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Multi-compartment vehicle routing problems: State-of-the-art, modeling framework and future directions

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ABSTRACT

Among the many extensions of the classical capacitated vehicle routing problem, multi-compartment vehicle routing problems have been studied extensively only in recent years. Vehicles with multiple compartments enable the joint delivery or collection of goods with differing characteristics in separate compartments that would otherwise need separate transportation with single-compartment vehicles. This enables greater flexibility in routing decisions and order assignment to tours. The versatile use of these vehicles is leading to increasing relevance in both research and industry, and consequently in an increasing number of related publications. The available studies, however, consider substantially different problem variants. As no survey on multi-compartment vehicle routing problems is available so far, the identification of common problem features and research opportunities has been difficult. This paper aims at overcoming this difficulty by proposing an extended typology for multi-compartment vehicle routing problems and extensively reviewing the existing literature. Although only few identical problems can be identified, common attributes among similar applications (regarding compartment flexibility, for example) are observed. Suggestions for future research directions are also proposed.

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1. Introduction

This paper deals with vehicle routing problems (VRPs) in which the loading space of the vehicles is or can be compartmented. Such multi-compartment vehicles (MCVs) are used to consolidate product flows in situations where different product types must be kept separated during transportation. Typical applications refer to the distribution of different petroleum products (e.g., diesel and super fuel) to petrol stations, the delivery of different kinds of temperature-sensitive groceries (e.g., frozen, fresh and ambient products) to supermarkets, or the collection of different waste types (e.g., different-colored glass waste) from containers at waste collection points. The utilization of MCVs allows for joint transportation of different product types on the same vehicle from a depot to the customers (e.g., petrol stations, supermarkets) or from the customers (e.g., waste collection points) to a depot, and provides a wide range of advantages, especially when products of different types have been ordered by the same customer or

have to be collected at the same collection point. In contrast to situations in which only single-compartment vehicles (SCVs) are available and each product type has to be transported on a separate SCV (see Fig. 1), the number of stops at customer locations or collection points, the number of necessary vehicles, and the total length of all tours may be reduced significantly with an MCV (see Fig. 2). Figs. 1 and 2 show that three dedicated SCV tours are required with seven stops in total, whereas with an MCV only a single tour with four stops may be necessary (assuming sufficient MCV capacity).

For the sake of simple explanation, in the following we will only refer to the distribution of products to customer locations. Nevertheless, our presentation likewise applies to the collection of goods from collection points, unless stated otherwise. As in classical vehicle routing, operational planning related to multi-compartment vehicle routing problems (MCVRPs), i.e., VRPs where the vehicles may have different compartments, requires the determination of a set of tours that specifies the customers to be visited on each tour and the sequence in which they have to be vis-

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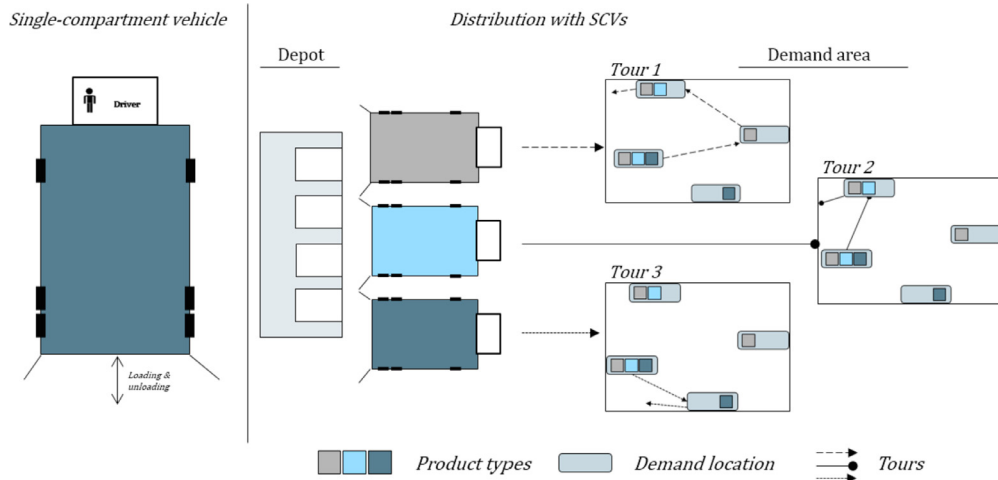


Fig. 1. Distribution process with SCVs.

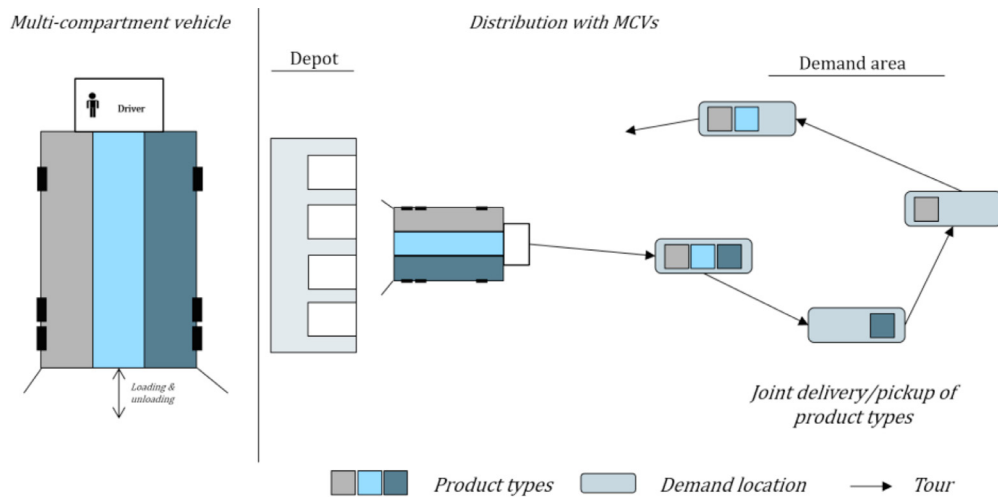


Fig. 2. Distribution process with MCVs.

ited. When planning the tours of MCVs one needs to deal with compartment-related particularities, such as the flexibility of the compartment setup, as well as delivery-related requirements, such as the options for consolidating different customer demands on the same delivery tour. We note that – in comparison to classic VRPs – additional specific constraints may need to be considered and additional decisions may have to be made. It has to be determined whether SCVs or MCVs should be used, and, in the latter case, what the compartment setup should be like and which product types and demands should be assigned to each of the compartments. These additional properties and decisions increase the problem complexity significantly.

MCVs have been in use for supplying petrol stations for decades, while respective technologies are relatively new for grocery distribution, where such vehicles have only been introduced a decade ago, although their number has been growing rapidly since then. Increasing availability of MCVs for applications in different industries has initiated MCVRP-related research, with the first studies tracing back to the 1980s. In total, 84 publications could be identified that focus on decision models and respective solution approaches for MCVRPs. The problems considered vary substantially with respect to the field of application and problem delimitation. However, their treatment may exhibit

different levels of detail and may be based on varying basic assumptions concerning objective functions and constraints. As a consequence, many MCVRP variants have been discussed, but no general problem formulation and model representation have been proposed so far. Not even a unique terminology has emerged over time.

