Cost-optimal truck-and-robot routing for last-mile delivery

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Abstract

During recent years, several companies have introduced small autonomous delivery robots and evidenced their technical applicability in field studies. However, a holistic planning framework for routing and utilizing these robots is still lacking. Current literature focuses mainly on logistical performance of delivery using autonomous robots, ignoring real world limitations, and does not assess the respective impact on total delivery costs. In contrast, this paper presents an approach to cost-optimal routing of a truck-and-robot system for last-mile deliveries with time windows, showing how to minimize the total costs of a delivery tour for a given number of available robots. Our solution algorithm is based on a combination of a neighborhood search with cost-specific priority rules and search operators for the truck routing, while we provide and evaluate two alternatives to solve the robot scheduling sub-problem: an exact and a heuristic approach. We show in numerical experiments that our approach is able to reduce last-mile delivery costs significantly. Within a case study, the truck-and-robot concept reduces last-mile costs by up to 68% compared to truck-only delivery. Finally, we apply sensitivity analyses to provide managerial guidance on when truck-and-robot deliveries can efficiently be used in the delivery industry.

KEYWORDS

autonomous vehicles, delivery robots, last-mile delivery, time windows, vehicle routing

1 | INTRODUCTION

Last-mile delivery describes the final step in the retail supply chain, that is, actual delivery to the customer. It is a key challenge for retailers and logistics service providers [22] and is responsible for a large share of logistics costs, often above 50% (see e.g., [18, 30, 33]). The last-mile service in urban areas is forecast to grow by 78% by 2030 [55], particularly driven by the growth of online shopping and home delivery [5, 24, 54]. Furthermore, the share of the world population living in urban areas will grow from 55% in 2018 to 68% by 2030, and with that the volume of freight transportation, emissions, and congestion within these areas [52]. Home deliveries are traditionally performed by diesel trucks, which have to visit each customer individually to hand over ordered goods, and thus intensify traffic and pollution problems. Moreover, failed deliveries (i.e., in cases where customers are not at home to receive a delivery) lead to additional customer visits and further increase traffic. Home deliveries usually require customer attendance. To ensure that the customer is at home, time windows are applied in practice (e.g., see [22]). This helps to avoid failed deliveries by arranging fixed service time windows a customer can select when ordering. In this way the delivery is scheduled at a time in which the customer is expected to be at home and can receive the order. The application of scheduled deliveries is steadily increasing for attended home deliveries (e.g., see [2, 28, 29, 50, 55]). However, the application...
of time windows reduces the efficiency of truck tours. Innovative last-mile solutions for retail fulfillment are required to address these challenges \cite{3, 21, 40}.

In recent years, many new technological concepts have emerged for last-mile deliveries. Delivery with autonomous robots in cities has already been realized (see \cite{48}). The small robots, for example, developed by Starship \cite{48}, Marble \cite{32}, and several others, transport an order (e.g., parcel or a grocery basket) to a single customer within an agreed time window. Robots navigate on sidewalks at pedestrian speed and can safely move in autonomous mode most of the time or switch to remote control in the event of problems. Once the robot arrives at the door, the customer is notified and can then unlock the freight compartment to retrieve the order. A further innovation, which received much attention in recent literature, is drone-based delivery \cite{41}. A well-studied concept in this field is the support of truck deliveries by an aerial drone. A fast-moving drone starts from and returns to the truck to perform individual deliveries on the route. However, robot delivery shows several advantages compared to drones for last-mile delivery in urban areas. First, air traffic is much more restricted and needs to be regulated. While robots can move anywhere on walkways and park safely on sidewalks, drones need to find safe drop-off and landing spots in an area with limited space. Second, robots are less sensitive to weather conditions and weight restrictions. Furthermore, robots move silently on pedestrian walks, and do not interfere with the privacy of residents. Finally, ground vehicles are more energy efficient than drones and also cheaper to produce, since less motor power is required and lightweight solutions are not needed. For large-scale application, the challenge is to benefit from the advantages of robots on the last-mile, but also compensate for their relatively low travel speed. One way to do this would be to create a dense warehouse infrastructure to reduce travel distances, but this involves high investments and is barely feasible in urban areas. The alternative is to combine several robots with faster means of transportation, such as trucks. The truck stops at drop-off points and releases the robots for delivery (see Figure 1). This concept is called truck-and-robot delivery.

Solving the resulting routing problem requires deciding on the truck route (i.e., a sequence of possible robot drop-off locations), and the assignment of each customer order to a location along the truck route, from which a robot is sent out to perform the actual delivery. The robots return to a robot depot after each delivery and from here they will be picked up for later tours. Due to the problem complexity in truck routing and robot scheduling, heuristics are needed to solve instances of relevant size. Current literature on truck-and-robot routing is focused on demonstrating logistical performance (e.g., with respect to late deliveries) for small problem sizes and some stylized assumptions. There is not yet any holistic approach to comprehensively evaluate delivery costs, robot availability, and meet time windows for attended home delivery. We propose an extended truck-and-robot routing problem that (i) is based on decision-relevant costs (for all aspects of truck and robot use), (ii) takes into account restrictions for the use of this system in practice, and (iii) considers trade-off decisions between cost and service quality. We formulate the complete problem as mixed integer program (MIP) and solve it by decomposition into truck routing and robot scheduling. Our solution approach is based on cost-specific priority rules and search operators for the truck routing, while we provide and evaluate two alternatives to solve the robot scheduling subproblem: an exact and a heuristic approach. This allows us to incorporate all problem extensions discussed and keep the computational effort at a minimum.

This paper is organized as follows. Section 2 provides an overview of related literature and highlights the differences to existing concepts. Section 3 describes the problem of a truck-and-robot system, detailing the concept with its costs and constraints. It then derives the mathematical problem formulation. Section 4 details our solution approach. In Section 5 we analyze the numerical performance of our approach and provide managerial insights. The final Section 6 summarizes our findings.
2 | REVIEW OF RELATED LITERATURE

Our problem is related to a growing body of literature on city logistics. This includes not only delivery robots but also drones, autonomous cars and parcel lockers. This section first discusses publications on related delivery concepts that have analogies to robot systems, before detailing the existing literature on robot delivery. It concludes by identifying the gap in research.

2.1 | Related last-mile delivery concepts

Drone-based delivery

Among different autonomous vehicles, aerial drones have received most attention in literature. In a first problem variant, drones start and end their tour at the distribution center (DC). This presumes that the DC lies in customer proximity (e.g., see [13, 37, 51]). In a second variant a drone starts from a truck visiting customers and picking up the drone at later customer stops during the tour, for example, [9, 37, 56]). The challenge is to find truck and drone routes that are cost-efficient, guarantee timely delivery and account for the drones’ maximum range. Existing approaches rely on a solution of the corresponding traveling salesman problem (TSP, i.e., determining a truck tour to visit all customers) that is then improved iteratively by reassigning customers to the drone. A possible resulting tour is shown in Figure 2. Recent publications allow the use of several drones combined with one truck. Murray and Raj [38], Phan et al. [42], and Moshref-Javadi et al. [35] propose a concept with several drones started from a truck at a customer location. Again, the drones meet the truck at one of its later stops. The authors consider up to four drones, which implies at least 20% truck deliveries (as every drone starts from a customer visited by truck). Murray and Raj [38] note that more drones per truck would reduce the system’s efficiency, since drones must keep some distance from other drones and thus take off and land sequentially. The solution approaches therefore rely on solving a TSP for a given set of customers visited by the truck. Chang and Lee [12], Moshref-Javadi et al. [36], and Salama and Srinivas [46] consider a truck with up to ten drones aboard visiting customer clusters. At the waiting point of each cluster, it launches drones for deliveries that return to the waiting truck. The clusters, their waiting points and the truck tour through these waiting points are subject to optimization. While this concept can reduce truck mileage significantly, it still relies on a TSP solution approach for truck routing, that is, the visited stops are given, and cannot handle time windows. Kim and Moon [26] study a hub concept in which a truck leaves parcels at a storage hub, from where drones deliver them. This has the potential to reduce truck mileage and eliminate the need to synchronize the truck and the drone movements, but it requires infrastructure investments into hubs. In summary, a major difference of the truck-and-robot concept is the number of customers served by drone/robot (see Figures 2 and 3). The truck routes therefore differ significantly from a TSP for customer visits, as individual robot depots and drop-off points may or may not be visited by the truck. The number of robots launched is higher than the number of drones considered in truck-and-drone literature. On the other hand, the truck does not have to be synchronized with returning robots, which adds flexibility to meet time windows (not considered by the mentioned drone publications) and achieve large reductions in truck mileage. For a more extensive overview on truck-and-drone routing we refer to Macrina et al. [31], Boysen et al. [10], and Rojas Viloria et al. [45].

As highlighted by Otto et al. [41], drones have some limitations in big cities, such as the capacity of the air space, availability of safe landing or drop-off space, noise regulation, and safety (e.g., flying above busy pedestrian zones) as well as privacy concerns (e.g., flying near homes while filming). Consequently, companies are now piloting this approach in rural areas rather than in cities [49], where drones can leverage their advantage of higher speed. The delivery robots used in our application have several advantages for the use in cities. They are quiet, inherently safe due to the low speed, robust against weather conditions and vandalism, and they can park without consuming electrical energy. Due to the different dynamics of driving...
versus flying, robots typically have a longer range, higher payload, and cheaper sensors and batteries. However, as they are slower than drones, existing concepts for drones cannot be applied directly to robots due to the underlying distribution process (see Section 3.2). The disadvantage of lower speed of robots is eliminated by the combination with trucks and depots. We further refer to Otto et al. [41] for a more detailed literature overview on drone delivery, the limitations of this approach and obstacles to overcome. To summarize, truck-and-drone concepts have several limitations in cities and their routing approaches cannot be applied directly for truck-and-robot routing. Robots have advantages in cities, while drones may be more beneficial for last-mile deliveries in rural areas.

