Su (1997)	Underlying skewness	(calls) − (puts)
Corrado and Su (1997)	Underlying kurtosis	+
	Time	
Dumas et al. (1998)	Time until expiration	+
Dumas et al. (1998)	Last trading day (dummy)	−
Dumas et al. (1998)	Shortest time to maturity in sample (dummy)	−
	Liquidity and information-related	
Bollen and Whaley (2004)	Volume measures	+
Bollen and Whaley (2004)	Net buying pressure	+
Easley et al. (2012)	Volume-synchronized probability of informed trading	+ (buyer) / − (seller)
Longstaff (1995)	Bid-ask spread	+ (buyer) / − (seller)
Longstaff (1995)	Round trip costs	+
	Miscellaneous	
Dennis and Mayhew (2000)	Beta CAPM	+
Bollen and Whaley (2004)	Buyer-/seller-motivated	+
Bollen and Whaley (2004)	Lagged implied parameters	+ (normal model) / − (delta model)

is impossible to calculate an implied volatility due to no convergence (0.28% of all trades regarding the normal distribution and 2.96% regarding the variance gamma process), and trades on the expiration day (1.97% of all trades), a sample of 972,907 trades remain. Note that the sample points of the regressions are further reduced, as we demand that the lagged parameters and calculation of the delta parameters must be from the same option series of the same day. After further adjustments in accordance with the research design (cutting all options with an implied second moment greater than one or a relative pricing error greater than 1% and applying Cook's distance as well as excluding data points with any missing information) the normal model results in 316,124 and the delta model in 250,266 usable observations for the normal distribution, if we do not cluster any further into put/call or buyer-/seller-motivated trades. Additionally, we require that at least one hour of trading must have passed for the underlying moments.

The sample also covers order book data besides trade information. Analogous to the underlying price, this data is also available as snapshots. An order book snapshot of actively traded options is created about every 35 seconds on average, whereas the gap between a trade and an order book entry is around 20 seconds. The order book snapshot must always be created before the corresponding trade. Every snapshot has an order book depth up to three levels and includes the corresponding bid and ask prices as well as the quantities. For every bid and ask price, the corresponding quantity (volume) is known as well. For the first two months, we use additional real order book data (difference information).

The underlying prices are obtained from XETRA snapshot data. This means that it is not trade-based, but that a price snapshot is frequently saved. The distance between two snapshots for one underlying varies between less than a second and around half a minute with an average of seven and a half seconds during the trading hours from 9:00 a.m. to 5:30 p.m. Additional data from other sources is required to calculate the implied volatility. First, the Svensson method with input parameters from the Deutsche Bundesbank is used to determine yield curves for each day. Second, we obtain dividend yields for each underlying from the Thomson Reuters Datastream. Based on this data, we present an example comparison of the actual used implied volatilities calculated using BAW and VG in the appendix (see figure B.1). It is noteworthy that the smile component is not as distinct when we use the VG process.

Table 2 presents an overview of the traded options in our dataset. It should be noted that nearly as many calls as puts are traded and that one trade can contain several contracts. Buyer- and seller-motivated trades are nearly balanced, with a little more buyer- (53.76%) than seller-motivated trades (46.24%). Details can be found in table A.1 in the appendix. The trades are arbitrarily distributed over time and do not follow a trend.

The majority of traded contracts have a short time to maturity. Table 3 highlights that the shorter the time to maturity, the more liquid the option series is.

The option term structure depends on the horizon. Up to three months, there are options for every month, up to a year for every quarter, up to three years for every half-a-year, and up to five years for every year. Options always expire on the third Friday of a month, and if not a business day the previous day is used.

To guarantee a correct assignment of lagged and delta data it is essential to sort the data first by option series and then chronologically ascending. To avoid jumps, we exclude all data from the regression with lagged terms from another day or option series than the dependent variable. We also review the correlation of the determinants, because high values can lead to problems. Additionally, this reveals relations between the different variables and helps to understand specific effects. Table A.8 in the appendix reports the correlation coefficients of the panel date for 23 parameters, which we use in the normal model (tables

Table 2: Number of trades over the year 2012 per underlying

Ticker symbol	Underlying	Calls	Puts	All
ADS	Adidas AG	6,397	8,730	15,127
ALV	Allianz SE	3,462	3,339	6,801
BAS	BASF SE	20,108	20,942	41,050
BAY	Bayer AG	19,927	21,581	41,508
BEI	Beiersdorf AG	22,475	24,005	46,480
BMW	Bayerische Motoren Werke AG	22,207	24,520	46,727
CBK	Commerzbank AG	13,930	16,360	30,290
CONT	Continental AG	1,329	1,484	2,813
DAI	Daimler AG	44,023	48,701	92,724
DB1	Deutsche Börse AG	3,672	4,541	8,213
DBK	Deutsche Bank AG	55,053	59,988	115,041
DPW	Deutsche Post AG	6,482	7,277	13,759
DTE	Deutsche Telekom AG	19,400	20,833	40,233
EOA	E.ON SE	33,436	31,747	65,183
FME	Fresenius Medical Care AG	29,938	35,565	65,503
FRE	Fresenius SE	2,188	2,719	4,907
HEI	HeidelbergCement AG	3,062	3,405	6,467
HEN3	Henkel AG	4,147	4,721	8,868
IFX	Infineon Technologies AG	5,601	6,732	12,333
LHA	Deutsche Lufthansa AG	5,781	6,382	12,163
LIN	Linde AG	2,322	2,557	4,879
LXS	LANXESS AG	5,617	6,784	12,401
MRK	Merck KGaA	3,182	3,867	7,049
MUV2	Münchener Rückversicherungs-Gesellschaft AG	8,606	11,497	20,103
RWE	RWE AG	30,223	30,408	60,631
SAP	SAP SE	21,529	20,139	41,668
SDF	K+S AG	9,507	9,197	18,704
SIE	Siemens AG	40,914	41,910	82,824
TKA	ThyssenKrupp AG	18,810	21,776	40,586
VO3	Volkswagen AG	3,311	4,561	7,872
Total		466,639	506,268	972,907

Table 3: Liquidity statistics for option trades relative to time to maturity

Time to maturity	$\leq$1 week	$\leq$1 month	$\leq$3 month	$\leq$6 month	$\leq$1 year	>1 year
Proportion of trades	10.0%	25.3%	31.1%	13.9%	14.1%	5.7%

The proportion of trades refers to the given time to maturity interval, excluding the previous interval.

6 and 7). In addition, a low correlation prevails in general, with a maximum coefficient of 0.82 for the bid-ask spread and round trip costs and a minimum coefficient of -0.68 for the time to maturity and the dummy for the shortest time to maturity. Furthermore, we test the determinants by applying an Augmented Dickey-Fuller test and reject a unit root.

4. Results

In this section we first present the results of the delta and normal models as well as a robustness checks. We focus on comparison of the two pricing methods with determinants like the net buying pressure of Bollen and Whaley (2004) and the mean-reversion process of the implied volatility and give an outlook with an out-of-sample approach.

4.1. Delta model results

We start to investigate the effects of determinants on the implied volatility with the delta model and set the results for the normal distribution of table 4 in reference to the results of the variance gamma process

assumption of table 5. If not otherwise reported, all factors are highly significant at the 1% level.
It is striking that an alternating volatility smile dependency is apparent in both tables. For normally distributed returns, we see a positive linear and squared Δ moneyness influence throughout (except for buyer-motivated puts) and a mixed influence of the Δ moneyness to the power of three, which is negative for calls and positive for puts if clustered in buyer-/seller-motivated trades. In contrast the smile component for the variance gamma process is less prominent for unclustered calls, while it reverses for unclustered puts. Here, we report a Δ moneyness to the power of three, which is mostly positive if we report a negative smile component and vice versa. These alternating effects may weaken the smile. The Δ moneyness to the power of three has a bigger magnitude than the squared Δ moneyness, resulting from the very low moneyness values near zero. The time component is negative for all pricing models, yet, never significant.
For the change in the underlying return and standard deviation, the results for the normal distribution are positive throughout, indicating an increasing implied volatility with higher returns or standard deviation. This statement coincides with the variance gamma assumption only for the standard deviation. For the change in returns, we have an alternating picture with smaller estimates for calls and bigger ones for puts, analogous to the linear moneyness results. This comes as no surprise, as the two measures are highly correlated. It should be noted that these changes are tiny, especially for the Δ returns, as the average absolute percentage change for returns is nearly zero.
The liquidity measures seem contradictory when comparing the two assumptions, yet are significant for all clusters. As an example, the Δ bid-ask spread for no clustering (all/all) has a negative estimate of -0.0025, and the variance gamma process has a comparable positive value of 0.0089. However, the round trip costs act in precisely the opposite way, with regression results of 0.0069 and -0.0136. Overall it is necessary to take a closer look at the buyer and seller cluster, as for the normal distribution an increasing Δ bid-ask spread has a negative influence on all seller-motivated trades. This indicates sellers receive less money for their options and vice versa for buyers, as they have to pay more. Analog conclusions can be drawn for the variance gamma process, whereby the round trip costs counteract.
Additionally, a control for the sale direction is equally important in both worlds. Due to the construction of the Δ buyer-/seller-motivated determinant, it has the value 1 if the trade direction changes from a seller trade to a buyer trade, -1 if it changes in the opposite direction, and 0 if the direction does not change (seller- following a seller-motivated trade or buyer- following a buyer-motivated trade). This quasi dummy is only useful and applicable for regressions, which are not clustered in seller- or buyer-motivated trades. This means that with an estimate regarding all options (non-clustered) of 0.0012 and 0.0018, respectively, the option price rises if we look at buyer-motivated trades.
We can replicate the results of Bollen and Whaley (2004) and show an influence of the net buying pressure, reported for at the money call (ATMC), and put (ATMP). Contrary to most other determinants, it is not highly significant for all variations. The measure is daily, which could weaken the influence. For the variance gamma process, it can be seen in the buyer-/seller clustered columns that if both ATMC and ATMP are jointly significant, ATMC is positive for seller-motivated trades and negative for buyer-motivated. In contrast for ATMP, it is the other way round (negative for the seller and positive for the buyer). To some extent, this pattern repeats itself for the implied volatility calculated based on the normal distribution assumption. If only one of the two is highly significant, it is positive. Looking at the significances, it appears that the ATMC factor is more informative for calls and the ATMP for puts, as expected.

The VPIN measure is always positive in the normal distribution case, and in general bigger for puts than for calls. Thus, apparently more informed trading leads to rising implied volatilities. It is negative for clustered calls in the variance gamma case. Besides the net buying pressure, we control for further daily constant factors.

Particularly remarkable are the highly significant negative autocorrelation terms for all variations. The regression results range from -0.365 to -0.237, suggesting a mean-reversion process.