Obtaining a clear view of the developments in the field is therefore not a straightforward task. Our paper is meant to support this issue. It presents the first comprehensive literature review that focuses entirely on MCVRPs. Section 2 will introduce consistent terminology and notation, define a conceptual MCVRP constituting a template for future research, and propose a respective model representation. Section 3 will introduce a typology based on attributes for the characterization of more specific MCVRPs. It will make it possible to categorize the different contributions and point out similarities and differences of the MCVRPs as well as identify the state-of-the-art in the field. Section 4 then discusses related MCVRP literature and provides details on the attributes used. Section 5 will provide a comparative analysis of the MCVRP literature with respect to the attributes introduced and proposed solution approaches. This section also includes a discussion of opportunities for future research. Finally, we will summarize our findings and contribution in Section 6.

2. A general multi-compartment vehicle routing problem and its formulation

This section introduces the general problem setting of MCVRPs and provides a corresponding general model. It serves as a basis for the introduction of further problem variants and modeling adjustments. Attributes for the definition of such variants will be discussed in Section 3. The capacitated VRP (CVRP) is the core element of MCVRPs. We will therefore first describe those characteristics of the CVRP that also apply to MCVRPs.

CVRP-related fundamentals. The (asymmetric) CVRP is defined on a directed, complete and weighted graph $G = (V, A)$, where the vertex set V represents the set of n customer locations ($\{1, \dots, n\}$) and a single depot ($\{0\}$). The corresponding set of arcs is denoted by $A = \{(i, j) | i, j \in V : i \neq j\}$ and a travel cost c_{ij} is associated with each arc $(i, j) \in A$. Furthermore, there is a demand of d_i units ($d_i \geq 0$) for a single homogeneous product type at every customer location i . A homogeneous set of vehicles is available at the depot for the fulfillment of demands. K denotes this set of vehicles, and each vehicle k has an identical (total) capacity Q . Consequently, the decisions to be made within the CVRP consist of assigning demands to vehicles and determining tours for all vehicles during which the deliveries are to be performed such that all demands are satisfied, the vehicles' capacities are respected, and the total travel costs are minimized.

Framework of the general MCVRP. MCVRPs mainly extend the CVRP with respect to two attributes. First, instead of a single product type, a set of product types $p, p \in P$, is considered, and the product types must not be mixed during transportation due to distinct transportation requirements. An order exists at each customer location i that consists of non-negative demands d_{ip} for multiple product types p , and each location may be visited several times to supply the different product types. Second, in an MCVRP, the loading space of a vehicle can be separated into a limited number m^{\max} of compartments, in each of which exactly one product type can be transported. We therefore define $M = \{1, \dots, m^{\max}\}$ as the set of compartments into which a vehicle's loading space can be separated. This feature enables the simultaneous transportation of several non-mixable product types in different compartments of a vehicle. The number and the capacity of individual compartments in a vehicle are not fixed with respect to the compartment-related attributes assumed in the general MCVRP, but can be adjusted flexibly (i.e., between 1 and m^{\max} compartments of different sizes may be used on each vehicle). The division into compartments is only limited by the total capacity Q of a vehicle's loading space. Moreover, each product type may be assigned to any compartment for transportation. Demands of multiple customers for the same product type may be assigned to the same compartment. We would like to note that in our context a homogeneous vehicle fleet represents a situation where all vehicles have the same total capacity (Q), while the setup of compartments may vary (i.e., the number of compartments and associated product types) between individual vehicles of the same fleet. With respect to delivery requirements, it is assumed that demands d_{ip} and d_{ir} for different product types p and r of one customer i , as well as individual demands for one product type p of different customers i and j , may be split across different vehicles. In this general MCVRP it has to be decided simultaneously (i) which compartments of which sizes will be used for which product type, (ii) which demands are assigned to which compartments, and (iii) in which sequence the different locations are to be visited by the vehicles such that (iv) the decision-relevant costs are minimized.

A model formulation for the general MCVRP. We introduce the following decision variables in order to formulate the mathematical model:

$u_{ipk} \in [0, 1]$: share of product p that is delivered by vehicle k to customer i , $i \in V, p \in P, k \in K$;

$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ is traveling from location } i \text{ to } j, \\ 0, & \text{otherwise,} \end{cases} \quad i, j \in V, k \in K$;

$y_{pkm} = \begin{cases} 1, & \text{if product type } p \text{ is assigned to compartment } m \\ & \text{of vehicle } k, \\ 0, & \text{otherwise,} \end{cases} \quad p \in P, k \in K, m \in M$.

The objective function and the constraints of the model can then be formulated as follows:

$$\text{Minimize } \sum_{i,j \in V} \sum_{k \in K} c_{ij} \cdot x_{ijk} \tag{1}$$

subject to

$$\sum_{k \in K} u_{ipk} = 1, \quad i \in V \setminus \{0\}, p \in P, \tag{2}$$

$$u_{jpk} \leq \sum_{i \in V} x_{ijk}, \quad j \in V \setminus \{0\}, k \in K, p \in P, \tag{3}$$

$$\sum_{j \in V} x_{ijk} \leq 1, \quad i \in V \setminus \{0\}, k \in K, \tag{4}$$

$$\sum_{j \in V} x_{ijk} = \sum_{j \in V} x_{jik}, \quad i \in V \setminus \{0\}, k \in K, \tag{5}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1, \quad k \in K, S \subseteq V \setminus \{0\}, |S| \geq 2, \tag{6}$$

$$\sum_{i \in V \setminus \{0\}} d_{ip} \cdot u_{ipk} \leq Q, \quad p \in P, k \in K, \tag{7}$$

$$\sum_{p \in P} y_{pkm} = 1, \quad k \in K, m \in M, \tag{8}$$

$$\sum_{i \in V \setminus \{0\}} u_{ipk} \leq |V| \cdot \sum_{m \in M} y_{pkm}, \quad p \in P, k \in K, \tag{9}$$

$$u_{ipk} \in [0, 1], \quad i \in V \setminus \{0\}, p \in P, k \in K, \tag{10}$$

$$x_{ijk} \in \{0, 1\}, \quad i, j \in V, k \in K, \tag{11}$$

$$y_{pkm} \in \{0, 1\}, \quad p \in P, k \in K, m \in M. \tag{12}$$

The objective function (1) minimizes the total travel cost of all tours. Constraints (2) ensure that the complete demand for a product type is fulfilled for each customer. A customer must be visited on a tour if at least one of their demands is assigned to the corresponding vehicle (constraints (3)). Furthermore, each vehicle is only allowed to visit each customer once at most (constraints (4)), and whenever a vehicle visits a customer location, it must also depart from it (constraints (5)). The elimination of subtours is ensured by constraints (6). Constraints (7) ensure that the demands assigned do not exceed the vehicle capacity. Please note that we only need to consider the total vehicle capacity as the compartment sizes can be adjusted flexibly. Moreover, constraints (8) and