Autonomous electric cars

Networks of autonomous electric cars are another approach to reduce costs and emissions in last-mile delivery. Agatz et al. [1] provide a valuable overview on advantages and challenges of autonomous vehicle platoons serving ad hoc passenger and freight transportation requests. The system consists of a pool of autonomous vehicles that can move individually or form a train (also called “platoon” or “flexible road train”) with several others. While this could improve traffic flow and reduce the use of private cars, it is computationally hard to assess due to its complexity. The idea of a platoon with vehicles joining and leaving on the route is similar to the truck-and-robot distribution, where robots are picked up and released. However, there are not yet satisfying modeling and routing approaches for these concepts [1]. This is due to the high problem complexity, given that the vehicles have a high speed, range to move on their own, and are able to join or leave a platoon anywhere on the route without stopping. To obtain insights on performance of autonomous vehicle platoons in urban freight delivery, Haas and Friedrich [19] apply traffic micro-simulation. While their results provide first insights into benefits and challenges, the system analyzed of three nodes and a roundabout does not provide a realistic problem size for application in practice. In addition, several technical and nontechnical issues of autonomous driving remain, such as reliability of sensors and real-time processing, testing and validation as well as regulatory and insurance-related questions [23]. To summarize, platooning with autonomous electric cars has some similarities with truck-and-robot delivery, but there are no routing approaches available that could be transferred.

Parcel lockers

Another related approach in last-mile delivery is the use of parcel lockers as hubs or storage locations. The most common way to use parcel lockers is to replace home delivery with pickup from a locker by the customer. This could be a way to reduce costs and emissions for less urgent deliveries that customers do not mind picking up close to their homes. For example, Veenstra et al. [53] solve the facility location of parcel lockers and the vehicle routing problem of delivering either to the customers or a nearby locker. Parcel lockers are already in use on a large scale, for example, by DHL [15]. In a further use case, lockers are applied as a micro-hub to exchange parcels between vehicles. Fikar et al. [17] show through agent-based simulation that this approach can improve service quality and reduce delays in food delivery with cargo bikes. Enthoven et al. [16] consider a two-echelon system of trucks and cargo bikes, using the lockers for cross-docking. Generally, our problem is similar to such two-echelon problems, but more complexity is added in our case as vehicles from the second-tier are transported by the first-tier vehicle. This leads to even more interdependencies and synchronization between the two vehicle types. The system of parcel lockers can be extended by the use of autonomous vehicles with several lockers that can park close to the customers and wait for them to retrieve their parcels. This is a promising concept for regular (next day) deliveries [33]. To summarize, delivery to parcel lockers is a promising solution for nonurgent deliveries and parcel lockers as micro-hubs have similarities to the truck-and-robot concept. However, we consider the truck-and-robot concept for urgent premium deliveries that has additional dependencies and requires specific routing approaches.

2.2 Autonomous last-mile delivery with robots

The literature on autonomous delivery robots is still very limited. It can be classified as hub-and-robot concepts and truck-and-robot concepts. While in the former robots move without an additional carrier, in the latter the support of delivery trucks is essential.

Hub-and-robot concepts

Bakach et al. [7] provide a cost assessment for a concept involving robots. They propose a two-tier last-mile delivery system where a truck brings goods to local hubs in which the goods are stored and loaded into robots. The robots then make pendulum tours to the customers. This is simulated with and without time windows. The key difference versus the problem considered in this paper is that the robots are not transported on the truck but stay around one fixed hub. Furthermore, the hubs have a different role to that of our depots, as goods are stored and automatically loaded into robots there. The authors propose an MIP to first define the lowest possible number of hubs across a set of problem instances. In the next step, for a given number of hubs, the customers are assigned to hubs and individual robots such that robot mileage is minimized, while the maximum
robot availability and range is limited. Note that this sequential approach is only possible if the flexibility of moving robots on the truck is given up. The authors provide a cost estimate for their solutions that indicates the potential to reduce operating costs by 70–90% compared to “truck only” deliveries. However, robot amortization and maintenance are not included in this comparison. Poeting et al. [44] assess the same concept as Bakach et al. [7], with only one robot per hub. A truck delivers most parcels directly to customers and only up to 3% of the parcels to the hubs for delivery by robots. Their focus is on planning a suitable truck tour such that customer time windows and hub opening hours are met. Poeting et al. [43] similarly propose an MIP to schedule robot pendulum tours from hubs with several robots such that customer time windows are met with minimal earliness and tardiness. They evidence that their MIP approach is suitable for up to 20 customers and evaluate the impact of the robot quantity. Sonneberg et al. [47] propose an MIP to plan cost-optimal tours with robots only and assess the benefits of additional robot compartments (that enable a robot to serve more customers on the same tour). As relevant costs they consider a rental fee per robot per day, labor costs for loading the robots and transport costs per distance unit. The model is applied to instances with ten customers. As expected, additional compartments drastically reduce the distance traveled, number of robots needed and total costs, since the robots no longer return to the hub after every delivery. In comparison, our problem covers a more general case and could be reduced to such hub concepts by setting the maximum number of robots aboard the truck to zero. Furthermore, we investigate whether an integrated, cost-optimal selection of truck-and-robot routes can achieve savings in operational costs and eliminate the need for expensive hub and storage facilities.

**Research gap and contribution**

Table 1 summarizes related literature. It includes characteristics on the use of support vehicles (SVs, i.e., robots or drones), that is, if SVs (i) are transported by truck (SV on truck), (ii) are stored in a location (SV storage), (iii) availability in the storage is limited (SV avail.), and (iv) SV return to the truck after delivery (SV return).
This paper is the first to consider a large but limited number of support vehicles, which can be picked up and transported by the truck, delivery time windows and total costs. The truck-and-robot concept is one of the most promising new last-mile delivery concepts in city logistics and is technically ready for implementation. Other approaches have considerable limitations for use in dense urban areas. First publications have demonstrated the computational performance and potential of the truck-and-robot system for smaller problem sizes. However, none of the current publications identifies and takes into account all decision-relevant costs to assess the profitability of the truck-and-robot concept (i.e., truck routing and robot usage costs). Moreover, practical constraints (as the limited availability of robots) and hard time windows (required for attended home delivery) have not yet been considered. We therefore contribute to the existing literature by closing this gap. We deduce and integrate empirically collected costs for both trucks and robots and extend the concept by incorporating the mentioned restrictions in practice. In this way we are the first to present a cost-based evaluation of this innovative last-mile delivery concept, which is a prerequisite for assessing possible applications and their profitability. As such, it is not only of academic interest but also of high practical relevance, as it enables field tests on a larger scale.

### 3 | PROBLEM DESCRIPTION

This section describes the technology required, the distribution processes involved as well as the associated costs for the truck-and-robot concept. It concludes by formulating the formal representation of the distribution problem.

#### 3.1 | Truck-and-robot related technology

The key principle of the truck-and-robot concept is that a truck acts as mother ship that can pick up, transport, load, and drop off delivery robots. Autonomous delivery robots are designed to carry out a single customer delivery at a time. The compartment is locked during transportation and can be opened at the customer location via an access code or using a smartphone app to retrieve the freight. The robots are equipped with several cameras, GPS, additional sensors for distance measurement and a mobile internet connection to move autonomously and prevent abuse or theft. This allows safe autonomous driving on sidewalks at pedestrian speed and, when needed, manual remote control. Once a parcel is delivered, the robot returns to a designated...
robot depot. The truck, in its role as mother ship, helps to reduce the distance traveled by the slow-moving robots. This reduces transportation times and increases the robots’ utilization. A typical truck setup could be to use the complete floor of the loading space for approximately 6–10 robots and the space above for shelves that carry the parcels. For instance, Daimler’s “Vans and Robots” setup (see Figure 4) provides space for 54 delivery boxes and up to eight robots. A ramp allows the robots to enter and leave the truck either from the back or the side. An employee is needed to manually drive the truck and load the robots with freight. We refer to Hoffmann and Prause [20] for more details on robot technology.

3.2 Truck-and-robot distribution concept

The truck-and-robot distribution is based on a combination of a delivery truck, robots and a network of small robot charging stations, also called robot depots (see Figure 3). The concept is intended for attended home deliveries in inner cities. The distribution process consists of a truck tour and various robot tours, one for each customer. The truck departs from a given starting point (e.g., the DC) with all parcels to be delivered on board and visits several drop-off locations (blue arrows in Figure 3) to release robots for delivery. A drop-off location is either a robot depot (denoted as “R” in Figure 3) or a designated drop-off point (denoted as “d” in Figure 3), where trucks are able to stop and unload robots. In the case of robot depots, the trucks are able to pick up new robots for later deliveries or release robots with deliveries right from the depot. Once a robot is released for delivery, it visits a single customer (green dotted lines in Figure 3). We consider attended home deliveries, and consequently the delivery (i.e., arrival at the customer location) has to happen within the time window agreed upon with the respective customer. As soon as a robot arrives at the customer location, the customer is notified and can pick up the delivery. In the event that a robot arrives early (e.g., before the assigned time window), it must wait until the customer is available to receive the delivery. This is another advantage as the cost of the robot’s waiting time is lower than the cost of a delivery person waiting or making multiple delivery attempts. After the parcel has been retrieved, the robot returns to the closest robot depot (not displayed in Figure 3 for sake of clarity), where it can recharge and wait for the next delivery tour. Since robots are comparatively slow, the advantage of this concept is that the truck does not have to wait for them. On its route, it will repeatedly pick up additional robots waiting at the depots, load them with parcels and drop them off close to the respective customers. As stated above, robots can also be loaded with parcels at depots and be sent to nearby customers without transporting them on the truck. The challenge of this concept is the definition of the truck route, that is, to decide on where to stop the truck to drop off the robots. Once a truck route is defined, it is necessary to decide where to start each customer delivery such that late deliveries and robot mileage are minimized, while respecting the truck’s maximum robot capacity and the robot availability of each depot along its way.