The R^2 is around 10% for both assumption variations, with the variance gamma process having a seemingly better fit for buyer-motivated trades and the normal distribution for both seller-motivated and non clustered trades. However, despite these R^2, all residuals of the normal distribution have a highly significant lower variance well below the 1% level, which was tested with the F-test. This contradiction can be explained by the inherent lower volatility of the dependent variable within the normal distribution context when compared to the variance gamma approach. This is a result from the initial fit. Consequently, the linear regression on the variance gamma parameter is – in the above mentioned cases – able to explain relatively more of the (larger) in-sample variance of the dependent variable; however, the residuals still bear a higher variance.

Also, the higher moments three and four (skewness and kurtosis, not separately reported), as well as the actual direct parameters of the variance gamma process (σ, θ, and ν), behave similarly in general, especially regarding the mean-reversion process of the negative AR(1) lag. Hereby, the net buying pressure is less significant.

4.2. Normal model results

In line with the delta model, we report and compare the panel regression results for the normal model with the normal distribution assumption (see table 6) and the variance gamma process (see table 7) in the following.

As expected, we see a highly significant volatility surface dependency with a negative smirk (linear moneyness) and at least for the normal distribution a positive smile with a negative spline (moneyness to the power of three). A somewhat surprising result is that this positive smile pattern repeats itself also for the variance gamma assumption. Thereby, the estimate for the smile (moneyness squared) over all trades is around 0.024 for both assumptions. For the variance gamma process however, it even changes to negative values for some buyer-motivated trades, flattening the smile with alternating results for the moneyness to the power of three. Overall, it seems like the smile dependency is more pronounced for the variance gamma process. Yet, it should be noted that this reverses if we perform a panel regression without the lagged implied volatility term. This reduction in the smile component can also be seen in the plotted implied volatilities over the moneyness in figure B.1 of the appendix. In both worlds the time factor is highly significant and negative throughout, with higher absolute values for the variance gamma process.

The time dummies consistently show that for shorter maturities, a rising implied second moment is to be expected because of parameter stationarity – in line with the behavior of the volatility surface. This is mostly true for the normal distribution as well, besides the negative connection of buyer-motivated trades to the shortest time to maturity dummy, which might suggest lower spreads.

The return factor is negative for the normal distribution assumption as expected; however, a positive effect is often noticeable for the variance gamma process. This might result from the negative relation between the returns and the oppositely-acting historical returns (see table A.8 in the appendix), which significantly and positively influences the implied factor in both assumptions, just like the underlying standard deviation. Due to the definition, the historical returns are negative if the current return is higher than the average past return. The negative skewness is an essential factor for the normal distribution, indicating tail risks. In general this is also true for the variance gamma process, yet the results are less significant. Here it is noteworthy that the third implied moment of the variance gamma process has a positive and more significant relation to the underlying skewness. For the fourth implied moment, the underlying kurtosis is in general not significant; however, the signs of the regression estimates become positive. Similar conclusions can be drawn for the direct factors of the variance gamma process (σ, θ, and ν).

All volume measures are insignificant, just like the overall net buying pressure. In contrast, the at-the-money net-buying-pressure for puts and calls is significant with a positive sign, which means that a higher buying pressure drives the prices. It is essential to review the results of the liquidity measures in detail, similar to the delta model. We see again the puzzle of these measures balancing each other out to a certain extent. The results of the VPIN measure within the normal distribution case show that the implied volatility is herewith positively connected for seller-clustered calls and buyer-clustered puts, and negatively connected for seller-motivated puts and buyer-motivated calls. This is also true for the VG process, with the exception of seller-motivated calls. In summary, the results surprisingly seem to indicate that for half of the cluster a higher probability of informed trading lowers the option prices. If we break down the results into buyer- and seller-motivated trades, we see at least for the normal distribution that the spread is negative for sellers and positive for buyers, beside buyer puts – indicating that sellers face a lower and buyers a higher price. This condition is not as clear for the variance gamma approach. The risk measure CAPM is a significant positive driver for both.

Once again, the new dummy factor buyer-/seller-motivated trade is an essential addition and nearly always highly significantly positive, which implies that control for an order book side is working and relevant.

The AR(1) process is again very dominant with an estimate near one for the normal distribution assumption and around 0.8 to 0.9 for the variance gamma process, thus highlighting the importance of the autocorrelation process.

The R^2 near one (normal distribution) indicates a very good fit, mostly driven by the lagged implied volatility. Analogous to the delta model, the R^2 for the variance gamma process is lower, yet still very high with values greater than 80%. The F-test on the residuals reveals once again a highly significant lower variance for the residuals of the normal distribution approach below the 1% level.

Similar results are observable for the regressions of the implied third and fourth moment and the factors of the variance gamma process, σ, θ, and ν (not reported).

Table 4: Delta model with implied volatility based on the normal distribution assumption

	Seller-motivated			Buyer-motivated			All		
	Call	Put	All	Call	Put	All	Call	Put	All
Intercept	6.51E-04***	-2.55E-04***	4.18E-04***	-2.23E-04***	-1.37E-04***	-2.08E-04***	3.14E-05***	-5.66E-04***	-1.05E-04***
Δ Moneyness	2.03E+00***	3.22E+00***	2.46E+00***	7.72E-01***	2.57E-01***	1.14E+00***	1.04E+00***	1.57E+00***	1.32E+00***
Δ Moneyness^2	1.96E+00***	4.57E+00***	2.70E+00***	2.21E+00***	-3.87E+00***	1.80E+00***	1.19E+00***	5.06E-01***	8.66E-01***
Δ Moneyness^3 $\cdot D_{M>0}$	-3.02E+01***	3.90E+02***	-1.94E+01***	-4.86E+00***	1.00E+03***	-7.81E+00***	-1.07E+01***	-8.30E+00***	3.02E+01***
Δ Underlying return	1.83E+00***	3.13E+00***	2.29E+00***	5.56E-01***	3.13E-01***	9.65E-01***	8.16E-01***	1.53E+00***	1.17E+00***
Δ Underlying volatility	9.44E-01***	2.19E+00***	1.14E+00***	1.33E+00***	1.01E+00***	1.16E+00***	1.24E+00***	1.65E+00***	1.28E+00***
$\Delta\sqrt{T}$	-7.71E-07	-8.66E-07	-8.01E-07	-2.43E-07	-4.85E-08	-2.20E-07	-3.83E-07	-4.64E-07	-4.16E-07
Net buying pressure ATMC (daily)	9.24E-05***	-7.38E-05*	5.15E-05***	-5.25E-05***	-2.86E-05	-4.64E-05***	9.30E-06	8.75E-05***	3.33E-05***
Net buying pressure ATMP (daily)	8.58E-05***	6.13E-05	8.61E-05***	1.03E-04***	3.57E-04***	1.43E-04***	-6.36E-06	1.60E-04***	3.85E-05***
VPIN (daily)	1.83E-04***	1.31E-03***	4.57E-04***	1.25E-04***	7.78E-04***	2.58E-04***	1.65E-04***	8.78E-04***	3.46E-04***
Δ Bid-ask spread	-7.76E-03***	-7.22E-03***	-8.15E-03***	7.82E-03***	7.72E-03***	7.05E-03***	-2.18E-03***	-1.25E-03***	-2.50E-03***
Δ Round trip costs	9.87E-03***	2.66E-03***	8.64E-03***	5.37E-03***	-1.64E-03***	4.85E-03***	7.59E-03***	1.96E-03***	6.89E-03***
Δ Buyer-/seller-motivated	–	–	–	–	–	–	1.36E-03***	1.06E-03***	1.24E-03***
$\Delta\sigma_{t-1}$	-2.71E-01***	-3.39E-01***	-3.02E-01***	-2.64E-01***	-2.37E-01***	-2.89E-01***	-3.01E-01***	-3.65E-01***	-3.34E-01***
Controlled for other daily parameters	yes	yes	yes	yes	yes	yes	yes	yes	yes
Controlled for firm fixed-effects	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	9.5%	14.0%	10.8%	9.6%	6.4%	9.9%	11.1%	15.3%	13.0%

Regression results for the model $\Delta\sigma_t = \beta_0 + \beta_1 \cdot \Delta\sigma_{t-1} + \beta \cdot Determinants + \varepsilon$.

All delta values are calculated as $\Delta x_t = x_t - x_{t-1}$.

Daily parameters besides ATMC, ATMP, and VPIN are not presented separately.

FE-panel regressions are used with firm clustered standard errors following Petersen (2008) and based on 250,461 observations.

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

– denotes that the respective regression parameter is not applicable.

The implied volatility σ is calculated based on a normal distribution assumption utilizing the work of Barone-Adesi and Whaley (1987).

Table 5: Delta model with implied second moment based on a variance gamma process

	Seller-motivated			Buyer-motivated			All		
	Call	Put	All	Call	Put	All	Call	Put	All
Intercept	9.28E-04***	-6.32E-04***	5.95E-04***	-4.57E-04***	2.13E-04***	-1.46E-04***	-1.70E-03***	-3.16E-04***	-1.26E-03***
Δ Moneyness	3.07E-01***	3.26E+00***	-4.48E-01***	-1.54E+00***	1.04E+00***	-1.53E+00***	-1.15E+00***	2.15E+00***	-9.46E-01***
Δ Moneyness^2	2.60E+00***	4.77E+00***	1.58E+00***	-3.67E+00***	-3.60E+00***	-3.17E+00***	3.24E-02***	2.42E+00***	-1.03E+00***
Δ Moneyness^3 $\cdot D_{M>0}$	-1.04E+00***	2.53E+02***	1.36E+02***	7.99E+01***	1.20E+03***	7.59E+01***	6.05E+01***	7.12E+00***	7.26E+01***
Δ Underlying return	9.69E-01***	3.22E+00***	9.83E-02***	-1.06E+00***	1.13E+00***	-1.12E+00***	-7.95E-01***	2.12E+00***	-6.28E-01***
Δ Underlying volatility	2.84E+00***	1.88E+00***	2.65E+00***	3.09E+00***	3.01E-02***	2.60E+00***	2.75E+00***	7.16E-01***	2.30E+00***
$\Delta\sqrt{T}$	-1.21E-06	-7.26E-07	-1.01E-06	-8.52E-07	-4.03E-08	-6.35E-07	-6.32E-07	-4.74E-07	-5.32E-07
Net buying pressure ATMC (daily)	6.88E-04***	-7.45E-06	5.06E-04***	-4.59E-04***	4.42E-06	-3.67E-04***	1.31E-04***	1.28E-04***	1.34E-04***
Net buying pressure ATMP (daily)	-3.91E-04***	2.19E-04***	-2.43E-04***	4.03E-04***	3.53E-04***	3.95E-04***	-4.67E-05	1.43E-04***	-9.30E-07
VPIN (daily)	-5.74E-04***	5.10E-04***	-3.95E-04***	-5.77E-04***	7.27E-04***	-2.42E-04***	3.63E-05***	3.96E-04***	1.31E-04***
Δ Bid-ask spread	6.36E-03***	-2.77E-03***	5.25E-03***	1.70E-02***	4.41E-03***	1.67E-02***	1.02E-02***	6.47E-04***	8.90E-03***
Δ Round trip costs	-1.80E-02***	1.94E-03***	-1.55E-02***	-2.69E-02***	-2.20E-03***	-2.46E-02***	-1.63E-02***	-1.89E-04***	-1.36E-02***
Δ Buyer-/seller-motivated	–	–	–	–	–	–	2.01E-03***	8.80E-04***	1.76E-03***
$\Delta\sigma_{t-1}$	-3.01E-01***	-3.10E-01***	-3.01E-01***	-3.06E-01***	-2.75E-01***	-3.05E-01***	-3.09E-01***	-3.34E-01***	-3.10E-01***
Controlled for other daily parameters	yes	yes	yes	yes	yes	yes	yes	yes	yes
Controlled for firm fixed-effects	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	9.8%	11.3%	9.8%	10.3%	9.1%	10.2%	10.3%	13.3%	10.4%

Regression results for the model $\Delta\sigma_t = \beta_0 + \beta_1 \cdot \Delta\sigma_{t-1} + \beta \cdot Determinants + \varepsilon$.