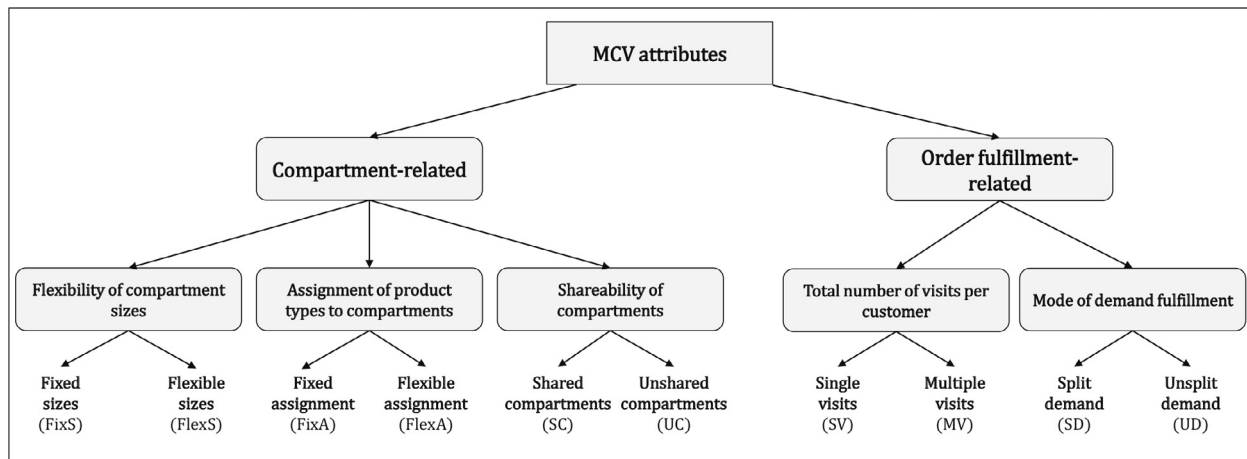


Fig. 3. Attributes of multi-compartment vehicle routing problems.

(9) consider the assignment of products to vehicles and compartments. The former ensure that only a single product type can be assigned to each compartment of a vehicle, while the latter connect the u_{ipk} variables to the y_{pkm} variables. Finally, the variable domains are denoted by constraints (10)–(12). A solution of this model represents a specification of the compartment setup (i.e., number and size of compartments) of each vehicle, an assignment of customer demands to vehicle compartments, and the determination of routes for each vehicle.

The general MCVRP and its formulation provide a basis for modeling different problem variants. Additional decision facts (e.g., concerning the question of whether and how orders should be split), additional constraints (e.g., loading constraints), and additional objective function components (e.g., loading and unloading costs) could be integrated into this basic formulation depending on the respective specific MCVRP. This is also true for classical VRP extensions such as multiple periods, time window constraints, weight constraints or route length restrictions. However, this paper focuses on attributes that are relevant to MCVRPs and that uniquely arise from using MCVs, i.e., attributes related to compartments and to the splitting of demands and deliveries. Such attributes and the corresponding model extensions will be presented and discussed in the following section. A unified terminology for MCVRPs will also be introduced as a result.

3. A typology of multi-compartment vehicle routing problems

The development of a typology for different MCVRPs requires considering highly heterogeneous problem variants and applications, since the problems discussed in the literature vary substantially. Comprehensive classifications for VRPs in general have already been suggested. Among others, Lahyani, Khemakhem, and Semet (2015b) recently introduced a taxonomy of rich vehicle routing problems, and Pollaris, Braekers, Caris, Janssens, and Limbourg (2015) defined categories of loading constraints for VRPs. Coelho and Laporte (2015) provided a classification of different MCVRP formulations in fuel distribution, but they only focused on two problem-specific attributes. Overall, none of these and no other review addresses MCVRP-specific attributes in a general manner and provides a comprehensive review of related literature.

We therefore introduce a typology for MCVRPs that considers problem attributes. We present a short description for each attribute and discuss necessary modifications of the general model ((1)–(12)). We will focus on attributes with respect to compartment-related specifications (see Section 3.1) and order fulfillment requirements (see Section 3.2). Fig. 3 gives an overview of the attributes discussed in the following. The typology developed

is used as a basis for the literature review (Section 4) to describe and distinguish problem characteristics.

3.1. Compartment-related attributes

With respect to the characterization of compartment-related properties, three types of attribute can be identified: (a) flexibility of compartment sizes, (b) assignment of product types to compartments and (c) shareability of compartments.

(a) Flexibility of compartment sizes. The size of the compartments can either be fixed (i.e., given) or flexible. The decision concerning the use of fixed or flexible compartment sizes determines the number of compartments of each vehicle. If fixed compartment sizes are used, the available vehicle capacity Q is separated into a distinct compartment setup with given capacities, and thus the number of compartments is predetermined. Otherwise, if flexible compartment sizes are used, the capacity of each vehicle can be split into up to m^{\max} compartments; the number of compartments therefore remains flexible as well.

In the general model ((1)–(12)), both the number and the sizes of compartments are flexible. We therefore introduce modifications of the model for the consideration of fixed compartment sizes. Additional constraints and parameter modifications are required for this case. First of all, let m^{\max} now equal the exact number of compartments in each vehicle (i.e., a fixed and not a maximum number of compartments). Furthermore, a fixed compartment size q_m ($q_m \leq Q$) for each compartment has to be introduced. Constraints (13) ensure that the given compartment capacities are not exceeded.

$$\sum_{i \in V \setminus \{0\}} \sum_{p \in P} d_{ip} \cdot u_{ipk} \leq q_m, \quad k \in K, \quad m \in M. \quad (13)$$

In general, whether compartment sizes are fixed or flexible can also be defined by the given compartment capacities (q_m) in a more implicit manner (Derigs et al. (2011)). If we consider a problem in which the sum of the given compartment capacities equals the vehicle capacity ($\sum_{m \in M} q_m = Q$), then the number of compartments must be fixed as the capacity share of each compartment is clearly defined. Not using a compartment m in this case means that the respective capacity q_m will remain unused in the vehicle as the remaining compartments are also fixed in size and cannot be adjusted. On the other hand, flexible compartment sizes can be implicitly indicated by allowing the sum of the theoretical compartment capacities to be larger than the total vehicle capacity ($\sum_{m \in M} q_m > Q$). This means that compartment capacities become potential capacities, as the full theoretical capacity of each com-

partment cannot be used, but instead the actual size of one compartment is dependent on the vehicle capacity taken up by the remaining ones. In this case, constraints (13) are required as long as the potential capacities of individual compartments are smaller than the vehicle capacity (i.e., $q_m < Q, \forall m \in M$). As soon as the potential capacities equal the vehicle capacity, constraints (13) become redundant and constraints (7) suffice as capacity restrictions. Such a setting allows full compartment flexibility with individual compartment sizes between 0 and the vehicle capacity Q , and is covered in our general model.