To summarize, the truck-and-robot concept presented relies on a network of depots and drop-off locations, robots fulfill the complete customer demand, and robots return to a robot depot, not to a truck, as they move at pedestrian speed.

3.3 Decision-relevant costs

In the following, we derive the decision-relevant costs for truck-and-robot delivery. This is necessary to extend the current literature with the cost-based approach and also obtain managerial insights about the benefits of the truck-and-robot concept in general.
Truck-related costs

One of the main drivers for potential cost savings within the truck-and-robot concept is the reduction of truck mileage compared to classical truck deliveries. The cost factors of the truck utilization are the driver’s salary (which is incurred at an hourly rate), fuel consumption, tolls, and truck amortization (which are proportional to the distance covered). These rates apply to the time and distance of the truck’s entire round trip back to the starting point. The truck-and-robot system primarily aims at reducing truck-related costs the following way: the truck does not have to drive to every individual customer and the driver does not need to spend time carrying parcels and waiting for customers.

Robot-related costs

The cost of robots is the primary cost increase compared to a traditional truck delivery. As is common for machines, an hourly machine rate for the robots is applied for the entire time the robot is loaded, travels to the customer, is unloaded and returns to the closest depot. This is also in line with a possible concept of a third-party service provider for robots. In this concept, robot manufacturers (e.g., [27, 48]) offer their robots as a service for logistics companies at a predefined hourly rate. It incurs for the entire time between a robot’s launch by the truck driver and its return to the closest depot after the delivery was made. Since we consider attended home deliveries as a premium shipping service (e.g., as offered by grocery stores and 6) a delivery cannot happen before the agreed time window. A robot must wait at the customer until the beginning of the time window in the event of an early arrival. The hourly cost rate covers amortization, maintenance, electrical energy, rent for the depot space and charging stations. The fact that for electric vehicles more than 50% of the total cost of ownership is incurred for amortization and only 22% for electrical energy highlights the need to consider usage time as the main cost driver for robots [8].

Service-related costs

Service-related costs account for any delayed deliveries. This means that a late delivery will incur hourly penalty costs if a consumer does not receive the parcel within the agreed time window. For instance, some companies offer a refund of premium shipping fees if an order arrives late (e.g., [6]). This also implies a trade-off decision between service dimensions (i.e., on time) and logistics costs.

3.4 Decision model

Based on the characteristics described, we present the truck-and-robot model to minimize total costs, while respecting a limited robot fleet. The basis of a truck-and-robot routing problem consists of the locations to be visited by the truck and robots. The available locations can be divided into the following sets:

- Set of robot depot locations $R$: At these locations, the truck can load additional robots up to the depot’s robot availability and/or launch available robots for delivery.
- Set of drop-off locations $D$: At these locations, the truck can only stop and launch the robots it has aboard. There are no additional robots available to load. This can be interpreted as a robot depot with a robot availability of 0.
- Set of customers $C$: Indicates the customer locations with a delivery request within the planning horizon being considered. These are visited exclusively by the robots.

Since the truck could potentially visit a drop-off or robot depot location $a, a \in D \cup R$, several times (i.e., releasing robots at different times), we allow this by duplicating locations. This results in the index sets $\hat{D}$ and $\hat{R}$ of duplicate locations. For simplicity, we refer to the set of all duplicate locations as $\hat{L} := \hat{D} \cup \hat{R}$. For every distinct location $a, a \in D \cup R$, we call the index set of its duplicates $I_a \subseteq \hat{L}$. Finally, we define $I_a^\delta$ as the set of indices in $I_a$ that are less or equal to $m \in I_a$.

Using these sets, a truck is characterized by its starting position $\gamma$, its initial number of loaded robots $\delta$ and its maximum robot capacity $K$. Subtours are not allowed and each truck stop is associated with a distinct arrival time $t_i, i \in \hat{L}$. The initial number of available robots in a robot depot is denoted by $r_a, a \in D \cup R$. These robots can be retrieved from and replenished into the depot by the truck. We do not consider the time when a robot has returned to the depots for robot availability. Robots often return to a depot only after the truck tour is completed due to their low speed compared to the truck. Waiting for robots to return is not economical due to the high costs of the truck driver. Furthermore, the return times would be hard to predict in practice as customers could take longer to retrieve their goods. The travel times between locations $i$ and $j, i, j \in \hat{L}$, are specified by $\theta_{ij}$ for the truck and between locations $i, i \in \hat{L}$, and customers $k, k \in C$, by $\theta_{ik}$ for the robots. Accordingly, $\lambda_{ij}$ indicates the travel distance of a truck between the corresponding locations. Please note that any fixed processing time that occurs at a stop (e.g., for walking around the truck, loading, and unloading the robots) can be modeled by adding it to the respective travel times. This processing time can be specific to every stop. The deliveries for all customers $k, k \in C$, have to happen within a defined time window of length $c$, which ends with the deadline $d_k$ (i.e., the time window of a customer is given by $[d_k - c, d_k]$). All customers are served exclusively by robots.
A solution is defined by the following decision variables. First, \( s_{ij} \) indicates whether the truck travels from location \( i \) to \( j \), \( i,j \in \hat{L} \). For every location duplicate \( i, i \in \hat{L} \), \( x_{ik} \) indicates whether a customer \( k \) is served from there (i.e., a robot is launched from this location to drive to the customer) or not. To ensure feasibility and assess actual costs, we additionally need the following auxiliary decision variables. If a robot arrives before the designated time window, it has to wait until the start of the time window (causing robot usage costs). The corresponding waiting time is denoted by auxiliary variable \( w_k, k \in C \). If the deadline is missed, the delay at customer \( k \) is indicated by auxiliary variable \( v_k, k \in C \). The auxiliary variable \( q_i \) indicates the number of robots on the truck upon departure from location \( i, i \in \hat{L} \). In line with this, auxiliary variable \( e_i \) indicates the quantity of robots taken out of a robot depot \( i, i \in \hat{R} \).

The cost of truck routing and robot scheduling is defined as follows:

- The truck costs incur for the time (at the rate \( c_t \)) and distance (at the rate \( c_d \)) of the entire round trip back to the starting point. This includes travel and processing times for loading the robots.
- Robots travel from depot/drop-off locations to customers and return to the closest depot afterwards, that is, the hourly cost (at rate \( c_r \)) for a robot incurs for the time from loading the robot to its arrival at the depot closest to the customer served. Service times for loading and unloading are included in the travel times.
- Deliveries can only happen after the beginning of the corresponding time window due to customer availability. An earlier arrival causes waiting time (and thus usage cost at the rate \( c_r \)) for the robot.
- Deliveries after the time window lead to hourly delay costs at the rate \( c_l \) as customers have to wait for their goods.

Table 2 summarizes the notation used. The objective function and the restrictions of the model are then formulated as follows.

### Table 2: Notation

<table>
<thead>
<tr>
<th>Index sets</th>
<th>Notation</th>
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<tbody>
<tr>
<td>( C )</td>
<td>Set of customers</td>
</tr>
<tr>
<td>( D )</td>
<td>Set of distinct drop-off locations</td>
</tr>
<tr>
<td>( R )</td>
<td>Set of distinct robot depot locations</td>
</tr>
<tr>
<td>( \hat{D} )</td>
<td>Set of drop-off locations including duplicates</td>
</tr>
<tr>
<td>( \hat{R} )</td>
<td>Set of robot depot locations including duplicates</td>
</tr>
<tr>
<td>( \hat{L} )</td>
<td>Set of all locations the truck can visit, including duplicates, ( \hat{L} := \hat{D} \cup \hat{R} )</td>
</tr>
<tr>
<td>( I_a )</td>
<td>Set of duplicate indices ( i, i \in \hat{L} ), of one distinct location ( a, a \in D \cup R )</td>
</tr>
<tr>
<td>( I_m^a )</td>
<td>Set of elements ( i \in I_a ) with ( i \leq m )</td>
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<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
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<tbody>
<tr>
<td>( d_k )</td>
<td>Deadline for customer ( k, k \in C )</td>
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<tr>
<td>( K )</td>
<td>Maximum robot capacity of a truck</td>
</tr>
<tr>
<td>( r_a )</td>
<td>Initial number of available robots in location ( a, a \in R )</td>
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<tr>
<td>( \gamma )</td>
<td>Starting position of the truck, with ( \gamma \notin \hat{L} )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Initial number of robots aboard the truck</td>
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<tr>
<td>( \epsilon )</td>
<td>Length of time windows</td>
</tr>
<tr>
<td>( \theta_{ij} )</td>
<td>Truck travel time from location ( i ) to location ( j, i,j \in \hat{L} )</td>
</tr>
<tr>
<td>( \theta_{ik} )</td>
<td>Robot travel time from location ( i, i \in \hat{L} ), to customer ( k, k \in C )</td>
</tr>
<tr>
<td>( \lambda_{ij} )</td>
<td>Distance between locations ( i ) and ( j, i,j \in \hat{L} )</td>
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<table>
<thead>
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<th>Notation</th>
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<tbody>
<tr>
<td>( c^l )</td>
<td>Cost of delays per time unit</td>
</tr>
<tr>
<td>( c^d )</td>
<td>Cost of truck per distance unit</td>
</tr>
<tr>
<td>( c^t (c^r) )</td>
<td>Cost of truck (robot) per time unit</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{ij} )</td>
<td>Binary: 1, if truck goes from location ( i ) to location ( j ); 0 otherwise</td>
</tr>
<tr>
<td>( x_{ik} )</td>
<td>Binary: 1, if customer ( k ) is supplied from location ( i ); 0 otherwise</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Auxiliary variables</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_i )</td>
<td>Number of robots taken out of depot location ( i, i \in \hat{R} )</td>
</tr>
<tr>
<td>( q_i )</td>
<td>Number of robots aboard the truck at departure from location ( i, j \in \hat{L} )</td>
</tr>
<tr>
<td>( t_i )</td>
<td>Arrival time of truck at location ( i, i \in \hat{L} )</td>
</tr>
<tr>
<td>( v_k )</td>
<td>Delay of delivery to customer ( k, k \in C )</td>
</tr>
<tr>
<td>( w_k )</td>
<td>Waiting time for robot at customer ( k, k \in C )</td>
</tr>
</tbody>
</table>
\[
\text{min } TC(S, X, E, Q, T, V, W) = \sum_{i \in L} \sum_{j \in L} (c^d \cdot \lambda_{ij} + c^t \cdot \theta_{ij}^r) \cdot s_{ij} + \sum_{i \in L} \sum_{k \in C} c^t \cdot \theta_{ik}^r \cdot x_{ik} + \sum_{k \in C} (c^t \cdot v_k + c^d \cdot w_k)
\]  