All delta values are calculated as $\Delta x_t = x_t - x_{t-1}$.

Daily parameters besides ATMC, ATMP, and VPIN are not presented separately.

FE-panel regressions are used with firm clustered standard errors following Petersen (2008) and based on 250,461 observations.

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

– denotes that the respective regression parameter is not applicable.

The implied second moment σ is calculated based on a Variance-Gamma process utilizing the work of Kienitz and Wetterau (2012); Lord et al. (2008).

Table 6: Normal model with implied volatility based on the normal distribution assumption

	Seller-motivated			Buyer-motivated			All		
	Call	Put	All	Call	Put	All	Call	Put	All
Intercept	5.60E-03***	1.96E-02***	9.42E-03***	6.24E-03***	1.51E-02***	8.64E-03***	5.82E-03***	2.16E-02***	1.03E-02***
Moneyness	-6.92E-03***	-1.39E-02***	-9.81E-03***	-3.74E-03***	-1.92E-02***	-7.93E-03***	-5.18E-03***	-2.11E-02***	-1.07E-02***
Moneyness^2	2.15E-02***	4.84E-02***	2.78E-02***	1.15E-02***	3.54E-03***	1.69E-02***	1.43E-02***	3.29E-02***	2.44E-02***
Moneyness^3 $\cdot D_{M>0}$	-1.11E-02***	-1.92E-02***	-1.40E-02***	-9.96E-03***	8.51E-02***	-9.82E-03***	-7.72E-03***	-1.02E-02***	-1.25E-02***
Underlying return	-1.99E-02***	-8.79E-03***	-1.81E-02***	-1.75E-02***	-1.01E-02***	-1.61E-02***	-1.63E-02***	-8.93E-03***	-1.54E-02***
History of returns (daily)	9.59E-04***	1.55E-02***	2.84E-03***	1.97E-03***	9.76E-03***	3.59E-03***	1.52E-03***	1.68E-02***	4.26E-03***
Underlying volatility	5.91E-01***	2.32E+00***	9.34E-01***	1.16E+00***	2.27E+00***	1.43E+00***	9.85E-01***	2.78E+00***	1.39E+00***
Underlying skewness	-2.54E-04***	-5.25E-04***	-3.06E-04***	-3.01E-04***	-2.99E-04***	-2.94E-04***	-3.28E-04***	-2.36E-04***	-2.84E-04***
Underlying kurtosis	-1.58E-05	-4.93E-05	-2.22E-05	-3.14E-05	-8.30E-05	-4.96E-05	-2.33E-05	-9.31E-05	-4.28E-05
$\sqrt{T}$	-9.96E-04***	-3.90E-03***	-1.76E-03***	-1.45E-04***	-1.37E-03***	-1.60E-04***	-5.02E-04***	-3.45E-03***	-9.68E-04***
Last trading day (daily)	9.47E-04***	7.55E-03***	2.93E-03***	1.54E-04***	5.52E-03***	1.05E-03***	1.57E-03***	9.64E-03***	3.74E-03***
Shortest time to maturity (daily)	-2.41E-05	1.05E-03***	2.79E-04***	-3.44E-04***	-1.07E-04*	-3.52E-04***	-5.45E-05**	6.78E-04***	1.11E-04***
Volume underlying (daily)	8.15E-13	2.21E-12	1.57E-12	-2.85E-14	-1.81E-12	-4.63E-13	5.37E-13	1.63E-13	8.19E-13
Volume per trade	3.72E-06	5.88E-06	4.71E-06	3.22E-06	2.80E-06	2.23E-06	3.53E-06	5.99E-06	3.81E-06
Trade vol. of opt. series (daily)	5.46E-08	4.39E-08	3.34E-08	-2.25E-08	-5.01E-08	-5.50E-08	-1.05E-08	-8.79E-09	-4.49E-08
Net buying pressure ATMC (daily)	2.53E-04***	5.52E-04***	3.38E-04***	2.91E-04***	5.44E-04***	3.69E-04***	1.72E-04***	8.20E-04***	3.79E-04***
Net buying pressure ATMP (daily)	2.15E-04***	-3.60E-06	1.93E-04***	3.18E-04***	3.65E-04***	3.49E-04***	2.13E-04***	1.84E-04***	2.36E-04***
Net buying pressure over all (daily)	-5.38E-07	-1.24E-06	-1.04E-06	-1.24E-06	-2.84E-08	-1.14E-06	-1.71E-07	-7.93E-07	-7.04E-07
VPIN (daily)	2.73E-04***	-9.57E-04***	1.23E-04***	-2.38E-04***	1.72E-04***	-6.24E-05***	-3.65E-06	-4.49E-04***	-2.50E-05***
Bid-ask spread	-2.38E-03***	-1.38E-03***	-1.32E-03***	1.92E-03***	-5.80E-04***	2.77E-03***	-1.09E-03***	-9.63E-04***	2.08E-04***
Round trip costs	1.28E-03***	7.20E-03***	1.51E-03***	4.05E-04***	6.03E-03***	9.40E-04***	1.13E-03***	8.36E-03***	1.76E-03***
Beta CAPM (daily)	9.67E-04***	3.49E-03***	1.47E-03***	9.53E-04***	1.34E-03***	1.13E-03***	9.17E-04***	3.21E-03***	1.54E-03***
Buyer-/seller-motivated	–	–	–	–	–	–	7.84E-04***	2.13E-04***	5.60E-04***
σ_{t-1}	9.77E-01***	9.21E-01***	9.63E-01***	9.71E-01***	9.38E-01***	9.62E-01***	9.73E-01***	9.11E-01***	9.55E-01***
Controlled for firm fixed-effects	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	98.0%	95.0%	97.1%	98.5%	97.1%	98.0%	98.1%	95.0%	97.1%

Regression results for the model $\sigma_t = \beta_0 + \beta_1 \cdot \sigma_{t-1} + \beta \cdot Determinants + \varepsilon$.

FE-panel regressions are used with firm clustered standard errors following Petersen (2008) and based on 316,636 observations.

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

– denotes that the respective regression parameter is not applicable.

The implied volatility σ is calculated based on a normal distribution assumption utilizing the work of Barone-Adesi and Whaley (1987).

Table 7: Normal model with implied second moment based on a variance gamma process

	Seller-motivated			Buyer-motivated			All		
	Call	Put	All	Call	Put	All	Call	Put	All
Intercept	3.47E-02***	7.08E-03***	2.41E-02***	3.13E-02***	7.30E-03***	2.17E-02***	3.02E-02***	8.55E-03***	2.09E-02***
Moneyness	-6.92E-03***	-6.74E-03***	-2.53E-02***	-3.47E-02***	-1.83E-02***	-2.36E-02***	-3.35E-02***	-1.61E-02***	-2.30E-02***
Moneyness^2	5.01E-02***	7.24E-02***	4.85E-02***	-4.02E-02***	2.60E-02***	-3.11E-03***	1.92E-02***	5.75E-02***	2.35E-02***
Moneyness^3 $\cdot D_{M>0}$	-9.72E-03***	-5.02E-02***	-1.71E-02***	8.96E-02***	6.19E-02***	2.11E-02***	1.22E-02***	-2.20E-02***	-2.81E-03***
Underlying return	6.36E-03***	1.16E-03***	1.43E-02***	-1.77E-02***	-2.93E-03***	3.04E-02***	-1.42E-02***	-2.32E-03***	1.46E-02***
History of returns (daily)	1.68E-02***	1.57E-02***	1.11E-02***	2.48E-02***	1.22E-02***	1.81E-02***	1.92E-02***	1.72E-02***	1.34E-02***
Underlying volatility	4.41E+00***	1.65E+00***	3.42E+00***	5.18E+00***	2.58E+00***	4.66E+00***	4.97E+00***	2.34E+00***	4.03E+00***
Underlying skewness	4.49E-04**	-4.20E-04***	1.13E-04	-2.72E-04	-2.43E-04***	9.98E-05	4.83E-05	-1.89E-04***	1.47E-04
Underlying kurtosis	-5.88E-05	-4.90E-05	-5.67E-05	-7.22E-05	-1.04E-04	-8.74E-05	-8.02E-05	-9.11E-05	-7.26E-05
$\sqrt{T}$	-2.71E-02***	-5.34E-03***	-1.93E-02***	-2.88E-02***	-3.28E-03***	-1.88E-02***	-2.63E-02***	-5.28E-03***	-1.79E-02***
Last trading day (daily)	2.93E-02***	8.30E-03***	2.16E-02***	3.76E-02***	8.52E-03***	2.45E-02***	3.15E-02***	1.21E-02***	2.23E-02***
Shortest time to maturity (daily)	7.60E-03***	6.30E-04***	4.33E-03***	8.71E-03***	1.18E-04*	4.66E-03***	7.84E-03***	5.37E-04***	4.23E-03***
Volume underlying (daily)	1.81E-12	2.94E-12	2.54E-12	5.65E-12	-1.07E-12	1.75E-12	2.99E-12	8.78E-13	1.94E-12
Volume per trade	7.20E-06	5.52E-06	8.59E-06	1.39E-06	3.88E-06	4.77E-06	3.53E-06	6.30E-06	6.34E-06
Trade vol. of opt. series (daily)	5.13E-07	1.03E-08	4.69E-07	-1.07E-07	-1.11E-08	-3.87E-08	5.47E-08	1.66E-08	1.17E-07
Net buying pressure ATMC (daily)	7.39E-04***	5.45E-04***	7.42E-04***	1.17E-03***	5.87E-04***	1.48E-03***	8.91E-04***	8.22E-04***	1.09E-03***
Net buying pressure ATMP (daily)	4.44E-04***	5.65E-05	6.15E-04***	1.64E-03***	3.34E-04***	5.91E-04***	1.07E-03***	1.93E-04***	5.77E-04***
Net buying pressure over all (daily)	-4.51E-06	-5.96E-07	-1.07E-06	-4.99E-06	1.09E-06	-2.11E-06	-4.51E-06	8.27E-09	-1.68E-06
VPIN (daily)	-2.44E-03***	-1.78E-03***	-1.51E-03***	-2.60E-03***	1.33E-04***	-1.45E-03***	-1.72E-03***	-8.89E-04***	-9.56E-04***
Bid-ask spread	9.59E-04***	-1.61E-03***	-5.11E-03***	-3.23E-03***	-2.67E-03***	-1.25E-02***	-1.39E-03***	-2.18E-03***	-8.15E-03***
Round trip costs	-1.51E-02***	7.10E-03***	-4.61E-03***	-1.60E-02***	8.54E-03***	-1.58E-03***	-1.45E-02***	9.38E-03***	-2.63E-03***
Beta CAPM (daily)	4.57E-03***	2.30E-03***	3.23E-03***	8.90E-03***	3.52E-04***	5.88E-03***	6.43E-03***	1.84E-03***	4.50E-03***
Buyer-/seller-motivated	–	–	–	–	–	–	1.23E-03***	-3.49E-05	9.20E-04***
σ_{t-1}	8.14E-01***	8.99E-01***	8.44E-01***	7.92E-01***	8.93E-01***	8.26E-01***	8.11E-01***	8.74E-01***	8.40E-01***
Controlled for firm fixed-effects	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	81.0%	92.0%	82.8%	79.1%	93.9%	80.9%	80.7%	91.5%	82.3%