(b) *Assignment of product types to compartments.* The assignment of product types to compartments may either be predetermined (*fixed assignment* of a specific product type to a specific compartment), or it can be a decision made depending on the problem (*flexible assignment* of all product types to any compartment). A fixed assignment of product types to compartments ensures that a given product type can only be assigned to a specific compartment, and therefore assignment to a vehicle is only possible if the compartment required is available on this vehicle. This is the case, for instance, if a compartment is dedicated to a specific kind of fuel (e.g., diesel fuel), and this compartment is not allowed to be filled with other types of fuel. In order to incorporate this case into the basic model, a parameter $a_{pkm} \in \{0, 1\}$ is introduced indicating whether compartment m can be used for transportation of product type p in vehicle k or not. Moreover, the following constraints (14) are added to ensure that a product type will only be assigned to a suitable compartment:

$$y_{pkm} \leq a_{pkm}, \quad p \in P, k \in K, m \in M. \quad (14)$$

In the special case where the number of product types is equal to the number of compartments ($|P| = m^{\max}$), the model can be further simplified, namely by setting the product type index equal to the compartment index. Consequently, the y_{pkm} -variables are no longer necessary, and constraints (8), (9) and (12) can be eliminated.

(c) *Shareability of compartments.* A compartment may either contain the demand quantities for a single product type of multiple customers (*shared compartments*), or just the demand quantity for a single product type of a single customer (*unshared compartments*). Shareability of compartments has also been suggested by Coelho and Laporte (2015) to differentiate between shared and unshared tanks in fuel distribution. The more general case of shared compartments is considered in our general model, while the second case requires the implementation of additional variables and constraints in the mathematical formulation. More precisely, we introduce auxiliary binary variables $\hat{u}_{ipkm} \in \{0, 1\}$ indicating whether the demand of customer i for product type p is assigned to compartment m of vehicle k or not. These additional variables are connected to variables u_{ipk} by adding constraints (15), while constraints (16) ensure that a compartment cannot be assigned to the demand of more than one customer. Constraints (17) define the new binary variables.

$$u_{ipk} \leq \sum_{m \in M} \hat{u}_{ipkm}, \quad i \in V \setminus \{0\}, p \in P, k \in K, \quad (15)$$

$$\sum_{i \in V \setminus \{0\}} \hat{u}_{ipkm} \leq y_{pkm}, \quad p \in P, k \in K, m \in M, \quad (16)$$

$$\hat{u}_{ipkm} \in \{0, 1\}, \quad i \in V \setminus \{0\}, p \in P, k \in K, m \in M. \quad (17)$$

Summary of compartment-related attributes. We use three types of compartment-related attributes in our typology. In summary, the following characteristics are used:

- Fixed compartment sizes (**FixS**) and flexible compartment sizes (**FlexS**),

- fixed assignment of product types to compartments (**FixA**) and flexible assignment of product types to compartments (**FlexA**),
- shared compartments (**SC**) and unshared compartments (**UC**).

3.2. Order fulfillment-related attributes

In the classical CVRP, the fulfillment of a single demand cannot be executed by several vehicles. However, problems with the split fulfillment of demands are relevant in practice and have been studied in the literature (known as VRPs with split deliveries; see, e.g., Archetti & Speranza (2012)). When multiple product types are considered, this terminology is not sufficiently precise as it may indicate split deliveries for a single product type to a customer or the separated delivery of multiple product types. We therefore suggest an extended terminology that distinguishes between two attributes, namely (a) the *total number of visits per customer* and (b) the *mode of demand fulfillment*.

(a) *Total number of visits per customer.* This attribute defines whether all demands of a single customer for multiple product types must be delivered by one vehicle only (*single customer visits*), or whether demands of a single customer for different product types may be delivered by different vehicles (*multiple customer visits*). More precisely, multiple customer visits enable the delivery to one particular customer with different vehicles for each product type ordered, while *single customer visits* ensure that each customer is only approached once. In the general model ((1)–(12)), the more general case of multiple customer visits is considered. Adapting this model to the case of single customer visits requires the following modification: In addition to constraints (4), constraints (18) are required to ensure that each customer is only visited by a single vehicle.

$$\sum_{j \in V} \sum_{k \in K} x_{ijk} \leq 1, \quad i \in V \setminus \{0\}. \quad (18)$$

(b) *Mode of demand fulfillment.* The differentiation of the demand fulfillment attribute indicates whether a single demand of a customer for a single product type must be delivered by one vehicle only (*unsplit customer demands*), or whether it may be split and delivered by more than one vehicle (*split customer demands*). Consequently, in the event of split customer demands, it is not just that different product types can be supplied by different vehicles, but the demands of one product type can also be split to be delivered by different vehicles. Similarly, Coelho and Laporte (2015) distinguish between split and unsplit (fuel) tanks at the customer sites. In order to incorporate unsplit customer demands in the model formulation, it is sufficient to define the decision variables u_{ipk} as binary ($u_{ipk} \in \{0, 1\}, i \in V \setminus \{0\}, p \in P, k \in K$).

Summary of order fulfillment-related attributes. The differentiation between the two types of split customer demands can be formulated in the following way: If demands for multiple product types p are regarded, the case will be referred to as *single* or *multiple customer visits*. If only a single demand for a single product type p is regarded, the case will be referred to as *split* or *unsplit customer demands* (c.f. classical split deliveries). Consequently, problems with single customer visits always consider unsplit customer demands, whereas problems with multiple customer visits may either consider split or unsplit customer demands.

Summarized, with respect to order fulfillment-related attributes, the following characteristics are introduced in our typology:

- Single customer visits (**SV**) and multiple customer visits (**MV**),

- split customer demands (**SD**) and unsplit customer demands (**UD**).

3.3. Further attributes

Many additional attributes have been proposed for the characterization of specific MCVRPs in the literature. For instance, MCVRPs with heterogeneous vehicles, multiple depots, stochastic demands, time windows or multiple periods have been discussed. However, these attributes can also be considered in the context of other VRP variants and are not exclusive to MCVRPs. We thus focus on those attributes that constitute the uniqueness of MCVRPs and refer to the literature for further well-known attributes characterizing VRP variants. With respect to the latter, we use the terminology of Lahyani et al. (2015b). In the following literature review, we categorize each MCVRP according to the newly introduced attributes and additionally provide information on further VRP attributes.

4. Literature overview and analysis

This section presents a comprehensive review of the literature on MCVRPs. It covers contributions in peer-reviewed scientific journals, handbooks, and conference proceedings. Routing of vehicles accounts for the core aspect in VRPs. As a result, only papers explicitly dealing with routing decisions are considered. Studies regarding multiple compartments that do not involve routing decisions are only exceptionally included here if the respective models are fundamental for MCVRPs or provide valuable extensions for future research.