subject to

\[\sum_{i \in L} x_{ik} = 1 \quad \forall k \in C\]  
\[\sum_{j \in \hat{L}} x_{jk} \leq M \cdot \sum_{i \in I \cup \{\gamma\}} s_{ij} \quad \forall j \in \hat{L}\]  
\[x_{ij}^r \leq 1 \quad \forall j \in \hat{L}\]  
\[\sum_{j \in \hat{L}} s_{ij} = \sum_{j \in \hat{L}} s_{ji} \quad \forall j \in \hat{L} \cup \{\gamma\}\]  
\[t_f = 0\]  
\[t_j \geq t_i + \theta_{ij}^r - M \cdot (1 - s_{ij}) \quad \forall j \in \hat{L}; i \in \hat{L} \cup \{\gamma\}\]  
\[v_k \geq t_i + \theta_{ik}^r - d_k - M \cdot (1 - x_{ik}) \quad \forall k \in C; i \in \hat{L}\]  
\[w_k \geq d_k - t_i - \theta_{ik}^r - e - M \cdot (1 - x_{ik}) \quad \forall k \in C; i \in \hat{L}\]  
\[q_f = \delta\]  
\[q_i \leq q_i + e_j - \sum_{k \in C} x_{jk} + M \cdot (1 - s_{ij}) \quad \forall i \in \hat{L} \cup \{\gamma\}; j \in \hat{R}\]  
\[q_j \leq q_i - \sum_{k \in C} x_{jk} + M \cdot (1 - s_{ij}) \quad \forall i \in \hat{L} \cup \{\gamma\}; j \in \hat{D}\]  
\[t_i \leq t_j \quad \forall a \in R; i, j \in I_a : i \leq j\]  
\[\sum_{h \in \hat{L} \cup \{\gamma\}} s_{hi} \geq \sum_{h \in \hat{L} \cup \{\gamma\}} s_{hj} \quad \forall a \in R; i, j \in I_a : i \leq j\]  
\[r_a - \sum_{i \in I_a} e_i \geq 0 \quad \forall a \in R; m \in I_a\]  
\[s_{ij} \in \{0, 1\} \quad \forall i, j \in \hat{L} \cup \{\gamma\}; i \neq j\]  
\[s_{ij} = 0 \quad \forall i \in \hat{L} \cup \{\gamma\}\]  
\[x_{ik} \in \{0, 1\} \quad \forall i \in \hat{L}; k \in C\]  
\[e_i \in \mathbb{Z} \quad \forall i \in \hat{R}\]  
\[t_i \geq 0 \quad \forall i \in \hat{L}\]  
\[q_i \in [0, K] \quad \forall i \in \hat{L}\]  
\[v_k, w_k \geq 0 \quad \forall k \in C\]  

The objective function (1) minimizes total costs \(TC\). The first term of the objective function considers the truck costs, which depend on the traveling segments \(s_{ij}\) selected and the corresponding costs for distance and travel time. The second term sums up the robot costs dependent on associated travel times, and the last term sums up the costs for delayed deliveries and for robot waiting times if the assigned time window is not met. Please note that the costs of the robots’ return to the closest depot do not depend on the scheduling decisions as they are known in advance, and thus added ex post. Every customer must be supplied by exactly one robot (see Constraints (2)), and robots can only start from locations where the truck has stopped (see Constraints (3)). Constraint (4) ensures that only one truck can start in \(\gamma\), and if it arrives at a location, it must also depart from there, which is denoted by Constraints (5). Constraints (5) also ensure the truck returns to the starting point \(\gamma\). Based on the tour, Constraints (6) and (7) define the arrival times at every truck stop. These constraints further prevent subtours as a feasible arrival time \(t_i\) for a location \(i, i \in \hat{L}\), visited twice, does not exist. Constraints (8) define the delay duration for every customer (in the event of late delivery) and Constraints (9) the robots’ waiting time. Waiting times occur in the event of early arrivals as we consider attended home deliveries and customers are assumed to be available only during the agreed time windows. Both constraints use the fact that the arrival time at a customer \(k\) supplied from location \(j\) (i.e., \(x_{jk} = 1\)) equals \(t_j + \theta_{jk}^r\). Constraints (10)–(12) keep track of the number of robots on the truck. The truck departs from the starting point with the initial number of robots. On later stops, robots
launched are subtracted and robots loaded in depots are added. Constraints (13) and (14) ensure without loss of generality that duplicates of the same location are visited in ascending index order. This fact is then used by Constraints (15) to ensure that the number of robots in a depot after visiting duplicate $m$ is not below 0. By enforcing ascending order of duplicates in Constraints (13) and (14), we ensure that no $e_i$ of unvisited stops is included in Constraints (15) for any $m$ that is visited. The variable $e_i$ of unvisited stops could otherwise imply infinite robot supply, as they are not constrained by Constraints (11). Finally, Constraints (16)–(22) define the variables.

A solution consists of a truck route from the starting point $\gamma$ through several depots and drop-off points and an allocation of each customer to one of the locations on that route, from which its parcel will be delivered by a robot. Consequently, the solution $\pi$ can be denoted as a tuple of locations $Y$, where $y(u) \in R \cup D$ is the $u$-th stop, and a matrix $X = (x_{uk})$, defining whether customer $k$ is supplied from stop $u$. Note that a location $a \in R \cup D$ can occur on the route several times and $y(1) = \gamma$ always holds.

The MIP presented is an NP-hard combinatorial problem [11]. The truck must visit a location up to $|C|$ times, which means we have to solve the problem for $|C| \cdot (|R| + |D|)$ duplicate locations. In a real-world application, for example, with $C = 50$, $R = 25$ and $D = 50$, these would amount to 3750 locations, which makes it intractable to solve in acceptable time with conventional solvers. For this reason, we propose a heuristic.

4 | SOLUTION APPROACH

In this section, we introduce our Truck-and-Robot Cost-optimal Routing approach (TRC). The task of finding an optimal truck tour is closely related to the classical vehicle routing problem (VRP), with the addition that the truck can visit a location several times or not at all, and that at the same time the robot schedules need to be defined. The proposed heuristic is therefore based on principles known from VRPs by generating an initial tour and improving the truck tour using a specific search procedure. The structure of our approach is inspired by Boysen et al. [11] as it has been shown that it is efficient for the truck-and-robot problem to minimize lateness. The specific algorithms and operators have been developed to reflect our setting and model. More precisely, we aim at minimizing total logistics costs, and therefore develop a tailored solution approach. This means that our approach uses a cost-specific start heuristic as well as specialized search operators to take into account the different cost aspects for the optimization. We extend the existing model by considering additional constraints for a limited availability of robots, and present a repair step for non-feasible routing solutions. Moreover, we introduce an innovative heuristic for the subproblem of scheduling robots.

Figure 5 summarizes our three-step approach. After an initial truck route has been determined using priority rules in Step 1, an iterative approach is applied that alternates Steps 2 and 3. Step 2a ensures the feasibility of the truck route with respect to robot availability before the robot scheduling subproblem is solved for each feasible truck tour in Step 2b to define the corresponding robot movements (i.e., which customers are served from which location on the truck tour). We present an exact and a heuristic approach to solve the robot scheduling. With the solution obtained, the remaining problem is to find the best truck tour, which takes place in Step 3 using a tailored heuristic based on a local search algorithm.

4.1 | Step 1: Initial solution for truck tours

To obtain a start solution for the truck tours we apply the priority rules “Move to the position that is cost-optimal for the highest number of customers per the location’s distance” (PR1) and “Move to the position from which most customers can be reached in time” (PR2). PR1 considers robot and delay costs but ignores truck-related costs. As a result, it tends to lead to longer truck routes than optimal. PR2 aims to minimize delays, as they can be a pivotal cost driver. With robot and truck costs ignored, it leads to shorter than optimal truck tours at the cost of inefficient robot use. Combined, the two rules incorporate all key cost drivers and provide starting points on both sides of the optimum (routes that are too long and too short) to the local search. PR1 and PR2 are detailed in the following.
PRI is based on the delivery costs of each customer \( k \) from a possible next drop-off or depot location \( a, a \in R \cup D \). These costs consist of the robot travel and waiting time and potential delay costs. These are driven by the arrival time of the truck at location \( a, a \in R \cup D \), and the robot travel time \( \delta_{ak} \) between location \( a \), and the customer \( k, k \in C \). Truck costs are not considered as they cannot be clearly allocated to a single customer. We select the next (first) truck stop as follows. Based on the current time and truck location, the delivery costs for every location-customer combination \( (a, a \in R \cup D; k, k \in C) \) is calculated. Next, for every customer, the delivery costs at potential next stops is compared to the delivery costs of stops already visited (including the current stop). If the delivery costs are minimal for one of the stops visited, the customer is assigned to that stop and will not be considered for the further assignment process. Finally, for every location \( a, a \in R \cup D \) that has not yet been visited, the number of customers with their cost minimum in this location is divided by the distance from the current truck position to derive a score. The location with the highest score is then selected as the next truck stop and the customers with their cost minimum in this location are assigned to it. The procedure is repeated until all customers \( k, k \in C \), are assigned to a location \( a, a \in R \cup D \).