Regression results for the model $\sigma_t = \beta_0 + \beta_1 \cdot \sigma_{t-1} + \beta \cdot Determinants + \varepsilon$.

FE-panel regressions are used with firm clustered standard errors following Petersen (2008) and based on 316,636 observations.

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

$-$ denotes that the respective regression parameter is not applicable.

The implied second moment σ is calculated based on a Variance-Gamma process utilizing the work of Kienitz and Wetterau (2012); Lord et al. (2008).

4.3. Further tests and robustness analysis

As mentioned, we also conducted all our analyses with the implied third and fourth moment as well as with the variance gamma process parameters σ, θ, and ν. To keep the scope of this paper somewhat moderate, we do not report the corresponding tables separately – just like the results of our additional model specifications, described in sections C.2 and C.3 in the appendix. They consist of an approach using only the AR(1) term and of all determinants without the dominant AR(1) term, based on the approach of Peña et al. (1999). We only use the simple autocorrelation process of the fourth normal model (NM4) as an out-of-sample benchmark.

Following the approach of Bollen and Whaley (2004), we perform all of our analysis on the individual firm level and aggregate the results with the median to control for potential outliers. As there have been no new insights we do not report the tables separately.

Additional smaller variations like heteroscedastic and autocorrelation robust standard errors, different time horizons, and much more have been tested without loss of generality. All the approaches are listed in table A.2 in the appendix.

We noted that even though the in-sample fits of the normal distribution and the variance gamma behave very well, the out-of-sample estimates of these two approaches are very diverse. Consequently, we present and compare the out-of-sample errors of the distributions using both the delta and normal model in table A.9 in the appendix as the last addition to our results. The errors are computed daily with a rolling out-of-sample estimation of the pricing parameters:

1. We exclude the day d_i of our sample D ($d_i \in D$) and estimate with the remaining data of the days $D' = D \not\ni d_i$ the regression coefficients $\beta_{D'}$ of the delta and normal models.
2. We calculate the out-of-sample implied parameters for the day d_i with the regression results from step 1 ($\beta_{D'}$) for the delta and normal models.
3. We determine a price with the option price models and use the out-of-sample implied parameters of step 2 as input.
4. We compute the relative out-of-sample errors from the model price of step 3. and the real price from the trading data.
5. We perform steps 1 to 4 for all days ($d_1, d_2, d_3, ... \in D$) in the sample to obtain the relative pricing errors for all recorded trades.

Due to the strong reaction of the variance gamma process on small changes, we estimate the implied moments (second, third, and fourth) and calculate the parameters σ, ν, and θ with the simplified method of moments, as described in equation 37, which smoothes the outcome and results in lower errors. Nevertheless, the process remains very sensitive to parameter variations, leading to very high relative out-of-sample errors for the variance gamma (often bigger than 1000%). For the normal distribution assumption, very small out-of-sample errors arise, which are around 1% and just a fraction of the bid-ask spread. While this shows a very good fit, they are not superior to a naïve forecast or to the tick size. As the benchmark values for tick size and spread are the same in both distribution assumptions, the same values are reported twice. We see that the delta model is superior to the normal model for the normal distribution assumption; however, the opposite is true for the variance gamma process. For the naïve forecast of the variance gamma process, a percentage error of 12.65% seems high, as it would be expected to be in the same range as the naïve forecast of the normal distribution at 1.21%. This is due to the smoothing usage of the simplified method of moments approach as described in this paragraph. The same naïve forecast benchmark with the direct variance gamma process parameters is indeed in the same range. Not reported are the additional model specifications, whereby it should be noted that models without an autocorrelation term perform worst. Clustering helps in general to improve pricing errors.

5. Discussion

In general, we can report the expected flattening of the moneyness smile when applying a variance gamma process. However, we find a highly significant positive smile pattern present for both distribution assumptions. When reviewing the different model specifications in detail, the moneyness smile is less pronounced if we incorporate an autocorrelation term. This is true not only for the delta model but also for the normal model. Therefore, it is essential to understand that some effects are covered by the autocorrelation. This is also true for most of the smile effects, which is why the smile results within the normal model are of the same magnitude for both option pricing methods, even if there is clearly a flattened smile as reported for the normal model without the autocorrelation (NM2, equation 20, see also figure B.1). It is an essential learning that the lagged term potentially covers several effects and this behavior will come up again with other determinants. Therefore, the tail risk measures skewness and kurtosis (drivers of the smile) are already incorporated in the autocorrelation term in the normal model. All of this suggests that additional factors must also be taken into account. Besides the aforementioned model assumptions, the smile pattern is often attributed to behavioral effects (Han, 2008; Poteshman, 2001), which are not entirely incorporated in the net buying pressure. Additionally, Tanha and Dempsey (2015) show that only the smile coefficients of in-the-money options react to a change of liquidity, volume, and momentum measures. This could drive the smile. We control for these effects implicitly within our models via a moneyness-depending dummy. Our results suggest that also in a high-frequency context, a combination of both tail risk measures and additional factors leads to the observed smile. Therefore, approaches that flatten the smile completely must be considered critically.

Contrary to the volatility smile, the influence of market microstructure and information flow determinants is unchanged and independent of the distribution assumptions. As these measures should not be affected by the pricing model the general suitability of the variance gamma approach is highlighted. Influence of information flow on all implied parameters is found in line with Bollen and Whaley (2004) for the net buying pressures at the money (ATMC and ATMP). New market information drives investors' market activity, the demand for options, therefore market prices, and consequently, the implied volatility. If many market participants intend to buy, the implied volatility rises, and vice versa if they want to sell. Attention should be drawn to the fact that call (put) buying pressure is negatively (positively) associated with skewness, also showing the influence of buying pressures on the pricing of tail risk. Thereby, information is interestingly not related to volume, but to buying pressures at the money. This price formation process not just widens the bid-ask spread but also drives prices, because the spread and net buying pressures are not correlated. The ATMC measure seems to be more informative for calls and the ATMP for puts. Regarding the market microstructure, the control for buyer- and seller-motivated trades is one of the most important extensions. It is particularly crucial for the delta models. Therefore, this determinant must be controlled for either by clustering or with a dummy approach. The positive influence on unclustered data is obvious, as buyer-motivated trades are more expensive than seller-motivated trades. This parameter is particularly important in the context of high-frequency trade-by-trade data for both distribution assumptions. The results of the VPIN measure seem puzzling. If we focus on calls, we see that a higher VPIN, which should capture a higher probability of informed trading, leads to a smaller spread (positive VPIN for seller- and negative VPIN for buyer-motivated calls). It seems counter intuitive that more informed trading leads to a lower spread, as we would expect market makers to widen the spread in high VPIN cases, to protect themselves from traders with superior information. Therefore, we argue that the VPIN does not measure informed trading in our case, but rather discount trading. If one order book side has a favorable price, market participants would use this opportunity and trade on this side; e.g., if the offered price to buy a call is relatively high, a market participant would sell his option, thus driving the seller-motivated price and leading to a high VPIN. The reverse causality is true for buyer-motivated call trades. The put results underline this argument. In the construction of the VPIN, we chose to reverse the VPIN buyer and seller volume (if an informed trader knows that the underlying is undervalued, he could buy calls or sell puts). Therefore, the results of an information measure should be the same for puts and calls, if indeed information is measured. However, if a price discount is measured, a reversion of the (reversed) VPIN put result – the case within our sample – would make sense. Therefore, we believe that the VPIN is not always a suitable tool to measure flow toxicity, which is an interesting addition to the debate of Andersen and Bondarenko (2014b), Easley et al. (2014), and Andersen and Bondarenko (2014a), as well as Collin-Dufresne and Fos (2015).