The MCVRP literature is very heterogeneous and differences arise from varying applications in the first place. We thus group all publications according to their area of application, since MCVRPs in the same field tend to consider more homogeneous problems. The categories refer to fuel distribution (Section 4.1), waste collection (Section 4.2), agricultural transportation (Section 4.3), grocery distribution (Section 4.4), and maritime transportation (Section 4.5). All other publications that do not distinctly fit into any of these categories are reviewed in Sections 4.6 (other applications) and 4.7 (conceptual work). We first provide a problem overview for each area of application and then subdivide the subsequent discussion according to pivotal problem attributes (e.g., multi- and

single-period problems). For each publication, we limit our discussion to additional problem attributes that are not included in our typology, and the respective solution approach. Our findings are summarized for each area of application using comprehensive tables. The tables include information on MCVRP-specific attributes as well as on the five most frequently found VRP-related attributes, namely multi-period contexts, stochastic demands, heterogeneous vehicle fleets, time windows, and tour duration or distance limitations.

4.1. MCVRPs in fuel distribution

4.1.1. Problem overview

The distribution of fuel to petrol stations was the first industrial application in which the utilization of MCVs was considered. One or several fuel types have to be transported from refineries or fuel distributors to petrol stations or other customers. The complete content of a compartment is usually delivered to a single customer (UC). Moreover, only a few publications consider problems where vehicles are equipped with flow meters by which the amount of fuel to be unloaded from a compartment may be monitored. The available literature in this area only considers vehicles with fixed compartment sizes (FixS). Most studies consider problems with flexible assignment of fuel types to compartments (FlexA) and problems in which each fuel station should only be visited once in order to fulfill its demands. This means that the most common analysis is of unsplit customer demands (UD) and single customer visits (SV). Table 1 summarizes the respective literature.

4.1.2. Related literature

Single-period problems. Brown and Graves (1981) were the first to study vehicles with multiple compartments. However, routing decisions are not explicitly considered since only a single customer may be assigned to any tour, where each vehicle may perform a sequence of single-customer trips subject to a shift length limitation. The study includes customer time windows, a heterogeneous fleet, multiple depots and technical accessibility constraints. In addition, some product types can only be assigned to specific compartments, and specific loading patterns must be respected when a vehicle's capacity is not fully used. The authors aim at minimizing the sum

Table 1
Characterization of MCVRP literature in fuel distribution (in chronological order).

Publication	MCVRS-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Brown and Graves (1981)	x			x		x		x		x			x	x	x
Brown et al. (1987)	x			x		x		x		x			x	x	x
van der Bruggen et al. (1995)	x		x	x	x			x		x	x		x	x	x
Avella et al. (2004)	x			x		x		x		x			x		x
Cornillier et al. (2008a)	x			x		x		x		x			x		x
Cornillier et al. (2008b)	x			x		x	x			x			x		x
Ng et al. (2008)	x			x		x	x			x			x		
Cornillier et al. (2009)	x			x		x		x		x			x	x	x
Cornillier et al. (2012)	x			x		x		x		x			x	x	x
Popović et al. (2012)	x			x		x		x		x	x				
Vidović et al. (2014)	x			x		x		x		x	x				
Coelho and Laporte (2015)	x		x	x	x	x	x			x	x		x		
Benantar et al. (2016)	x			x		x		x		x			x	x	x
Urli and Kilby (2017)	x			x	x		x			x		x	x	x	x
Hsu et al. (2018)	x			x		x		x		x			x		x
Yahyaoui et al. (2018)	x		x		x		x			x					
Kaabachi et al. (2019)	x		x		x		x			x					x
Share in %	100	0	24	88	29	76	35	65	24	82	35	0	71	47	71

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

of transportation costs and penalty costs for shift length violations. Brown, Ellis, Graves, and Ronen (1987) analyze a similar problem in which multiple customers can be visited on each tour. In extension to transportation costs, they take balanced workloads and delivery quantities into account in the objective function. The authors develop a dispatching system that decomposes the problem into subproblems, which are then solved sequentially. Ng, Leung, Lam, and Pan (2008) present an MCVRP with a heterogeneous fleet and up to three customers per tour. Demands are deterministic, but the actual delivery quantities may be adjusted within certain thresholds. The multi-objective problem consists of maximizing the quantities delivered as well as minimizing the number of trips and the number of customers visited during a trip. It is solved with a specialized heuristic. Avella, Boccia, and Sforza (2004) consider a heterogeneous fleet in which compartments are unshared between customers and may only be filled completely or not at all. Furthermore, each vehicle may perform several trips while a shift length limitation for each vehicle driver must be respected. The authors propose a simple heuristic approach and a branch-and-price algorithm. Cornillier, Boctor, Laporte, and Renaud (2008a) focus on the so-called tank-truck loading problem with a heterogeneous fleet, in which the assignment of demand quantities to compartments has to be determined for a given assignment of customers to vehicles. Each customer may only be visited once and at most two customers can be visited on any duration-constrained tour. The authors introduce a complete enumeration and a column generation approach. A similar problem is studied by Cornillier, Laporte, Boctor, and Renaud (2009), in which the number of customers per tour is restricted to four. In addition to Cornillier et al. (2008a), time windows, multiple trips, shift length limitations, and the option of overtime are taken into account. The goal consists of maximizing the total profit, which is determined by the difference between revenues generated from the quantities delivered and the sum of variable transportation and variable overtime costs. The authors develop two matheuristics (MHs) for solving the problem. Cornillier, Boctor, and Renaud (2012) additionally consider multiple depots. They propose an exact method in which all feasible tours are determined in a first step, and a subset of these tours is selected by means of a mathematical model in a second step. Benantar, Ouafi, and Boukachour (2016) investigate an MCVRP with known demands, which may be rationed downwards to a certain extent. Their study incorporates time windows for the customers and geographical accessibility constraints. A heterogeneous fleet of owned and rented vehicles is also considered as given, and consequently rental fees are included in the objective function. The authors suggest a tabu search (TS) approach to solve the problem. Hsu, Walteros, and Batta (2018) deal with a problem in which delivery quantities are limited by tank capacities and fuel levels at the customer sites. Since compartments are unshared and must always be filled completely by the distributor, it may be necessary to return residual contents if the customers' tanks cannot take a full compartment. The objective function aims at maximizing the total profit, which derives from the difference between revenues from fuel deliveries and the costs of returning residual contents. The authors develop a variable neighborhood search (VNS) as a solution approach. Yahyaoui, Kaabachi, Krichen, and Dekdouk (2018) examine shared compartments and a fixed assignment of product types to compartments. They propose a VNS as well as a genetic algorithm (GA). A similar problem with additional tour length limitations is considered by Kaabachi, Yahyaoui, Krichen, and Dekdouk (2019) and solved by a hybrid artificial bee colony algorithm and a hybrid VNS.