PR2 considers only customers \( k, k \in C \), that can be reached from a given drop-off or depot location \( a, a \in R \cup D \), before their deadline \( d_k \). It selects the location \( a, a \in R \cup D \), with the highest number of customers that can be reached on time as next stop. At every stop, all customers who can be reached before the deadline terminates are served, and the selection of the next location \( a, a \in R \cup D \), is repeated with the remaining customers. This procedure terminates as soon as there are no more customers that can be served in time. This rule was proposed by Boysen et al. [11] and proved to be effective for our approach as well.

4.2 Step 2a: Feasibility and robot availability

Both priority rules ignore robot availability at any given location \( a, a \in R \cup D \), and an unlimited number of customers can be served. This may result in non-feasible routes. In contrast to the routing approach by Boysen et al. [11], we must ensure that the total number of available robots \( NR \) (initial number of robots on the truck \( \delta \) plus all robots at depots visited on the tour \( r_a, a \in R \)) is equal to or larger than the number of customers \( |C| \). To do so, we post-process the results obtained by PRI and PR2 by adding the closest additional depots if necessary. The post-processing is described in Algorithm 1.

4.3 Step 2b: Robot scheduling for given truck route

As a next step, we solve the corresponding scheduling of robots for the given truck route. We propose two alternatives. The first one is based on the exact solution of a scheduling MIP, the second one develops a heuristic.

Alternative 1: Exact solution of MIP for robot scheduling

An MIP for the subproblem of robot scheduling assigns customers to the truck stops (i.e., depots and drop-off points on the route). This is essential to evaluate the total costs. Using the MIP, we provide the basis for the solution evaluation and thus for the acceptance of improved solutions within the improvement heuristic. This complements the truck tour to a full solution, that is, it provides the total costs for a given tour and the starting point of each robot delivery. In contrast to the complete problem, we do not need duplicates of robot drop-off \((D)\) and depot locations \((R)\). Table 3 presents the additional notation of truck tour parameters and decision variables.

For a given truck route \( Y \) and the corresponding index set of stops \( U \), the time of each truck stop \( t_u, u \in U \), is determined by Equations (23) and (24). The parameter \( y(u) \) indicates the actual location of the \( u \)-th stop, that is, \( y(u) \in R \cup D \). Based on the time of each stop, we precalculate the total cost \( c_{uk}^T \) of supplying a customer \( k \) from the corresponding stop \( u \) as defined in

<table>
<thead>
<tr>
<th>Algorithm 1. Truck route post-processing for feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> (Truck tour ( Y ), number of customers (</td>
</tr>
<tr>
<td>feasible = false;</td>
</tr>
<tr>
<td>while not feasible do</td>
</tr>
<tr>
<td>NR = ( \delta + \sum_{a \in Y} r_a );</td>
</tr>
<tr>
<td>if ( NR &lt;</td>
</tr>
<tr>
<td>append closest depot ( a \in R, a \notin Y );</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>feasible = true;</td>
</tr>
<tr>
<td>end if</td>
</tr>
<tr>
<td>end while</td>
</tr>
<tr>
<td>return ( Y )</td>
</tr>
</tbody>
</table>
TABLE 3  Additional parameters and variables for the MIP

<table>
<thead>
<tr>
<th>Truck tour parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
</tr>
<tr>
<td>$y(u)$</td>
</tr>
<tr>
<td>$t_u$</td>
</tr>
<tr>
<td>$c_{uk}^T$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision and auxiliary variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{ak}$</td>
</tr>
<tr>
<td>$q_u$</td>
</tr>
<tr>
<td>$r_{ua}$</td>
</tr>
</tbody>
</table>

(25). The costs include the robot usage cost per time $(c^r)$ depending on the travel time from the truck stop to the customer, the potential waiting time for the beginning of the delivery time window, and the time to return to the closest depot $\min_{a \in R}(\theta_{ak})$. Delay costs are also included.

$$
t_1 = 0
$$  
(23)

$$
t_u = t_{u-1} + \theta_{y(u), y(u-1)}^r + \sum_{v \in R}(\theta_{y(v), k})^r + \min_{v \in R}(\theta_{y(v), k}) + c^r \cdot (t_u + \theta_{y(u), k} - d_k)^+
$$  
(24)

$$
c_{uk}^T = c^r \cdot (\theta_{y(u), k} + (d_k - c - t_u - \theta_{y(u), k})^+ + \min_{a \in R}(\theta_{y(a), k})) + c^r \cdot (t_u + \theta_{y(u), k} - d_k)^+
$$  
(25)

The auxiliary variable $r_{ua}$ indicates the number of robots available at location $a, a \in R \cup D$, after the $u$-th stop and $q_u$ indicates the number of robots aboard the truck after departing from $u$. The resulting MIP for minimizing the costs of robot scheduling for a given truck tour follows.

$$\min F(Q, X, R) = \sum_u \sum_{a \in U \cup C} x_{ak} \cdot c_{uk}^T$$  
(26)

subject to

$$\sum_{u \in U} x_{ak} = 1 \quad \forall k \in C$$  
(27)

$$r_{ua} = r_{ua-1} \quad \forall a \in R, u \in U : a \neq y(u)$$  
(28)

$$r_{ua} \leq r_{u,a-1} + q_{u-1} - q_u - \sum_{k \in C} x_{ak} \quad \forall a \in R \cup D, u \in U : a = y(u)$$  
(29)

$$q_0 = \delta$$  
(30)

$$r_{a0} = r_a \quad \forall a \in R$$  
(31)

$$r_{au} = 0 \quad \forall a \in D, u \in U$$  
(32)

$$x_{ak} \in \{0, 1\} \quad \forall u \in U, k \in C$$  
(33)

$$r_{au} \geq 0 \quad \forall a \in R, u \in U$$  
(34)

$$0 \leq q_u \leq K \quad \forall u \in U$$  
(35)

Constraints (27) ensure that exactly one robot is launched to each customer. Constraints (28) and (29) keep track of the number of robots in the depots and on the truck after every truck stop. Constraints (30) and (31) define the initial quantity of robots in each depot and on the truck. Constraints (32) ensure that robots cannot be stored at drop-off locations. Constraints (33)–(35) define the range of variables.

Due to the problem structure, the corresponding LP solution satisfies the integer constraints in most cases. The problem can therefore be solved many times within the entire heuristic at affordable computational effort. Nevertheless, the MIP for the robot scheduling constitutes a potential bottleneck in terms of computation time. For this reason, we propose an alternative heuristic approach in the following. Numerical analysis will be applied to assess the performance in terms of computation time versus solution quality of both variants.

Alternative 2: Heuristic for robot scheduling

The robot scheduling heuristic (RSH) relies on (i) calculating a lower bound of the robot schedule cost, (ii) fixing the number of started robots per stop, and (iii) swapping customers between stops.
Step 3: Local search for improving truck routing

(i) The RSH starts with assigning each customer to the cost-optimal truck stop without considering robot availability. Here we need to differentiate two cases: First, if the resulting solution is above the total costs of the current best-known solution, the given truck tour cannot be part of a better solution. It will be disregarded and the search continues with Step 3. Second, if this solution is below the total costs of the current best-known solution, we further test if this robot schedule is feasible with respect to robot availability in the depots and on the truck. If so, this robot schedule and the given truck tour are the new best-known solution. If not, we search for a feasible robot schedule for the given tour in step (ii).

(ii) Customers are sequentially assigned to stops on the tour using regret-insertion. The regret is defined as the cost difference of the best and second-best possible stop to which a customer can be assigned. Since stops can run out of robots simultaneously (e.g., several consecutive drop-off points as soon as the truck runs empty), we consider as possible second-best stops only those after the best one, starting at the first depot. The customer with the highest regret is assigned to the best possible stop. After each assignment, the robot availability and truck capacity have to be updated, and new regret values are calculated. This step is repeated until all customers are assigned and therefore a feasible robot schedule is obtained.

(iii) Finally, we improve the robot schedule by swapping pairs of customers if this leads to a cost decrease. Please note that this step does not impact robot availability and therefore feasibility remains. We further focus on the customers currently not assigned to their best stop to speed up the search. This step concludes the robot scheduling and provides the total costs of the current solution.

In the following, we denote the approach based on this heuristic as TRC-RSH and the approach based on exact robot scheduling as TRC. In the numerical analysis, we will show which of the alternatives is beneficial under which conditions.

4.4 Step 3: Local search for improving truck routing

A local search (LS) procedure is applied to improve the solutions. The truck tours obtained are post-processed in each iteration to obtain feasible tours (Step 2a), and the corresponding robot scheduling (Step 2b) is solved. This means that any truck tour obtained within the LS corresponds to exactly one complete solution \( Y = (Y', X) \), which is the result of the feasibility check and the subsequent robot scheduling. Please note that if \( Y \) is feasible, \( Y' = Y \) holds. This enables us to find the best possible objective value for any given truck tour as the respective robot scheduling solution is optimal. We denote the complete solutions (i.e., truck tour after feasibility check and corresponding robot schedule) for the initial tours \( Y_{pr1} \) and \( Y_{pr2} \) as \( \pi_{pr1} \) and \( \pi_{pr2} \). Each of these two start solutions is then evaluated \( \kappa / 2 \) times by the LS procedure as given in Algorithm 2. The best solution is chosen across all \( \kappa \) search cycles. The LS is applied for \( k^{th} \) iterations with the respective starting solution. The following operators within the LS are used in order to improve the given truck route:

- **Best-cost removal**: The stop whose removal leads to the best estimated total cost is removed. This operator aims at reducing unnecessary detours early on the tour, which may cause delays on later stops. The total costs are estimated without solving the robot scheduling, using the following procedure. First, for all customers assigned to the remaining stops, the new arrival time is calculated based on the new, shorter truck tour. This new arrival time can lead to an updated robot waiting time or delay time, and thus a change in cost. Customers who were previously assigned to the stop that has been removed must be redistributed. We do this by calculating the hypothetical robot and delay cost (based on arrival time) for every combination of stops on the truck route and customers to be redistributed. All of these customers are then assigned to the stop where their respective cost is minimal (without enforcing the number of available robots at that stop). Finally, the truck time and distance are updated together with their corresponding costs. After assessing these hypothetical new total costs for the removal of every stop, the stop with minimal cost is selected for removal.