A negative impact of time is evident, in contrast to Dumas et al. (1998). However, they adjust their moneyness by $\frac{1}{\sqrt{T}}$, leading to a steeper smile for longer time horizons. The negative effect of time, especially for deep-in and deep-out-of-the-money options, can be seen in Cont and da Fonseca (2002) or Fengler et al. (2006). Moreover, Hull and White (1987) find that the Black-Scholes formula causes an overpricing that increases with time to maturity. Even though the time component is not significant in the delta model, the signs are consistently negative. Further time-related parameters are the dummies for the shortest time to maturity and the last trading day. Many studies, like that of Day and Lewis (1992), only focus on options with the shortest time to maturity for liquidity reasons. As we perform a dummy control approach, we do not have to exclude data. However, we also use a robustness test, excluding options with time to maturity longer than a month without any differences. Time dummies have the same effects as the time to maturity for the VG process (higher implied second moment with a shorter time to maturity); however, for the normal distribution assumption the influence of the shortest time to maturity reverses for buyer-motivated trades. Even if this might hint at narrowing spreads for shorter maturities, we see correlation-wise no such connection. Counterintuitively, the time measures are correlated in such a way that shorter maturities are connected to higher spreads (see table A.8 in the appendix). This might be explained by increased trade of deep in and out of the money options (see figure B.3 in the appendix), as we see a shrinking spread with time for (deep-)in-the-money options and a widening spread for (deep-)out-of-the-money options. But the main driver is, in fact, the definition of a relative bid-ask spread. On average option prices decrease with time, while the absolute spread is relatively unaffected by time leading to the reported negative correlation between spread and time, which becomes positive for absolute spread measures. Therefore, the spread is in both cases (relative and absolute) not only a liquidity measure but also another time measure. This is especially true for the variance gamma distribution due to the diminished influence of the AR(1) process. As less movement of the dependent variable can be explained by the lagged term, the time influence of the bid-ask spread becomes more pronounced. Additionally, it is difficult to see a clear pattern in the influence of the liquidity parameters, bid-ask-spread and round trip costs, as their signs switch based on different clusterings and model assumptions, respectively. While

Peña et al. (1999) only report a positive influence of the spread on the smile, we see that the two measures are interleaved and counterbalance each other to a certain extent. However, it can be seen for the normal distribution that a wider spread has a generally negative influence on seller-motivated trades and a positive influence on buyer-motivated trades. To some extent this is also true for the VG process, but the effect is not as clear due to entanglement with time to expiration. However, we conclude that datasets should be clustered into buyer- and seller-motivated trades so that the liquidity parameters can unfold their full potential. Consequently, we report that with a higher spread, the implied volatility is lower for seller-motivated trades, thus they must be sold for lower prices. For buyer-motivated trades the spread has a positive influence; therefore, trading becomes more expensive for buyers.

Furthermore, the moments of the underlying returns are very effective measures. The negative influence of returns on implied volatility is vastly reviewed in the literature and ascribed to either the leverage or volatility feedback hypotheses. The first hypothesis argues that with a decline in share prices, the leverage of a firm rises as the equity value decreases. As an investment becomes more risky the volatility rises. The second hypothesis has a reversed causal relationship: with an increase in volatility, investors expect to be compensated with lower prices. In general, our high-frequency data shows this negative relation between implied volatility and underlying returns as well. However, if we look at the delta model results, all estimates for the normal distribution are positive and differ from the expectations. This is also true for some clusters of the VG process. For the delta model, this results from the positively correlated Δ Moneyness. If we exclude the moneyness terms, the influence of the underlying return becomes always negative for the normal distribution, which can occur because of leverage or volatility feedback effects (Bollerslev, 2006). The results of the normal model point to the leverage effect. While we report only negative relations between implied volatility and returns in a normal distribution setting, the picture for the variance gamma process is intermixed. What we report for the VG process is a transition of the risk from the leverage effect to the higher moments, which the variance gamma process enables. In comparison, we find no indication for the volatility feedback hypothesis, which necessarily predicts a negative connection between implied volatility and return.

For the history of returns (a short momentum effect) the classic picture holds. Due to the construction of the factor, a positive value implies that a current return is lower than the average. Therefore, all returns lower than the average have a positive impact on the implied volatility. Regarding the other moments, we start with the underlying second moment. Even if the Black and Scholes (1973) assumption of a linear volatility does not hold, it should be evident that the current underlying volatility influences the implied volatility in the same manner. The skewness is more complex to review. For the normal model, a solely negative effect is present in the Black-Scholes setting, which is puzzling. We would expect, as seen for the variance gamma assumption that we have a negative sign for puts and a positive sign for calls. Herewith, fat tail risks are priced accordingly. Interestingly, the significance of the third moment decreases in the variance gamma context, whereas it rises again if we review the implied third moment, which evinces a sensible model behavior and foundation. Even if the influence of the fourth moment, kurtosis, is not significant for any regression (also regarding the implied fourth moment), the regression coefficients become positive, as expected from a robust model.

The beta of the CAPM captures the underlying's sensitivity to the market risk of the underlying (Copeland and Weston, 2000). Hence, we control for a more general risk by using it. With higher market risks the implied volatility rises, which itself is viewed and used as an uncertainty measure and predictor (Baker et al., 2016). Consequently, a positive relation of historic risk (beta) to future expected risk (implied volatility) is consistent with the expectations.

The last and most important determinants to be reviewed are the lagged implied volatilities. For the normal model it is obvious that the process is nearly entirely autocorrelated, in line with Cont and da Fonseca (2002). For the variance gamma distribution the autocorrelation is not quite so pronounced. Resulting effects have already been shown, such as the temporal characteristics of the bid-ask spread. The reduction of the AR(1) term is probably caused by the broader scattering of the numerically-computed implied parameters – also important for the out-of-sample pricing, discussed later. The delta model provides further insightful results. The highly significant negative autocorrelation on a sub-second time scale in this model could be explained by trades alternating between higher ask and lower bid prices, like in the model of Roll (1984). Nevertheless, the significant negative AR-process prevails, if we cluster separately for buyer- and seller-motivated trades. This mean-reversion process is due to the order book dynamics and algorithmic trading. When reviewing the order book dynamic, we see that a mean reversion process in the prices and, consequently, in the implied volatility is induced by so-called flickering quotes or fleeting orders (a rapid order submission and cancellation pattern). Figure B.4 presents a representative six minutes example of order book prices at the top of the book. In this example not a single trade occurred, thus the price movements are purely dynamics of the order book. The trades clustered for one side (bid or ask) do not alternate between the bid (bottom) and ask (top) prices, but between new (better) offers and (bad) offers after a subsequent cancellation. These flickering quotes drive our highly-significant mean-reversion process.

One shortcoming of our approach should be mentioned. Even if we try to use high-frequency data when applicable, we still rely on some determinants with a daily-frequency (e.g., the CAPM coefficients).

Table 8 gives an overview of the results and comparison with expectations. Despite some possible weaknesses, the added value of our analysis outweighs it. We show the resilience and strength of our unique high-frequency trade-by-trade data, and apart from confirmation of existing literature, we introduce new important order book based determinants. All in all, the broad and unique dataset gives a good overview and covers many different effects.

The last but most crucial point is the direct comparison of the standard Black-Scholes inspired models using the normal distribution and advanced approaches with, e.g.: Lévy processes such as the variance gamma process in our case. We have shown that using fast Fourier methods, it is easily possible to adapt a Lévy process to (American) option pricing. The models behave as expected, meaning there was a less distinct volatility smile, and the influence of higher moments (third or fourth) on the second moment became weaker while the respective underlying moment for the implied third and fourth model was stronger. The obvious downsides are that using advanced stochastic processes, especially for calculating implied moments for American options, is very computationally expensive. Furthermore, some data points did not converge to a useful solution. Additionally, of course, advanced methods are harder to cope within a trading environment. Instead of one intuitively understood relative price parameter (like implied volatility), in our case at least three parameters must be taken into account. Leaving all these points aside, the biggest disadvantage can be seen in the out-of-sample test. While the variance gamma process allows for flexibility, it reacts drastically to the smallest parameter variations. We have a very stable approach with our delta model, but even with this, the pricing errors are tremendous. For a business case with probably even more uncertainty in parameter forecasting, this problem worsens and results in a not applicable

Table 8: Results of the determinants and expected influence on implied volatility

Source (developed from)	Determinants	Expected influence on implied volatility	Results	Confirmation
	Momentum and Smile			
Wallmeier (2015)	Moneyness	+ / - (different for smirk, smile and spline)	+ / − (different for smirk, smile and spline)	✓
Bollerslev (2006)	Underlying return	−	−	✓
Mixon (2002)	Momentum effects/history of returns	+	+	✓
Black and Scholes (1973)	Underlying volatility	+	+	✓
Corrado and Su (1997)	Underlying skewness	+ (calls) − (puts)	−	✗
Corrado and Su (1997)	Underlying kurtosis	+	+	✓
	Time			
Dumas et al. (1998)	Time until expiration	− (corrected scaling of Dumas et al. (1998))	−	✓
Dumas et al. (1998)	Last trading day	+	+	✓
Dumas et al. (1998)	Shortest time to maturity in sample	+	+	✓
	Liquidity and information-related			
Bollen and Whaley (2004)	Volume measures	+	+ / − (not significant)	✓
Bollen and Whaley (2004)	Net buying pressure	+	+	✓
Easley et al. (2012)	Volume-synchronized probability of informed trading	+ (buyer) / − (seller)	− (buyer call) / + (seller call)	✗
Longstaff (1995)	Bid-ask spread	+ (buyer) / − (seller)	+ / −	✗
Longstaff (1995)	Round trip costs	+	+ / −	✗
	Miscellaneous			
Dennis and Mayhew (2000)	Beta CAPM	+	+	✓
Bollen and Whaley (2004)	Buyer-/seller-motivated	+	+	✓
Bollen and Whaley (2004)	Lagged implied parameters	+ (normal model) / − (delta model)	+ (normal model) / − (delta model)	✓

outcome. On the other hand, the out-of-sample prices of the normal distribution setting are – despite the shortcomings of the model – very stable and result in a tiny error in the range of one tick. This means that this approach is usable in a working environment and beats the variance gamma process in this context easily. Compared to the forecasting error found by Chen et al. (2016), who model the implied volatility surface using a nonlinear Kalman filter approach, we report smaller errors within the normal distribution setting. As further research, the complete analysis could also be applied to index options. For this type of options, a very interesting research subject would be to compare the impact of a Black-Scholes setting against a Lévy process on different determinant dependencies. The smile of index options is generally steeper for index options (Branger and Schlag, 2004; Elyasiani et al., 2020), so we would expect that the influence offloading of moment and moneyness parameters is more evident, while the market microstructure determinants should prevail. Additionally, the computational complexity of index options is reduced, as the option style is mostly European. Furthermore, a theoretical paper on why index options have a more prominent smile could enlighten the discussion.

In the end, we are not able to deliver a superior forecast with all of our high-frequency data. Even if we have a very rich dataset with adequate relevant information in the determinants, the naive forecast is at least equally good. From this follows that even within a high-frequency trading environment and potential co-location of traders, no excess profits can be made. This might be due to high-frequency traders adjusting prices and expectations very fast, leading to efficient prices. All things considered, it is not possible for non-insiders to make a profit, even if all public information is considered. Therefore, as a closing thought, we conclude that the markets are semi-strong efficient.