Multi-period problems. An MCVRP with many case-specific properties is studied by van der Bruggen, Gruson, and Salomon (1995) in a multi-period context. This considers various aspects in-

cluding decisions about the opening and closing of intermediate depots, the composition of the vehicle fleet, the determination of periodic delivery patterns to customers, and routing. The authors propose a hierarchical decomposition procedure in which the actual routing problem is solved at the lowermost stage, and which aims at minimizing the sum of variable transportation costs, overtime costs, and fixed vehicle costs. Cornillier, Boctor, Laporte, and Renaud (2008b) examine a multi-period MCVRP with a heterogeneous fleet. It needs to be ensured that the replenishment is performed in such a way that safety stocks and tank capacities are respected. Two customers may be visited at most on each tour, and multiple trips, shift duration and the option of overtime are considered. The objective function includes revenues generated by the fuel deliveries, variable transportation costs, drivers' wages, and overtime costs. The authors develop a multi-phase heuristic that solves the occurring subproblems iteratively. A similar problem is studied by Popović, Vidović, and Radivojević (2012). Each compartment must always be filled completely or not at all, each petrol station may only be visited once per each period, and the number of customers that can be visited on a single tour is limited to three. Variable transportation costs as well as inventory holding costs are taken into account. The authors propose a VNS. Vidović, Popović, and Ratković (2014) limit the maximal number of customers per tour to four and include fixed vehicle costs. They introduce a heuristic that combines an MH component to determine an initial solution and a variable neighborhood descent algorithm. Coelho and Laporte (2015) deal with four variants of multi-period MCVRPs, which differ according to the combination of split and unsplit customer demands as well as shared and unshared compartments. They aim at minimizing transportation and inventory holding costs, and develop a branch-and-bound algorithm for small and a branch-and-cut algorithm for larger problem sizes. The authors demonstrate that the most flexible setting, i.e., split customer demands and shared compartments, results in the smallest total cost. Urli and Kilby (2017) study both a single-period and a multi-period MCVRP. Although the delivery quantities are deterministic for each day of the planning horizon, a multi-period approach is applied to decide about the composition of the heterogeneous vehicle fleet. For each vehicle that can be acquired for the fleet, a certain fixed cost occurs in each period. The authors suggest a large neighborhood search (LNS) based on a constraint programming model.

4.2. MCVRPs in waste collection

4.2.1. Problem overview

MCVs are frequently considered in waste collection, in which different types of non-mixable waste types are picked up at collection points. However, MCVRPs in waste collection have only been studied since 2010. In contrast to applications in fuel distribution, instead of the delivery of product types to customers, here the collection of different product (i.e., waste) types from customers or waste collection points is examined. Consequently, MCVs are empty at the departure from a depot and their load will increase during the tour. Nevertheless, the general structure of such problems remains identical to problems in which product types have to be distributed. In contrast to problems in fuel delivery, most studies in waste collection deal with fixed assignments of product types to vehicles (FixA) as different types of waste may have different compartment requirements or need different compression techniques. Moreover, a single compartment can always be used to collect one type of waste from several customers (SC). Similar to problems in fuel distribution, the number and size of the compartments are fixed in most cases (FixS). With respect to the number of customer visits, multiple visits (MV) are studied more often.

Table 2
Characterization of MCVRP literature in waste collection (in chronological order).

Publication	MCVRP-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Muyldermans and Pang (2010a)	x		x		x		x			x					
Muyldermans and Pang (2010b)	x		x		x		x			x					
Reed et al. (2014)	x		x		x			x		x					
Henke et al. (2015)		x		x	x			x		x					
Oliveira et al. (2015)	x		x		x			x		x					
Elbek and Wøhlk (2016)	x		x		x			x		x	x	x			
Rabbani et al. (2016)	x		x		x			x		x			x		x
Farrokhi-Asl et al. (2017)	x		x		x		x			x			x		x
Gajpal et al. (2017)	x		x		x			x		x					x
Rabbani et al. (2017)	x		x		x		x			x					x
Kiilerich and Wøhlk (2018)	x		x	x	x		x	x		x	x				
Henke et al. (2019)		x		x	x			x		x					
Zbib and Laporte (2020)	x			x	x			x		x			x		
Share in %	85	15	77	31	100	0	38	69	0	100	15	8	23	0	31

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

Furthermore, across all publications single waste types are always collected completely by one vehicle if a customer is visited, and therefore customer demands are not split (UD). Table 2 summarizes the literature in this area.

4.2.2. Related literature

Single-period problems. [Muyldermans and Pang \(2010b\)](#) consider an MCVRP in domestic waste collection aiming at the minimization of variable travel costs. The authors introduce a guided local search algorithm for the problem. They analyze the problem characteristics for which the use of MCVs should be preferred to SCVs when single customer visits are considered. MCVs turn out to be beneficial, for instance, if the number of product types increases, the vehicle capacity increases or the demand sizes decrease relative to the vehicle capacity. Additionally, [Muyldermans and Pang \(2010a\)](#) were the first to explore an arc-oriented routing problem with MCVs. In this problem, the demand for the collection of different waste types occurs on the arcs instead of the nodes of the network. As a solution procedure, the authors adapt their guided local search from the node-oriented problem. [Reed, Yiannakou, and Evering \(2014\)](#) study a related problem and develop an ant colony optimization (ACO) approach. [Oliveira, Ramos, and Martins \(2015\)](#) deal with a similar problem and suggest a cluster-first, route-second approach, grouping waste containers according to their collection frequency. A problem in a slightly different context, namely the collection of different types of glass waste, is studied by [Henke, Speranza, and Wäscher \(2015\)](#). As a special feature of the problem, the number of compartments is flexible (but limited), and compartment sizes can only be varied in discretized steps. The authors introduce a VNS and show that the computing times increase with a growing number of product types, a growing number of demands, and a decreasing maximal number of compartments. For an extension with continuously flexible compartment sizes, [Henke, Speranza, and Wäscher \(2019\)](#) propose a branch-and-cut algorithm and analyze the impact on the total costs of using discretely or continuously flexible compartment sizes. The vehicle fleet mix problem of [Rabbani, Farrokhi-Asl, and Rafiei \(2016\)](#) consists of owned and rented vehicles. Owned vehicles must always return to one of multiple depots, whereas rented vehicles do not need to return. Furthermore, tour duration limits exist and a distinct disposal facility has to be visited for each waste type. The sum of transportation costs, costs for renting vehicles, and service costs for loading and unloading must be minimized. The authors introduce several GAs. [Rabbani, Farrokhi-Asl, and Asgarian \(2017\)](#) extend this problem to a network design problem by integrating the determination of locations for depots and treatment

facilities. The latter must be visited at the end of each tour in order to dispose of the collected waste. The authors also apply a GA for the solution of the problem. A similar network design problem was introduced by [Farrokhi-Asl, Tavakkoli-Moghaddam, Asgarian, and Sangari \(2017\)](#). The authors do not differentiate between owned and rented vehicles, but use a heterogeneous fleet of owned vehicles. They propose a GA and a particle swarm optimization algorithm. [Gajpal, Abdulkader, Zhang, and Appadoo \(2017\)](#) examine an MCVRP for household waste collection that takes ecological aspects into account. The authors consider alternative fuel-powered vehicles with a limited fuel tank capacity, thus resulting in distance-constrained tours. They present an adapted savings algorithm and an ACO algorithm. Only recently, [Zbib and Laporte \(2020\)](#) have studied an arc-oriented MCVRP with a heterogeneous fleet of vehicles. A special characteristic of their problem is that the actual capacity of a compartment depends on the type of waste collected and a corresponding compression factor for that type of waste. The authors aim at minimizing variable transportation costs, and solve the problem using a matheuristic, which decomposes the problem in three sequentially solved sub-problems.