- **Depot insertion**: After a random stop of the current tour, the unused depot that leads to the minimal deviation is inserted. This operator contributes to the degree of freedom for the robot schedule as this increases the number of available robots and their possible starting locations.

- **Random removal**: A random stop is removed from the current tour.

- **Random insertion**: A random stop is inserted at a random position of the current tour.

- **Random swap**: Two random stops of the tour are swapped.

To enlarge the search area in the event of local minima, two instead of one of these operations are performed sequentially within a single iteration if no improvement has been made during the last 50 LS cycles and three operations after 100 LS cycles without improvement. Our tests have shown that this enables the best-known solutions to be improved, and that the best solutions were also found faster and more robustly across all \( \kappa \) cycles.
Algorithm 2. Local search procedure

Input: Starting solutions π_{pr1}, π_{pr2}; number of iterations κ
π_{best} = null; // best solution found
for i = 1 to κ do
    if i ≤ κ/2 then
        π = π_{pr1}.clone();
    else
        π = π_{pr2}.clone();
    end if
    π_{local} = LS(#iterations: κ_{ls}, start solution: π); // local search with κ_{ls} cycles
    if π_{best} = null or Z(π_{local}) ≤ Z(π_{best}) then
        π_{best} = π_{local}; // save best result
    end if
end for
return π_{best}

5 | NUMERICAL ANALYSIS

This section analyzes the performance of our solution approach and provides managerial insights. Section 5.1 describes the generation of our problem instances, which are available online at http://www.vrp-rep.org/datasets/item/2020-0005.html. Using these instances, we show the efficiency of our TRC approach in Section 5.2. This section compares it first to MIP solutions using Gurobi for small instances (see Section 5.2.1). We further investigate the efficiency of the exact versus heuristic approach for the robot scheduling (see Section 5.2.2). Additionally, we assess the efficiency with the state-of-the-art approach from literature in Section 5.2.3. Section 5.2.4 compares the performance to other instances provided in recent literature. In Section 5.3, we compare the truck-and-robot system’s performance to traditional truck deliveries. Section 5.4 details the cost structure of the solutions and resulting implications for the system’s ability to meet changing demand settings. We have implemented our algorithm and the corresponding benchmark approach in Python (using PyCharm 2018.3.5 Professional Edition) with Gurobi (version 8.0.1) as a solver for the robot scheduling MIP. All computations were executed on a 64-bit PC with an Intel Core i7-8650U CPU (4 × 1.9 GHz), 16 GB RAM and Windows 10 Enterprise.

5.1 | Instance generation and parameter setting

This section details the generation of the problem instances. We consider Munich as a model delivery area for deriving actual delivery situations. We assume a delivery area that resembles half of the city center (a square area with side length of 4 km) and in which |C| = 50 customers are served per tour. For a realistic spatial distribution, we randomly choose 50 of all known building locations in the northern half of Munich using Open Street Maps (see [39]) and distribute the |R| = 25 depots in an equidistant manner. The drop-off points are uniform-randomly distributed and the truck is assigned to one random depot or drop-off location as a starting point. For the deadlines we assume the company offers different delivery times for customers, depending on their location, and assigns orders to trucks such that early deadlines are closer to the truck’s starting position and later deadlines further away (see similar approach in Boysen et al. [11]). To simulate this, every deadline is calculated as \( \delta_k = t_{k_{\min}} \cdot \rho_k \), where \( t_{k_{\min}} \) is the time needed to directly go from the starting point to the customer by truck (excluding any handling times) and \( \rho_k \) is a factor drawn from a uniform distribution in the interval \([\rho_{\min}, \rho_{\max}]\) (in our example \([5, 8]\)) for every customer individually. This procedure assumes a given vehicle allocation such that viable tours can be planned. The initial number of robots is set to \( n_r = 10 \) for every depot \( a, a \in R \), if not stated otherwise. We further use the setting for the remaining parameters as given by Boysen et al. [11]. The truck can carry up to \( K = 8 \) robots and is fully loaded at the beginning (\( \delta = 8 \)). The average speed is 30 km/h for the truck and 5 km/h for the robots. The handling time per stop is assumed to be 40 s. The truck costs are estimated using fuel, labor, investment costs and amortization, resulting in a distance cost of \( c_{\text{d}} = 0.20 \text{ €}/\text{km} \) and a time cost of \( c_{\text{t}} = 30 \text{ €}/\text{h} \). For the delays, we refer to the common offer to refund the premium shipping fee in the event of late delivery. In our example, a fee of 5 € is refunded for a one hour delay. Robot costs are calculated based on the target purchasing price of 2000 € as reported on Condliffe [14]. Amortization time is assumed to be five years. The robots are in operation 50 weeks per year, six days per week and eight hours per day. Furthermore, the utilization rate is 50% and there is a markup for maintenance, electricity, etc., of 50%. Using these estimates, we derive a cost rate of \( c_{\text{c}} = 0.50 \text{ €}/\text{h} \), with \( c_{\text{c}} = 2000 \text{ €}/(5 \cdot 50 \cdot 6 \cdot 8 \text{ h} \cdot 50\% \text{ utilization}) \cdot (1 + 50\% \text{ markup}) \). Lastly, we set the run-time parameters of our heuristic to \( \kappa = 16 \) search cycles and
TABLE 4  Empirically estimated values applied as default values in a problem instance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>C</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>D</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>R</td>
<td>$</td>
</tr>
</tbody>
</table>

Constraints

- $K$: Truck’s maximum robot capacity, 8
- $r$: Initial number of available robots per depot, 10
- $\delta$: Initial number of robots aboard the truck, 8
- $\epsilon$: Length of time windows, 10 min
- $\mu$: Fixed processing time at every stop, 40 s
- $[\rho_{\min}, \rho_{\max}]$: Deadline factor interval, [5, 8]
- $\omega_t$: Average truck speed, 30 km/h
- $\omega_r$: Average robot speed, 5 km/h

Cost factors

- $c^d$: Distance cost of truck, 0.20 €/km
- $c^t$: Time cost of truck, 30 €/h
- $c^r$: Cost of robots, 0.50 €/h
- $c^l$: Cost of delays, 5.00 €/h

$\kappa = 500$, as this robustly produced results that were close to the best-known one across all $\kappa$ cycles. The search cycles were executed in parallel using multiprocessing. Table 4 summarizes the parameters applied that have been collected empirically. We apply this data throughout all experiments except for a further comparison on lateness in Section 5.2.4, where we build the experiments upon the data instances of Boysen et al. [11].

5.2  Efficiency of suggested solution approach

5.2.1  Comparison to exact solution of MIP

In our first performance comparison, we assess the solution quality and runtime of our TRC approach by solving the MIP presented exactly (see Equations (1)–(22)) using Gurobi. The MIP solution using Gurobi is only feasible and computationally tractable for small instances. We use instances with 9, 12, 15, and 18 customers and 16 depots each. The number of drop-off points is derived as $2 \cdot |C|$. The number of robots per depot $r$, the robot capacity of the truck $K$, and initial number of robots aboard $\delta$ are set at $|C|/3$. The remaining parameters are as described in Table 4. We consider two duplicates of each location, that is, the truck can visit a location at most twice. This has proved to be sufficient to find the optimal solution. The runtime limit of Gurobi is set at 30 min. Table 5 compares TRC to the MIP for 20 instances per problem size, showing the percentage of instances for which the optimal and best-known solutions were found, the average gap of solutions to Gurobi’s lower bound, the runtime and average savings obtained by applying TRC instead of Gurobi. The comparison shows that the MIP solution with Gurobi becomes computationally intractable for more than 12 customers. TRC in contrast remains fast, identifies better solutions on average and in total finds 14 out of 19 proven optima (i.e., 18% of all 80 instances).

5.2.2  Comparison of alternative solution approaches

Next, we compare the performance of TRC and TRC-RSH, that is, the efficiency if RHS is used for robot scheduling instead of the MIP in Step 2b. Table 6 summarizes the results across 20 instances per parameter setting.
TABLE 6 Performance comparison of TRC-RSH versus TRC

<table>
<thead>
<tr>
<th>Instances</th>
<th>Average change due to RSH (%)</th>
<th>Standard deviation of objective value change (ppt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customers</td>
<td>Robots per depot</td>
<td>Computation time</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>–71</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>–56</td>
</tr>
<tr>
<td>75</td>
<td>10</td>
<td>–46</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>–29</td>
</tr>
<tr>
<td>125</td>
<td>10</td>
<td>–23</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>–14</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

Applying TRC-RSH is up to 71% faster, in particular for smaller test cases. However, the advantage of computation time decreases with a growing number of customers. With high number of customers and high number of robots, the TRC becomes as fast as the TRC-RSH. Whereas there is no significant difference in solutions between TRS-RSH and TRC for instances with up to 100 customers and 10 robots per depot, the objective values deviate increasingly and we obtain slightly better or worse solutions by TRS-RSH. This can be attributed to the stochastic search within our LS. Since RSH evaluates some truck tours with higher-than-actual costs, it can avoid local minima that prevent finding a better solution. On the other hand, it can also prevent the LS from finding the best solution. Both effects occur increasingly with larger instances, as can be seen in the increasing standard deviation. This effect is reinforced with very tight robot availability (e.g., see the example with 100 customers and 5 robots per depot). A lower robot availability reinforces RSH’s disadvantages in robot scheduling and therefore leads to worse objective values. On the other hand, when robots are not limiting at all, RSH’s time advantage disappears, since the trivial robot schedule to assign every customer to their cheapest stop is often feasible.