6. Conclusion and outlook

In this study we compare the classical Black-Scholes setting with the (pure jump) variance gamma process, which accounts for higher moments. Using high-frequency tick-by-tick data of over one million American option trades, we are able to review the different influences of several implied volatility determinants with the two distribution assumptions. We show that the variance gamma process weakens the smile problem of the normal distribution, even if it does not resolve it completely, indicating that the smile incorporates not just tail risks but also other effects, like a behavioral component. The influence of the underlying moments (return, skewness, and kurtosis (Corrado and Su, 1997; Mixon, 2002; Bollerslev, 2006)) reflect the expectations – like growing importance of the higher moments for higher moment related variance gamma parameters – and reveal interesting conclusions such as an indication of the leverage effect in the underlying return. Other determinants known from the low-frequency literature, like time (Dumas et al., 1998; Peña et al., 1999; Wallmeier, 2015), liquidity and information flow (Longstaff, 1995; Bollen and Whaley, 2004; Easley et al., 2012), and market microstructure are not significantly influenced by the pricing model choice. A further learning is that the time effect influences the relative bid-ask spread, which is more pronounced for the variance gamma process. With the help of our detailed and novel dataset, we can develop new significant parameters like a control for buyer- and seller-motivated trades. Furthermore, we reveal a mean-reversion process of the implied volatility on a sub-second scale, driven by the market microstructure. Our results are proven by different robustness checks. In addition, the power of the deterministic implied volatility model can be seen from the small out-of-sample errors.

We also provide an out-of-sample test of the pricing models. The seemingly advanced variance gamma process reacts drastically to small parameter variations, which is not applicable for any market maker, even if it is more robust compared to other distributions like CGMY (Fiorani, 2004). By contrast, the simpler pricing methods of the normal distribution with a simple forecast of the implied volatility delivers very good and stable pricing errors in the range of the tick size, well below the bid-ask spread. It is manageable, intuitive, and easy to talk about, as it just uses one implied parameter.

Future additional research should be done on the order book phenomena of fleeting orders and their causes. Even if there are some explanations like the search for hidden liquidity (Hasbrouck and Saar, 2009), this does not apply to our data as there are no hidden orders possible. This would be in line with the objective of O'Hara (2015), who emphasizes that market microstructure literature must reflect the new reality of high-frequency trading. The excellent quality of our data allows for other order book-based research. With this dataset, the lead-lag relationship between stocks and their derivatives when new information is introduced to the market can be analyzed further, as done by Huth and Abergel (2014) and Judge and Reancharoen (2014). This could reveal the role of hedge activities, wherein the usage of order book data promises a better understanding of market connections. Additionally, the ongoing debate of the put-call parity (see, for example, Kamara and Miller (1995), Hsieh et al. (2008), and Cremers and Weinbaum (2010)) can be challenged. Lastly, the effect of crises on the different determinants, like the COVID-19 pandemic (Papadamou et al., 2020) as well as index options would be interesting to analyze in the future. All in all, our findings support the information content of different determinants, which notably does not contradict efficient markets. Echoing the last scene of the Coen brothers' neo-Western crime thriller: We awake from a dream, report a bit disenchanted on new approaches that tend to backfire in the free and open out-of-sample world, and simultaneously are confident in the justness of the predecessor, which shines the way for us. In spirit of the tale, it still is "a country for old distributions".

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A. Tables

Table A.1: Proportion of buyer- and seller-motivated trades per underlying

Underlying	Buyer-motivated	Seller-motivated
Adidas AG	52.73%	47.27%
Allianz SE	51.89%	48.11%
BASF SE	53.82%	46.18%
Bayer AG	52.92%	47.08%
Beiersdorf AG	52.37%	47.63%
Bayerische Motoren Werke AG	52.53%	47.47%
Commerzbank AG	55.73%	44.27%
Continental AG	48.78%	51.22%
Daimler AG	53.78%	46.22%
Deutsche Börse AG	54.95%	45.05%
Deutsche Bank AG	55.13%	44.87%
Deutsche Post AG	55.41%	44.59%
Deutsche Telekom AG	55.04%	44.96%
E.ON SE	56.37%	43.63%
Fresenius Medical Care AG & Co. KGaA	56.67%	43.33%
Fresenius SE & Co. KGaA	58.04%	41.96%
HeidelbergCement AG	55.56%	44.44%
Henkel AG & Co. KGaA	53.93%	46.07%
Infineon Technologies AG	55.21%	44.79%
Deutsche Lufthansa AG	56.35%	43.65%
Linde AG	52.75%	47.25%
LANXESS AG	54.69%	45.31%
Merck KGaA	50.77%	49.23%
Munich Re AG	51.11%	48.89%
RWE AG	55.41%	44.59%
SAP SE	51.13%	48.87%
K+S AG	53.26%	46.74%
Siemens AG	51.44%	48.56%
ThyssenKrupp AG	54.84%	45.16%
Volkswagen AG	51.94%	48.06%
Total	53.76%	46.24%

Trades with a price above the present mid bid-ask spread are classified as buyer-motivated and trades with a price below as seller-motivated, respectively.

Table A.2: Analyzed specifications and robustness tests

Usage of LASSO picked determinants
Usage of single regressions
Usage of maturities shorter than 1 month
Usage of maturities longer than 1 month
Usage of base model on hourly frequency
Usage of random effects panel regression
Usage of clustered Newey-West standard errors (HAC)
Usage of OLS errors in panel regressions
Usage of specifications of the normal model
Usage of specifications of the delta model
Usage of different time horizons for daily parameters (30, 60 and 100 days)
Usage of different time horizons for intra-day parameters (1, 30, 60 minutes and whole day)
Usage of only order book snapshots

Table A.3: Moneyness categories defined by Bollen and Whaley (2004)

Category	Labels	Range
1	Deep in-the-money call Deep out-of-the-money put	$0.875 < \Delta_C \leq 0.98$ $-0.125 < \Delta_P \leq -0.02$
2	In-the-money call Out-of-the-money put	$0.625 < \Delta_C \leq 0.875$ $-0.375 < \Delta_P \leq -0.125$
3	At-the-money call At-the-money put	$0.375 < \Delta_C \leq 0.625$ $-0.625 < \Delta_P \leq -0.375$
4	Out-of-the-money call In-the-money put	$0.125 < \Delta_C \leq 0.375$ $-0.875 < \Delta_P \leq -0.625$
5	Deep out-of-the-money call Deep in-the-money put	$0.02 < \Delta_C \leq 0.125$ $-0.98 < \Delta_P \leq -0.875$

Δ_C is the call option delta and Δ_P the put option delta.
Categories, labels and range definitions follow Bollen and Whaley (2004).

Table A.4: Information criteria of different AR-processes for optimal lag length of the normal model

Lag structure	AIC	AICc	BIC
AR(0)	-1.06E+06	-1.06E+06	-1.06E+06
AR(1)	-2.41E+06	-2.41E+06	-2.41E+06
AR(2)	-1.93E+06	-1.93E+06	-1.93E+06
AR(3)	-1.59E+06	-1.59E+06	-1.58E+06
AR(4)	-1.33E+06	-1.33E+06	-1.33E+06
AR(5)	-1.13E+06	-1.13E+06	-1.13E+06
AR(6)	-9.75E+05	-9.75E+05	-9.75E+05
AR(7)	-8.54E+05	-8.54E+05	-8.54E+05
AR(8)	-7.53E+05	-7.53E+05	-7.53E+05
AR(9)	-6.65E+05	-6.65E+05	-6.65E+05
AR(10)	-5.93E+05	-5.93E+05	-5.93E+05

Table A.5: F-test results of different AR-processes for optimal lag length of the normal model

AR(1) vs.	AR(2)	AR(3)	AR(4)	AR(5)
p-value of F-test	0.93	0.19	0.11	0.42

Table A.6: Information criteria of different AR-processes for optimal lag length of the delta model

Lag structure	AIC	AICc	BIC
AR(0)	-1.89E+06	-1.89E+06	-1.89E+06
AR(1)	-1.93E+06	-1.93E+06	-1.93E+06
AR(2)	-1.59E+06	-1.59E+06	-1.59E+06
AR(3)	-1.33E+06	-1.33E+06	-1.33E+06
AR(4)	-1.13E+06	-1.13E+06	-1.13E+06
AR(5)	-9.76E+05	-9.76E+05	-9.76E+05
AR(6)	-8.55E+05	-8.55E+05	-8.55E+05
AR(7)	-7.54E+05	-7.54E+05	-7.54E+05
AR(8)	-6.66E+05	-6.66E+05	-6.66E+05
AR(9)	-5.94E+05	-5.94E+05	-5.94E+05
AR(10)	-5.32E+05	-5.32E+05	-5.32E+05

Table A.7: F-test results of different AR-processes for optimal lag length of the delta model