Multi-period problems. [Elbek and Wøhlk \(2016\)](#) were the first to study a multi-period problem in waste collection. Their problem includes many application-specific aspects. Two types of waste, glass and paper, have to be collected from containers in an urban area. Each vehicle carries two large containers, i.e., compartments, one for each waste type. The wastes collected need to be transported to an intermediate facility. There, the compartment for paper can be emptied, and the compartment for glass waste can be exchanged with an empty compartment if it is filled completely. Furthermore, when a certain amount of waste has been accumulated at the intermediate facility, it needs to be transported to distinct recycling facilities. Apart from variable transportation costs, variable service costs for emptying the compartments are considered. The authors introduce a VNS. [Kiilerich and Wøhlk \(2018\)](#) provide model formulations and large-scale problem instances for arc-oriented routing problems with multiple compartments. The authors include variable travel costs and fixed vehicle costs in the objective function.

4.3. MCVRPs in agricultural contexts

4.3.1. Problem overview

As a third area of application, MCVRPs can be identified in the context of the collection of agricultural products, e.g., milk, olive oil, or livestock. The attributes of such problems are quite similar to problems in fuel distribution, i.e., the number and sizes of

Table 3
Characterization of MCVRP literature in an agricultural context (in chronological order).

Publication	MCVRP-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Ruiz et al. (2004)	x			x		x		x		x			x		x
El Fallahi et al. (2008)	x		x		x		x			x					x
Oppen and Løkketangen (2008)	x			x	x		x			x	x		x		x
Caramia and Guerriero (2010)	x			x	x		x			x			x		x
Kandiller et al. (2015)	x			x	x		x			x					x
Lahyani et al. (2015a)	x			x	x		x			x	x		x		
Sethanan and Pitakaso (2016)	x			x		x	x			x			x		x
Tasar et al. (2019)	x			x	x		x	x		x					x
Share in %	100	0	13	88	75	25	88	25	25	88	25	0	63	0	88

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

compartments are usually fixed (FixS), the assignment of product types to compartments is mostly flexible (FlexA), and demands are unsplit in most cases (UD). However, compartments may often be used for collecting agricultural products from more than one customer (SC), and single customer visits are assumed in the majority of problems studied (SV). Table 3 summarizes MCVRP literature in the agricultural context.

4.3.2. Related Literature

Single-period problems. Ruiz, Maroto, and Alcaraz (2004) introduce an MCVRP with a heterogeneous fleet for the distribution of different types of animal feed. Six customers at most may be visited on a tour, a maximum duration of each tour is given, and geographical accessibility restrictions are to be taken into account. Apart from the minimization of travel costs, the authors aim at maximizing vehicle capacity utilization. They propose a combined decomposition and set partitioning approach. Another problem with tour length limitations for animal feed distribution is studied by El Fallahi, Prins, and Wolfler Calvo (2008). The authors develop a memetic algorithm with path relinking and a TS. Kandiller, Eliyi, and Tasar (2015) look at a feed distribution problem with service times and duration constraints and solve it with commercial optimization software. Tasar, Türsel Eliyi, and Kandiller (2019) explore variants of the MCVRP with tour length limitations for which they also consider the times for loading and unloading. The basic problem variant is characterized by unsplit customer demands and single trips per vehicle. It is extended by allowing split customer demands and multiple trips. The authors introduce a VNS. Caramia and Guerriero (2010) deal with the collection of milk from farms. They consider a problem that incorporates aspects of the truck and trailer routing problem. A duration-constrained tour for a vehicle may consist of a main tour in which customers without any accessibility restriction are visited by the complete vehicle, and some subtours in which the trailer is uncoupled and customers with accessibility restrictions are visited by the truck only. The goal consists of minimizing both the total distance and the number of vehicles used. The authors propose an MH that iteratively solves two relaxed optimization problems: (i) the problem of assigning customers to vehicles, and (ii) the problem of sequencing the vehicles. Another distance-constrained MCVRP in this context is studied by Sethanan and Pitakaso (2016). They consider a heterogeneous fleet and cleaning activities, which are necessary before a compartment can be filled again. Cleaning costs are therefore taken into account in addition to variable travel costs. They propose variants of differential evolution algorithms as solution approaches.

Multi-period problems. Oppen and Løkketangen (2008) study a multi-period problem setting with a heterogeneous fleet for livestock collection. Interdependencies between periods and collections occur at the depot, i.e., the slaughterhouse, which faces

inventory and production capacity constraints. Interestingly, the number of compartments depends on the animal type to be collected and the sequence in which the collection is performed. This is due to the fact that some compartments are arranged above each other, and their usage depends on the size of the animals collected. Lahyani, Coelho, Khemakhem, Laporte, and Semet (2015a) apply a branch-and-cut algorithm for a multi-period MCVRP with heterogeneous vehicles in the context of collecting olive oil of different quality grades. The authors also consider the cleaning of compartments between periods. In addition to transportation and cleaning costs, fixed vehicle costs are taken into account.

4.4. MCVRPs in grocery distribution

4.4.1. Problem overview

In recent years the use of MCVs for the distribution of groceries has become a relevant topic as MCVs can contribute to a cost-efficient supply of different product assortments with specific temperature requirements. An example would be the supply of convenience stores or supermarkets. This comprises frozen products, fresh foods (e.g., meat or dairy products) and classical products for the daily routine requiring an ambient temperature (e.g., household articles or cosmetics). The distribution process usually involves the supply of multiple outlets on each tour, and each customer is supplied with their specific demands. A setting of shared compartments (SC) and unsplit customer demands (UD) is therefore implied in the corresponding literature. Most studies in grocery distribution concern flexible compartment sizes (FlexS), as modern vehicles are either equipped with adjustable walls and individually controllable temperature zones or additional cooling units, e.g., boxes, that can be loaded into a vehicle. As each product type has specific temperature requirements, the assignment to compartments is always taken as fixed (FixA), except for one publication. Finally, single as well as multiple customer visits are studied (SV, MV). Table 4 summarizes related publications.

4.4.2. Related literature

Single-period problems. Chajakis and Guignard (2003) study two MCVRP variants for the supply of convenience stores with three product types. In the first variant, the complete loading area of the vehicle corresponds to a compartment that is only for ambient products. However, one may add further boxes, i.e., further compartments, of a given capacity for frozen and refrigerated products. In the second variant, a movable bulkhead separates the temperature zones. Furthermore, a box for refrigerated products may be placed within the frozen area. In both variants, vehicles have weight constraints in addition to the typical volume capacity constraints, and cooling costs depending on the number of used boxes

Table 4
Characterization of MCVRP literature in grocery distribution (in chronological order).