In conclusion, TRC-RSH has the potential to save computation time in cases where robot availability is not extremely low. As one of our contributions is to consider robot availability, we will rely on TRC in our further experiments as it provides more robust results and performs better for scenarios with more restricted robot availability. In practice, the choice between TRC-RSH and TRC can be made based on the number of customers and the availability of robots.

5.2.3 Comparison to benchmark approach

We compare our solution approach to the state-of-the-art approach by Boysen et al. [11]. Their approach (denoted as TRL) aims at minimizing the number of customers who receive their order late, that is, it only evaluates whether orders are delivered late, but does not account for the absolute time of delay. The multi-start local search procedure of TRL relies on a neighborhood search with standard operators for VRPs. We reimplemented this approach and use the parameter setting as indicated by the authors for the “large dataset.” We adapt our TRC approach to minimize the number of late deliveries as well (denoted as TRC lateness), and set the local search limit to $\kappa_{ls} = 2000$. TRL lateness and TRC lateness are ultimately compared with TRC applied for its intended total cost objective (denoted as TRC cost). For both TRL and TRC lateness, the total costs are obtained by an ex post calculation on the final solution using the cost factors stated.

Figure 6 shows the average results (of a sample of 20 instances per data point) for 25–125 customers.

Performance comparison for minimizing lateness

The results show that our TRC approach also performs very well if applied for the altered objective of minimizing lateness. In comparison to TRL, it is able to match the given results both in terms of computation time and objective value. The higher computation times for TRC lateness can be attributed to cost-specific starting rules and search operators. As its rules and operators are focused on total cost, it takes TRC slightly longer to converge to solutions with the minimal number of delays. The increase in runtime for all methods is mostly driven by the fact that the MIP for robot scheduling must be solved for longer truck tours with an increasing number of customers.

Impact of total cost perspective

Considering TRC applied to the cost objective, we see that its computation times are comparable to TRL. Looking at the solution structure, the comparison between TRC with cost objective and TRL further highlights the impact of a total cost perspective. TRC cost leads to a higher number of delays. However, the average delay duration (of all delays $>0$) is drastically reduced compared to the TRL results. This is because for TRL, every delayed customer is “lost” and there is no incentive to shorten an
unavoidable delay. In contrast, TRC shortens unavoidable delays. Comparing the total costs achieved by all approaches, we see that minimizing lateness while ignoring the other cost factors leads to a significant increase in actual costs. By way of example, in the case of 75 customers, TRC cost reduces total costs by 46%, average delay duration by 93% and truck mileage by 49% compared to TRL. The difference in total costs is lower when comparing TRC cost to TRC lateness since the latter already applies our cost-specific operators and therefore respects total costs while minimizing lateness. For the large data set of 125 customers, the total costs of both approaches are almost equal. The reason for this is the impact of robot availability, which we detail in the following.

Impact of limited robot availability

In TRC cost we take into account the limited availability of robots at depots (i.e., 10 robots per depot), which represents an additional restriction. This means that the more customers are to be served, the more constraining the limited robot availability gets for the routing decision. In the case of 125 customers, for instance, the truck must visit at least 12 depots to supply all customers (see increased truck distance and delay duration). The restriction reduces the cost advantage at first glance but leads to a more realistic truck tour where the maximum number of robots started at a single depot is reduced from 74 to 16. Thanks to our combination of cost optimization and limited robot availability, the maximum number of robots started remains constant.
TABLE 7 Performance of our TRC approach compared to the TRL approach of Boysen et al. [11] based on the 200 instances of the “large dataset” of Boysen et al. [11]

<table>
<thead>
<tr>
<th>Criteria</th>
<th>TRL lateness</th>
<th>TRC lateness</th>
<th>Change TRC versus TRL (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of best known solutions found</td>
<td>196</td>
<td>186</td>
<td>−5.1</td>
</tr>
<tr>
<td>Avg. objective value (number of delays)</td>
<td>0.99</td>
<td>1.03</td>
<td>+4.0</td>
</tr>
<tr>
<td>Avg. total costs (ex post evaluation) [€]</td>
<td>27.49</td>
<td>25.25</td>
<td>−8.1</td>
</tr>
<tr>
<td>Avg. truck distance (km)</td>
<td>10.09</td>
<td>8.49</td>
<td>−15.9</td>
</tr>
<tr>
<td>Avg. delay per customer (s)</td>
<td>14.38</td>
<td>10.33</td>
<td>−28.2</td>
</tr>
<tr>
<td>Avg. computation time (s)</td>
<td>217.10</td>
<td>563.13</td>
<td>+159</td>
</tr>
</tbody>
</table>

as the number of customers goes up. This makes the concept economically feasible, as depots must be affordable and easy to integrate into existing traffic space and the robot fleet size should be realistic to keep investment costs as low as possible.

5.2.4 Performance comparison based on instances from literature

We further assess the algorithmic performance with data instances of Boysen et al. [11]. The so called “large dataset” comprises 200 instances with 40 customers each. We compare our TRC to the TRL approach of Boysen et al. [11] based on their lateness objective (see Section 5.2.3).

The comparison in Table 7 confirms the results from Section 5.2.3. TRC lateness finds objective values comparable to TRL lateness. Out of 200 instances, 182 instances were solved with identical objective values by both approaches. TRC found a better solution for four instances and a worse solution for 14 instances. Due to the problem- and cost-specific search operators used in our TRC approach, it takes more effort to find lateness-optimal solutions, but it is able to find pareto improvements with respect to the associated costs: total costs, covered truck distance, and the absolute time of delays are decreased despite the objective of minimizing the number of delays. TRC’s computation times are 5–6 min higher. As already seen in Figure 6, TRL is faster for smaller instances (as it is the case here with 40 customers), but the relative difference is much lower for larger instances with more customers. Both approaches are consequently comparable with respect to time-efficiency.

5.3 Analysis of truck-and-robot performance

A detailed cost comparison with traditional delivery modes is a prerequisite for the adoption of truck-and-robot systems. As such, we analyze different parameter settings for the truck-and-robot concept and compare the TRC results with the solution for classical truck-only delivery, modeled as a vehicle routing problem (VRP) with time windows. We solve the VRP (see Appendix A) using the Gurobi solver in Python with a time limit of three hours. We adapt our problem setting to solve the VRP in reasonable time and apply the following to reduce computational effort. First, there are no waiting times. We assume that the driver can always leave the parcel at the door should he/she arrive before the time window, which reduces the time windows to deadlines. This drastically increases flexibility for the truck routes. Furthermore, we define the same processing time of 40 sec. per customer, which is an optimistic time for manual deliveries. Finally, we allow the use of up to four vehicles without incurring any fixed cost per vehicle. The solutions show that less than four vehicles are used in more than 90% of the cases and thus additional vehicles do not provide better solutions. These simplifying assumptions work in favor of the truck-only deliveries.

5.3.1 Influence of varying robot fleet size and general observations

First, we study the impact of robot availability. Figure 7 shows how the truck-and-robot concept solved by our TRC (labeled TRC cost) performs compared to the VRP subject to robot availability per depot \( r_a, a \in R \). We analyze the impact on total costs, truck distance, the average delay per customer, and the number of delays. The reported numbers are again average values from 20 instances per data point. Since truck-only delivery does not use any robots, its performance is not affected by a change in robot availability. Furthermore, since the truck-only delivery could not be solved to proven optimality within the defined time limit of three hours in all cases, we report the values of the best-known solution (labeled VRP) and the lower bound of the total cost (labeled VRP LB).

In our default case with 10 robots per depot, TRC leads to a cost saving of between 59% and 68% (compared to the lower bound and the best-known solution, respectively). In comparison to the best-known VRP solution, the truck distance is reduced by 82%. This correlates with the reduction in CO\(_2\) emissions when diesel trucks are applied. While the number of delays increases by 27%, the delay duration per customer is reduced by 68%. This shows that the VRP does not avoid delays even if
four delivery trucks are used. In contrast, the truck-and-robot approach is able to adapt its solution in order to minimize the total delay time by accepting smaller delays for a few more customers, and thus reducing total costs. Finally, in the case of truck-only delivery, an average of three trucks is required to serve all customers compared to only one truck using the truck-and-robot concept.

While cost and truck mileage for the truck-and-robot system are very competitive even for a small robot fleet, a minimum number of robots is required per depot (in our case 7). Above this minimum fleet size, the improvements in total costs achieved are small, but a reduction of the robot fleet below this level leads to a significant increase in total costs. The number of delays as well as total time of delay is minimal for ten robots per depot. Neither an increase nor a decrease leads to better solutions. The reasons for this are twofold. First, an increase in the number of robots per depot above 10 causes more delays, as the trade-off between truck cost and delay cost leads to shorter truck tours that cause delays due to longer robot trips. Second, the number of available robots per depot defines the shortest possible truck tour such that the truck can pick up enough robots. If the number of robots is reduced, the truck tour becomes longer and the probability of late deliveries from later stops rises. This effect also explains the substantial increase in truck distance, delay cost for the case there are three robots per depot. Even if all customers were located in the same area, the truck would have to visit \((|C| - \delta)/r\) depots, that is, 14 depots in this case. This leads to high truck cost and a late robot launch for the customers served last. The effects of robot availability observed highlight the need to carefully size the robot fleet for the cost-optimal supply of customers. On the other hand, our results indicate that the concept could be piloted with a very small robot fleet for a use case in which small delays are still acceptable. In summary, our results show that the proposed truck-and-robot concept outperforms classic truck delivery in terms of both cost and service criteria.