AR(1) vs.	AR(2)	AR(3)	AR(4)	AR(5)
p-value of F-test	0.71	0.60	0.38	0.44

Table A.8: Correlation statistics for the panel data

	Moneyness	Moneyness^2	Moneyness$^3 \cdot D_{M>0}$	Underlying return	History of returns (daily)	Underlying volatility	Underlying skewness	Underlying kurtosis	$\sqrt{T}$	Last trading day (daily)	Shortest time to maturity (daily)	Volume underlying (daily)	Volume per trade	Trade vol. of opt. series (daily)	Net buying pressure ATMC (daily)	Net buying pressure ATMP (daily)	Net buying pressure over all (daily)	VPIN (daily)	Bid-ask spread	Round trip costs	Beta CAPM (daily)	Buyer-/seller-motivated	σ_{t-1}
Moneyness	1	0.27	0.23	-0.01	0.13	0.05	0.00	0.00	-0.02	0.00	0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.01	-0.03	0.00	0.00	-0.17
Moneyness^2	0.27	1	0.80	-0.01	0.04	0.05	0.00	-0.01	0.23	-0.02	-0.12	0.01	0.08	0.07	0.00	0.01	0.00	0.00	0.07	0.11	0.08	0.00	0.11
Moneyness$^3 \cdot D_{M>0}$	0.23	0.80	1	0.00	0.02	0.02	0.00	0.00	0.04	0.00	-0.02	0.00	0.03	0.03	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.02
Underlying return	-0.01	-0.01	0.00	1	-0.27	-0.04	0.21	0.01	-0.03	0.02	0.02	-0.07	0.00	0.01	0.10	-0.04	0.01	0.02	0.00	0.00	0.04	0.02	-0.01
History of returns (daily)	0.13	0.04	0.02	-0.27	1	0.08	-0.07	-0.01	0.04	-0.03	-0.02	-0.06	0.02	0.02	-0.17	-0.04	-0.01	-0.01	0.04	0.02	-0.19	0.00	0.00
Underlying volatility	0.05	0.05	0.02	-0.04	0.08	1	0.00	0.31	-0.05	0.02	0.03	0.28	-0.02	0.00	0.05	0.05	-0.01	-0.05	0.02	0.01	0.32	0.01	0.35
Underlying skewness	0.00	0.00	0.00	0.21	-0.07	0.00	1	0.07	-0.01	0.01	0.00	0.00	0.01	0.01	0.02	-0.03	0.00	0.00	0.00	0.00	-0.02	0.01	-0.02
Underlying kurtosis	0.00	-0.01	0.00	0.01	-0.01	0.31	0.07	1	-0.02	0.00	0.02	0.02	-0.01	-0.01	0.01	0.01	0.00	0.00	0.03	0.02	-0.06	0.00	-0.02
$\sqrt{T}$	-0.02	0.23	0.04	-0.03	0.04	-0.05	-0.01	-0.02	1	-0.20	-0.68	-0.02	0.13	0.05	-0.01	0.01	0.00	0.00	-0.18	-0.15	-0.04	0.03	-0.14
Last trading day (daily)	0.00	-0.02	0.00	0.02	-0.03	0.02	0.01	0.00	-0.20	1	0.19	0.02	-0.02	-0.01	0.00	-0.02	0.01	-0.03	0.12	0.13	0.02	-0.03	0.27
Shortest time to maturity (daily)	0.01	-0.12	-0.02	0.02	-0.02	0.03	0.00	0.02	-0.68	0.19	1	-0.02	-0.08	-0.03	0.00	0.00	0.00	0.00	0.20	0.20	0.05	-0.03	0.15
Volume underlying (daily)	0.00	0.01	0.00	-0.07	-0.06	0.28	0.00	0.02	-0.02	0.02	-0.02	1	0.01	0.03	0.09	0.09	0.01	-0.08	-0.02	-0.01	0.17	0.01	0.11
Volume per trade	0.00	0.08	0.03	0.00	0.02	-0.02	0.01	-0.01	0.13	-0.02	-0.08	0.01	1	0.53	0.01	0.01	0.00	-0.01	-0.07	-0.08	-0.02	-0.01	-0.01
Trade vol. of opt. series (daily)	-0.01	0.07	0.03	0.01	0.02	0.00	0.01	-0.01	0.05	-0.01	-0.03	0.03	0.53	1	0.01	0.02	0.00	-0.02	-0.03	-0.03	-0.01	-0.01	-0.02
Net buying pressure ATMC (daily)	-0.01	0.00	0.00	0.10	-0.17	0.05	0.02	0.01	-0.01	0.00	0.00	0.09	0.01	0.01	1	0.14	0.03	0.00	-0.03	-0.02	0.06	0.03	0.06
Net buying pressure ATMP (daily)	0.00	0.01	0.00	-0.04	-0.04	0.05	-0.03	0.01	0.01	-0.02	0.00	0.09	0.01	0.02	0.14	1	-0.02	-0.01	-0.02	-0.01	0.03	0.04	0.03
Net buying pressure over all (daily)	0.00	0.00	0.00	0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.03	-0.02	1	-0.02	-0.01	0.00	-0.01	0.01	-0.02
VPIN (daily)	-0.01	0.00	0.00	0.02	-0.01	-0.05	0.00	0.00	0.00	-0.03	0.00	-0.08	-0.01	-0.02	0.00	-0.01	-0.02	1	-0.01	0.00	-0.06	0.00	-0.05
Bid-ask spread	0.01	0.07	0.01	0.00	0.04	0.02	0.00	0.03	-0.18	0.12	0.20	-0.02	-0.07	-0.03	-0.03	-0.02	-0.01	-0.01	1	0.82	-0.04	-0.05	0.10
Round trip costs	-0.03	0.11	0.01	0.00	0.02	0.01	0.00	0.02	-0.15	0.13	0.20	-0.01	-0.08	-0.03	-0.02	-0.01	0.00	0.00	0.82	1	-0.04	-0.06	0.11
Beta CAPM (daily)	0.00	0.08	0.02	0.04	-0.19	0.32	-0.02	-0.06	-0.04	0.02	0.05	0.17	-0.02	-0.01	0.06	0.03	-0.01	-0.06	-0.04	-0.04	1	0.01	0.48
Buyer-/seller-motivated	0.00	0.00	0.00	0.02	0.00	0.01	0.01	0.00	0.03	-0.03	-0.03	0.01	-0.01	-0.01	0.03	0.04	0.01	0.00	-0.05	-0.06	0.01	1	-0.04
σ_{t-1}	-0.17	0.11	0.02	-0.01	0.00	0.35	-0.02	-0.02	-0.14	0.27	0.15	0.11	-0.01	-0.02	0.06	0.03	-0.02	-0.05	0.10	0.11	0.48	-0.04	1

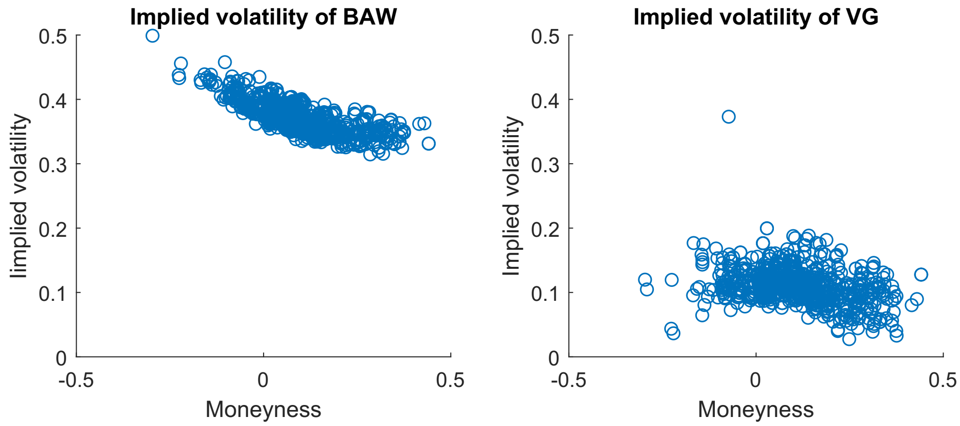
Correlation coefficients for the determinants, which are used for the normal model.

Table A.9: Out-of-sample pricing error

		Put	Call	All	Put	Call	All
			BAW			VG Moment	
Normal model	Buyer	0.96%	1.05%	0.97%	119.08%	>1000%	844.07%
	Seller	1.14%	1.19%	1.03%	110.73%	>1000%	624.96%
	All	1.07%	1.25%	1.15%	118.45%	>1000%	713.92%
Delta model	Buyer	0.72%	0.84%	0.75%	109.69%	>1000%	908.50%
	Seller	0.78%	0.90%	0.82%	99.92%	>1000%	633.63%
	All	0.78%	0.84%	0.76%	111.40%	>1000%	795.98%
Benchmark	Spread	4.33%	4.10%	4.17%	4.33%	4.10%	4.17%
	One Tick	0.92%	0.91%	0.92%	0.92%	0.91%	0.92%
	NM4: $\sigma_t = \sigma_{t-1}$			1.21%			12.65%

Values are the median of absolute out-of-sample percentage errors $|\frac{y-\hat{y}}{y}|$. The out-of-sample values $\hat{y}$ are calculated by excluding one whole day, estimating the models and using the results to calculate the values $\hat{y}$ for the excluded day. This is done in a rolling manner for all days. The value for the bid-ask-spread is calculated in the same manner with the bid and ask prices $|\frac{Ask-Bid}{(Bid+Ask)/2}|$ for comparison. For the VG process we estimated the moments and used the simple method of moments to calculate the VG input parameters, as this yielded more stable results. To benchmark we report the spread and one tick as percentages as well as a naive forecast with $\sigma_t = \sigma_{t-1}$.

B. Figures



Shown are call options on Deutsche Bank AG six to three months before maturity with the same maturity date

Figure B.1: Exemplary comparison of implied volatility computed with BAW and VG

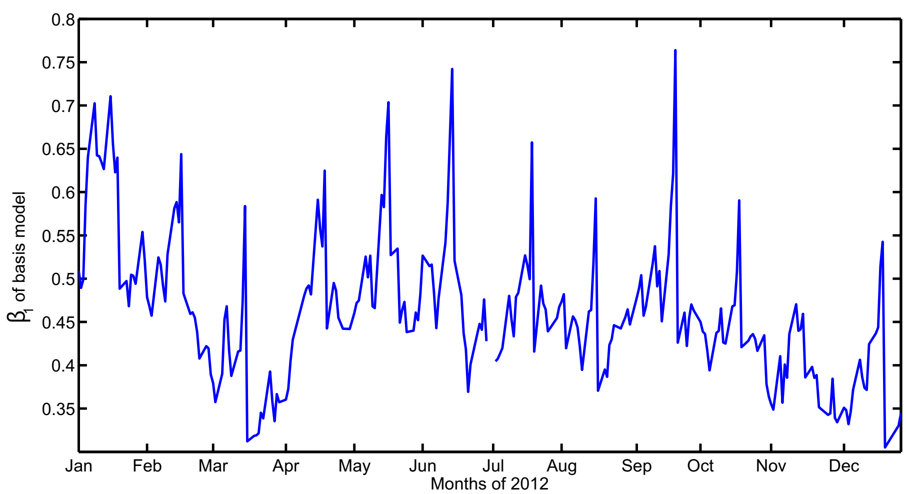


Figure B.2: Cyclical behavior of one β of the foundation model in the example of Deutsche Bank AG and β_1

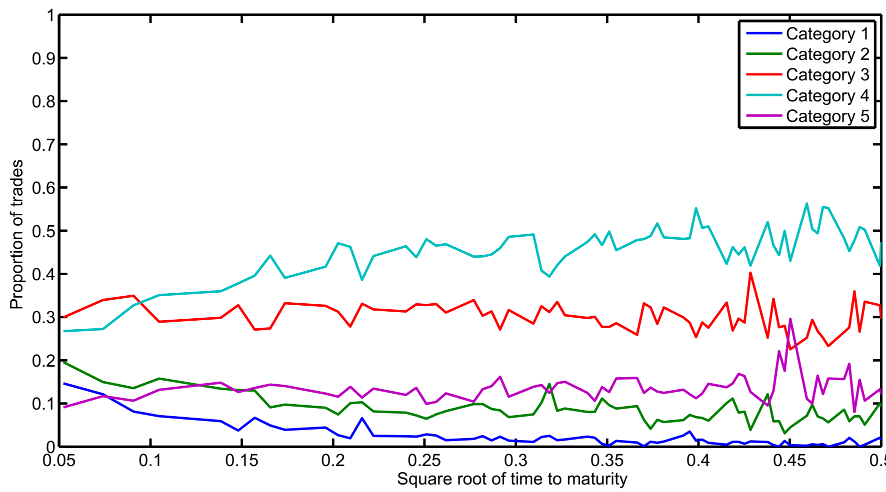


Figure B.3: Proportion of traded moneyness categories over time to maturity at the example of Deutsche Bank AG