### 5.3.2 Influence of varying depot density

Since the availability of robot depots is essential for the truck-and-robot concept, we show the impact of changing depot density. For this experiment, we keep the total number of available robots as close as possible to our default setting in Section 5.1 by adapting the number of robots per depot to 28/16/10/7 for the instances with 9/16/25/36 depots, respectively. The results are summarized in Figure 8.

The results show that the impact of depot density on cost and truck distance is small. In our analysis, the number of robots per depot increases as the number of depots declines, which leads to shorter truck tours (see also Section 5.3.1). In detail,
mileage is reduced by 24% in the case of nine depots versus our default case with 25. Total costs vary between −2% (25 vs. 36 depots) and +6% (25 vs. 9 depots) only, and do not reveal a clear trend. Note that due to the equidistant arrangement of depots assumed, a change in their number affects all depot coordinates, and thus a solution with fewer depots may be better if those depots happen to be closer to the random customer locations. This explains the cost minimum for 16 depots. The most striking observation is the increase in the number of delays together with total time of delay. If fewer depots are available, the best solution accepts an increase in delays in favor of shorter tours. This observation is in line with our results from Section 5.3.1. In summary, fewer depots can be attractive with regard to cost and emissions but significantly reduce on-time deliveries. This highlights the advantage of the truck-and-robot concept with depots versus other approaches that involve only one large robot hub in which robots are loaded and launched.

5.3.3 Influence of varying robot costs

The cost of a single robot is an important input factor for the truck-and-robot distribution, and may vary significantly for different situations. This depends on market dynamics, volumes produced and technical details of the different robot models. We therefore analyze the impact of varying robot costs on the overall problem. The results are illustrated in Figure 9.

As could be expected, total costs show a linear increase as the robot cost rate rises. However, even if our basic rate of 0.50 €/h is quadrupled to 2.00 €/h, total costs increase by only 131% and remain competitive compared to truck-only delivery. This means that even with significantly higher robot costs, the truck-and-robot concept enables distribution with lower costs compared to the lower bound of the VRP. The robot cost increase is only partly mitigated via longer truck tours (i.e., the truck visits more stops close to customers such that robot time is reduced). In the case where robot costs are 2.00 €/h, we found that robot time was reduced by 34% compared to the base case, leading to a 164% increase in overall costs for robots. Additional deviations on the truck tour to reduce robot time would lead to delays for customers who are served from late stops and are thus inefficient. Up to 300% of the original cost rate, we can again observe the positive effect of longer truck tours on delays (also see Section 5.3.1). Our two delay measures decrease by 9% each with higher robot costs as more distance is covered by the truck and this reduces robot travel times and consequently delays. The increase in the truck distance of 77% is still acceptable considering the great reduction in travel distance compared to traditional truck delivery.
5.4 Cost impact of changing demand

The number of orders may increase as the service becomes more popular with consumers. Consequently, we also analyze the impact of changing demand on the cost structure. Figure 10 provides a breakdown of the costs depending on the number of customers served. Please note that waiting cost and costs for robot travel times are summarized as robot costs.

The truck usage cost (both for time and distance) increases linearly with the number of customers. As the number of customers quintuples from 25 to 125, so does the truck cost (+450%). The robot costs (including a growing share of 20–38% waiting costs), on the other hand, only increase moderately. Consequently, a change in order volumes within a certain range does not change the robot fleet needed or its utilization but mostly the shift and tour length of the truck. In the event of lower order volumes, the truck-and-robot concept therefore helps to save variable truck costs (e.g., fuel and personnel cost) as the use
of robots enables short delivery tours. Furthermore, the delay cost increases moderately for scenarios with up to 100 customers as well. As a consequence, the truck-and-robot concept robustly reveals good service performance for the corresponding scenarios. Afterwards, the delays increase significantly for supplying another 25 customers (125 customer case), meaning that a higher number of robots per depot would be necessary to decrease the number of delays (see Section 5.3.1). In conclusion, these results underline the flexibility and advantages of the truck-and-robot system but also identify the limits for on-time deliveries and the need to adapt the robot system to the given requirements.

6 | CONCLUSION

The truck-and-robot concept is an innovative solution for last-mile delivery. Our model extends this concept by identifying relevant costs and therefore enabling total cost evaluation. Furthermore, we include a setting with practical relevance where the available robot fleet is limited. A specialized heuristic is presented to address this problem based on problem-specific construction heuristics, search operators for truck tours and an MIP to find optimal robot schedules for given truck tours and robot availability. Additionally, we introduce a heuristic alternative for the robot scheduling step that further decreases runtimes. In numerical experiments we analyze the main characteristics of the truck-and-robot concept and their impact on total costs. We show the efficiency of our approach via comparison with a benchmark method and highlight the need for a total cost perspective. Furthermore, we compare the truck-and-robot approach to classic delivery by trucks. In summary, the findings from the experiments can provide guidance for planning and operating a truck-and-robot system. A truck-and-robot system can reduce costs by up to two-thirds compared to classic truck delivery and at the same time reduce the truck fleet required. The savings potential depends mostly on the truck driver labor costs and the robot purchase prices. However, even for significantly higher robot costs in the early stage of the innovation cycle we show that

(i) the concept is attractive for companies, as total costs are below the cost of conventional truck-only delivery,
(ii) high level of on-time deliveries is not opposed to cost-optimal routing, and
(iii) truck distance and thus local emissions are reduced by more than 60% as a by-product (without incurring additional costs).

For a given delivery area, a certain minimum number of robots is required to ensure high service quality. Beyond that number, improvements due to additional robots are small. Depot density has little impact on cost and emissions but moderate impact on service quality.

There are numerous opportunities for future research. To begin with, the approach presented can help evaluate the effects of urban planning decisions on a truck-and-robot system. Certain zones could be forbidden for the truck or the robots, for example, while in other zones robots could be allowed to travel at higher speed. Such constraints can be modeled by adapting the corresponding index sets and travel times. Moreover, the robot fleet needed could be further reduced by allowing robots to travel between depots such that robot availability is increased at locations visited by the truck. Robot availability could be further improved by considering the robots’ return from customers to depots (with stochastic arrival times). To enable this, decisions on the robot movements must be made simultaneously with the delivery routing decisions. In addition, deliveries without time windows or for bulky goods that require manual delivery can be included in the planning problem. For the heuristic, this means customers are then potential or required stops on the truck route. Another interesting aspect is the allocation of customers to trucks and the start time of the truck, which we assume as given. In practice, these decisions must be made such that they enable our method to find efficient tours. Lastly, solving truck-and-robot problems requires tailored solution approaches and innovative algorithms. There are further approaches for related VRP variants that could be adapted and tested for the truck-and-robot concept.

ACKNOWLEDGMENTS

We owe a debt of gratitude to the industry experts who have provided empirical data and practical insights as well as encouraged our research topic. We further want to thank the editors and reviewers for their valuable advice on improving this work. Open access funding enabled and organized by Projekt DEAL.
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How to cite this article: Ostermeier M, Heimfarth A, Hübner A. Cost-optimal truck-and-robot routing for last-mile delivery. *Networks*. 2022;79:364–389. https://doi.org/10.1002/net.22030

APPENDIX A: A MIP MODEL FOR THE VRP WITH DEADLINES

We introduce the following MIP model as a benchmark for the truck-and-robot concept. It minimizes the cost of traditional truck-based manual delivery, assuming the same cost factors as in the truck-and-robot case. We assume that the driver can leave the parcel at the door in the event that he/she arrives before the time window, which reduces the time windows to deadlines and favors the VRP. Furthermore, we assume the same processing time of 40 s per customer, which is again on the optimistic side and favors the VRP. We additionally introduce the number of available vehicles, which also reduces the travel time.

The model is formulated as follows:

\[
\begin{align*}
\min & \quad \sum_{i \in C} \sum_{j \in C} s_{ij} \cdot c_{ij}^\text{VRP} + \sum_{k \in C} v_k \cdot c^\ell \\
\text{s.t.} & \quad \sum_{j \in C} c_{ij}^\text{VRP} = 1, \quad \forall i \in C, \\
& \quad s_{ij} \in \{0, 1\}, \quad \forall i, j \in C, \\
& \quad v_k \geq 0, \quad \forall k \in C, \\
& \quad t_k \geq 0, \quad \forall k \in C. 
\end{align*}
\]

(A1)

This leads to the objective function (A1), which incorporates the cost of truck travel and delays. Constraint (A2) ensures that every customer is visited exactly once. (A3) and (A4) keep track of the arrival times at customers and (A5) derives the delay from the arrival time. Constraints (A6) and (A7) establish flow constraints for the trucks at every stop. Equations (A8) and (A9) define the solution space.
subject to

\[
\sum_{i \in C \cup \{ \gamma \}} s_{ik} = 1 \quad \forall k \in C \quad (A2)
\]

\[
\theta_{rk} - M \cdot (1 - s_{rk}) \leq t_k \quad \forall k \in C \quad (A3)
\]

\[
t_i + \theta_{ij} - M \cdot (1 - s_{ij}) \leq t_j \quad \forall i, j \in C \quad (A4)
\]

\[
t_k - d_k \leq v_k \quad \forall k \in C \quad (A5)
\]

\[
\sum_{i \in C \cup \{ \gamma \}} s_{i k} = \sum_{i \in C \cup \{ \gamma \}} s_{ki} \quad \forall k \in C \quad (A6)
\]

\[
\sum_{i \in C} s_{ik} \leq f \quad (A7)
\]

\[
s_{ij} \in \{0, 1\} \quad \forall i, j \in C \quad (A8)
\]

\[
t_k \geq 0; \ v_k \geq 0 \quad \forall k \in C \quad (A9)
\]