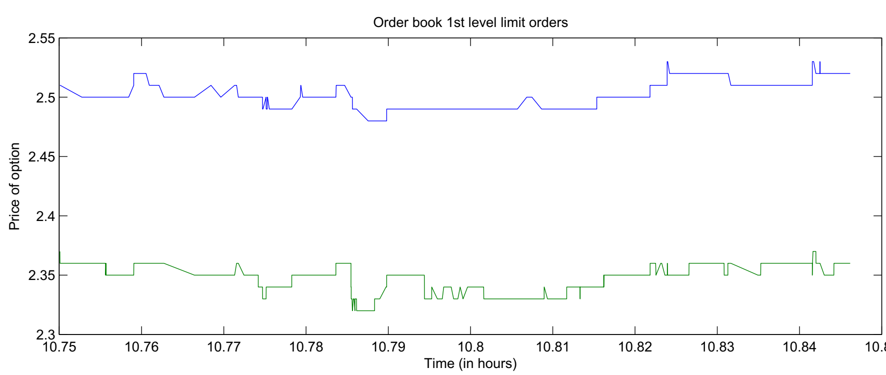


Figure B.4: Six minute excerpt of an order book at the example of one Munich Re option on the 2nd January 2012

C. Models

C.1. Model foundation

We review different specifications for the foundation model. We use the AIC criteria to evaluate the best model from the following alternatives. Equation 17 has the minimum AIC and is therefore used as our model foundation, as expressed in chapter 2.

$$\text{AIC}=-1.0177\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \varepsilon, \tag{8}$$

$$\text{AIC}=-1.0382\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot \sqrt{T} + \varepsilon, \tag{9}$$

$$\text{AIC}=-1.0282\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot T + \varepsilon, \tag{10}$$

$$\text{AIC}=-1.0219\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot T^2 + \varepsilon, \tag{11}$$

$$\text{AIC}=-1.0242\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^2 \cdot D_{M>0} + \varepsilon, \tag{12}$$

$$\text{AIC}=-1.0538\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^2 \cdot D_{M>0} + \gamma_4 \cdot \sqrt{T} + \varepsilon, \tag{13}$$

$$\text{AIC}=-1.0404\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^2 \cdot D_{M>0} + \gamma_4 \cdot T + \varepsilon, \tag{14}$$

$$\text{AIC}=-1.0304\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^2 \cdot D_{M>0} + \gamma_4 \cdot T^2 + \varepsilon, \tag{15}$$

$$\text{AIC}=-1.0273\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^3 \cdot D_{M>0} + \varepsilon, \tag{16}$$

$$\text{AIC}=-1.0583\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^3 \cdot D_{M>0} + \gamma_4 \cdot \sqrt{T} + \varepsilon, \tag{17}$$

$$\text{AIC}=-1.0448\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^3 \cdot D_{M>0} + \gamma_4 \cdot T + \varepsilon, \tag{18}$$

$$\text{AIC}=-1.0341\text{E+06} \qquad \sigma = \gamma_0 + \gamma_1 \cdot M + \gamma_2 \cdot M^2 + \gamma_3 \cdot M^3 \cdot D_{M>0} + \gamma_4 \cdot T^2 + \varepsilon, \tag{19}$$

where σ is the implied volatility, M is the moneyness, defined as $\frac{X}{S} - 1$, with the strike price X and the spot price S. M is derived from the absolute moneyness of Dumas et al. (1998). T is the time to maturity (in days). Furthermore, the dummy $D_{M>0}$ is 1, if the moneyness is greater than zero and 0 otherwise and T is the time until expiration in days. The regression coefficients are denoted as γ and ε is an error term.

C.2. Additional normal models

The additional reviewed specifications of the normal model for robustness reasons are as follows. The second normal model (NM2) excludes the autocorrelation term,

$$\sigma(t) = \beta_0 + \beta \cdot Determinants + \varepsilon, \tag{20}$$

whereas the third normal model (NM3) is a pure AR(1) process,

$$\sigma(t) = \beta_0 + \beta_1 \cdot \sigma(t-1) + \varepsilon, \tag{21}$$

and the fourth (NM4) is a presumed pure autocorrelated process with a correlation coefficient of one,

$$\sigma(t) = \sigma(t-1) + \varepsilon \quad \text{(no regression).} \tag{22}$$

C.3. Additional delta models

The additional reviewed specifications of the delta model for robustness reasons are as follows. The second delta model (DM2) is a pure AR(1) process,

$$\Delta\sigma(t) = \beta_0 + \beta_1 \cdot \Delta\sigma(t-1) + \varepsilon, \tag{23}$$

with $\Delta\sigma(t) = \sigma(t) - \sigma(t-1)$, and the third delta model (DM3) only considers the determinants without an autocorrelation,

$$\Delta\sigma(t) = \beta_0 + \beta \cdot Determinants + \varepsilon. \tag{24}$$

C.4. Pricing American style options under normal-distribution

The adjusted option pricing model of Barone-Adesi and Whaley (1987) is used for the dividend paying American style equity options. The implied volatility is numerically calculated.
The option price for calls $C(S,T)$ is

$$\begin{aligned} C(S,T) &= c(S,T) + \frac{1 - N(d_1(S^*))e^{-d}}{q_2 {S^*}^{q_2 - 1}} S^{q_2}, && \text{when } S < S^* \\ C(S,T) &= S - X, && \text{when } S \geq S^* \end{aligned} \tag{25}$$

and the price for puts $P(S,T)$

$$\begin{aligned} P(S,T) &= p(S,T) - \frac{1 - N(-d_1(S^*))e^{-d}}{q_1 {S^*}^{q_1 - 1}} S^{q_1}, && \text{when } S > S^* \\ P(S,T) &= X - S, && \text{when } S \leq S^*. \end{aligned} \tag{26}$$

with $c(S,T)$ and $p(S,T)$ being the price of an European Call and Put, respectively, with time to maturity T, spot price S, strike price X, zero-coupon rate r and dividend yield d. Furthermore,

$$q = \begin{cases} \frac{-(N-1) - \sqrt{(N-1)^2 - 4\frac{M}{K}}}{2} = q_1 \\ \frac{-(N-1) + \sqrt{(N-1)^2 - 4\frac{M}{K}}}{2} = q_2 \end{cases} \tag{27}$$

with the substitutions $M = \frac{2r}{\sigma^2}$, $N = \frac{2b}{\sigma^2}$ and $K(T) = 1 - e^{-rT}$. As known from Black and Scholes (1973) we use $d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$ and $N(x)$ as the normal cumulative distribution function.

C.5. Pricing American style options and Lévy-processes

We calculate the implied option parameters under a Lévy-process following and utilizing the work of Lord et al. (2008) and Kienitz and Wetterau (2012) by inverting and solving for the option price. The value of an American option is determined using Bermuda options with different exercise dates, the convolution method and the 4-point Richardson extrapolation as suggested by Fang and Oosterlee (2009). The extrapolation gives the American call value using Bermuda options with M exercise dates $B(M)$ as

$$C(d) = \frac{1}{21}\left(64B(2^{d+3}) - 56B(2^{d+2}) + 14B(2^{d+1}) - B(2^d)\right). \tag{28}$$

In the following we present a short summary and overview of the valuation of Bermuda calls.
The Bermuda call value B at a given exercise date t_m is the maximum of the intrinsic option value c, the possible instant payout h and zero.

$$B_{t_m} = \max[c(t_m), h(t_m), 0] \tag{29}$$

The possible payout is thereby the maximum of the spot price at t_m minus the strike and zero.

$$h(t_m) = \max[S_0\, e^x - X, 0] \tag{30}$$

Thereby, x is a vector representing a uniform grid of possible future returns at time t_m. It is constructed in such a way that the Nyquist relation is considered.

$$x(t_m) = \ln \frac{S(t_m)}{S(t_0)} \tag{31}$$

Analogously,

$$y(t_m) = \ln \frac{S(t_{m+1})}{S(t_0)}. \tag{32}$$

Finally, the intrinsic option value at grid point p determined using the fast Fourier transformation and the convolution method is given as

$$\begin{aligned} c(x_p(t_m)) = & e^{-r\cdot(t_{m+1}-t_m)-\alpha\cdot x_p(t_m)+i\cdot u_0\cdot(y_0(t_m)-x_0(t_m))} \cdot (-1)^p \cdot \\ & FFT\{e^{i\circ j\circ(y_0(t_m)-x_0(t_m))\circ\Delta u} \circ \varphi(-(u_j - i\alpha))\} \cdot \\ & IFFT\{w \circ b(y(t_m))\} \end{aligned} \tag{33}$$

Hereby, $\circ$ is the Hadamard (element-wise) product, w is a modified weight vector with the form $w' = (0.5;-1;1;-1;...;1;-0.5)$, j is simply a vector with the form $j' = (0,1,2,...,N-1)$ and the frequency domain u is defined as $u = (j - \frac{N}{2}) \cdot \Delta u$ with $\Delta u = \frac{2\cdot\pi}{N\cdot(y_1(t_m)-y_0(t_m))}$. All three vectors have the same size N just like b, x or y. φ is the characteristic function, in our case the characteristic function of the variance-gamma process with input σ, θ, ν among others. However, it is easy to incorporate any Lévy process with a characteristic function by simply using the respective one. α is a dampening factor, which we set as 0.5. FFT and $IFFT$ denote the fast Fourier transformation and the inverse fast Fourier transformation, respectively. For convergence it is important that the discontinuity lies on the x grid. For a plain vanilla option the discontinuity would be the point $S = X$. Therefore, equation 33 is calculated twice for each exercise point, with an adjusted grid for the second time. Additionally, the calculation of the option value is an iterative process, as the damped option value of the previous period b from equation 29 is needed in equation 33. After each time step the x grid is set equal to the y grid and the calculation starts over until we reach the last time step t_0. For the first time step t_M the value of the Bermuda call is simply $B_{t_M} = \max[h(t_M), 0]$. The final Bermuda value is $B = c(x_{N/2+1}(t_0))$.

C.6. Moments of the variance-gamma process

We use the second (M_2), third (M_3) and forth moment (M_4) of the variance-gamma process (e.g. volatility, skewness, and kurtosis) in our analysis. These moments are calculated as described by Rathgeber et al. (2019):

$$M_2 = \sigma^2 + \theta^2 \nu \tag{34}$$

$$M_3 = \frac{2\theta^3\nu^2 + 3\sigma^2\theta\nu}{M_2^{3/2}} \tag{35}$$

$$M_4 = 3 + \frac{3\sigma^4\nu + 12\sigma^2\theta^2\nu^2 + 6\theta^4\theta^3}{M_2^2} \tag{36}$$

C.7. Simplified method of moments

For the out-of-sample pricing we estimate the aforementioned moments and use these to calculate the parameters of the variance-gamma process (σ, θ, and ν). The parameters are estimated using the simplified method of moments (SMoM). According to Rathgeber et al. (2019) the calculations are:

$$\begin{aligned} \sigma_{SMoM} &= \sqrt{M_2} \\ \theta_{SMoM} &= \frac{M_3 \sigma_{SMoM}}{3 \nu_{SMoM}} \\ \nu_{SMoM} &= \frac{M_4}{3} - 1 \end{aligned} \tag{37}$$