Publication	MCVRP-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Chajakis and Guignard (2003)	x	x	x		x			x		x					
Eglese et al. (2005)		x	x		x			x	x					x	x
Hsiao et al. (2017)		x		x	x		x			x				x	
Hübner and Ostermeier (2018)		x	x		x			x		x					
Ostermeier and Hübner (2018)		x	x		x			x		x					
Ostermeier et al. (2018)		x	x		x			x		x					
Chen et al. (2019)	x		x		x		x			x				x	
Chen and Shi (2019)	x		x		x		x			x				x	
Martins et al. (2019)		x	x		x			x		x	x			x	
Share in %	33	78	89	11	100	0	33	67	11	89	11	0	0	56	11

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

are taken into account. The authors introduce mathematical models and Lagrangean relaxations that reflect the assignment component of the problem only and neglect routing aspects. Eglese, Mercer, and Sohrabi (2005) study a real-world grocery distribution problem with time windows, tour duration constraints, and backhauls. In addition to temperature zones, they distinguish product types according to storage at hub depots or distribution centers. The authors also take into account the loading and unloading sequence of product types to prevent blocking situations. In their objective function, they consider fixed vehicle costs, variable travel costs as well as penalty costs for time window or capacity violations. The problem is solved using a simulated annealing-based approach. Hübner and Ostermeier (2018) investigate the influence of loading and unloading costs on the distribution with MCVs. Their model evaluates routing decisions based on loading, unloading and transportation costs. They present an LNS. Ostermeier and Hübner (2018) consider the selection of both SCVs and MCVs for the optimal fleet mix and show that a mixed fleet is superior in all scenarios analyzed. Ostermeier, Martins, Amorim, and Hübner (2018) study loading constraints for the use of MCVs. The authors present a branch-and-cut approach as well as an extended LNS framework. They show that loading constraints are relevant even for small problem sizes and state that most loading issues can be solved by minor changes to the overall routing. Hsiao, Chen, and Chin (2017) consider a problem in grocery distribution in which both compartment sizes and compartment temperatures can be adjusted flexibly. The impact of temperature and storage time on food quality is explicitly taken into account, i.e., higher temperatures or longer transportation times lead to a decline in quality and consequently to shortages with respect to customer demands. The extensive objective function captures travel time-dependent wages, travel distance-dependent fuel costs, idle time costs, storage and cooling costs, travel time-dependent emissions costs, shortage costs, and costs for quality substitution. They present a biogeography-based optimization approach. Chen, Liu, and Langevin (2019) discuss a problem in perishable food distribution with time windows. The number of customers to be visited by a single vehicle is limited in order to ensure the freshness of products. The objective function considered consists of variable and fixed transportation costs as well as fuel consumption costs. The authors introduce an adaptive LNS (ALNS) algorithm. Chen and Shi (2019) study an MCVRP with time windows in the context of urban/last-mile delivery. The problem is motivated by customers having demands for multiple products that should not be transported in a single compartment because of odor or mutual contamination. Two variants of particle swarm optimization are proposed.

Multi-period problems. The first publication in grocery distribution that deals with multiple planning periods is presented by Martins,

Ostermeier, Amorim, Hübner, and Almada-Lobo (2019). The authors study an MCVRP with product-specific time windows. This means that all deliveries for a single product category are carried out within the same time window across the complete planning horizon. Furthermore, penalties apply if the determined time windows are not met on a delivery day. The objective function aims at minimizing transportation costs and penalties. Martins et al. (2019) introduce an ALNS with specialized daily and weekly operators to solve the corresponding problem.

4.5. MCVRP in maritime transportation

4.5.1. Problem overview

The use of multiple compartments is also quite common in ship routing problems (SRP), where large ships or tankers transport different types of products (e.g., oils, chemicals) within different compartments. In most applications, several supply ports (i.e., ports where products are picked up for transportation) and demand ports are visited on a ship's schedule. With respect to MCVRP attributes, most SRPs include a setup with fixed compartment sizes (FixS). Compartments are shared between customers (SC) in the majority of studies and follow a flexible assignment of products (FlexA). A slightly larger number of problems allow for multiple visits (MV) and unsplit demands (UD) for the supply of ports. The distribution fleet usually consists of heterogeneous ships with different characteristics and capacities (see Table 5).

SRPs with multiple compartments consider further problem characteristics that – in part – go beyond truck routing. For instance, travel times compromise several days or weeks and are subject to a high degree of uncertainty (e.g., due to weather conditions), and thus often only a few stops on a route are planned ahead in time. In addition, various constraints restrict the routing options (e.g., incompatibilities between ships and ports), a depot rarely exists, and a ship route may not be a round trip (Fagerholt & Christiansen, 2000a). SRPs therefore often comprise fewer routing combinations compared to truck routing problems. Given the long planning horizons, we define an SRP as multi-period if a port can be supplied with the same product multiple times within the planning horizon.

4.5.2. Related literature

Single-period problems. Bausch, Brown, and Ronen (1998) was the first study published in the context of maritime transportation with multiple compartments. The authors consider a real-life application for the supply of liquid bulk products by different kinds of vessels to select cost-minimal schedules. Fagerholt and Christiansen (2000a) present a case study for the distribution of mineral fertilizers. They formulate a multi-ship pickup and delivery

Table 5
Characterization of MCVRP literature in maritime transportation (in chronological order).

Publication	MCVRP-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Bausch et al. (1998)	x		x		x		x			x			x	x	
Fagerholt and Christiansen (2000a)		x		x		x	x			x			x	x	
Fagerholt and Christiansen (2000b)		x		x		x	x			x			x	x	
Jetlund and Karimi (2004)		x		x		x	x			x			x	x	x
Al-Khayyal and Hwang (2007)	x		x		x			x	x		x		x		
Kobayashi and Kubo (2010)	x			x	x		x			x			x	x	
Christiansen et al. (2011)	x			x	x			x	x		x		x		
Siswanto et al. (2011)	x			x	x			x	x		x		x		
Agra et al. (2013)	x		x	x	x			x	x		x		x	x	
Christiansen et al. (2015)	x			x	x			x		x	x		x	x	
Santosa et al. (2016)	x			x	x			x	x		x		x	x	
Foss et al. (2016)	x			x	x			x	x		x		x		
Christiansen et al. (2017)	x			x	x					x	x		x	x	
Wang and Li (2018)	x			x		x	x			x				x	
Siswanto et al. (2019)	x			x	x			x	x		x		x	x	
Share in %	80	20	20	87	73	27	47	53	47	53	60	0	93	73	7

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

problem with time windows and use a (heuristic) set partitioning approach to minimize transportation costs. These include variable sailing costs, port fees and costs for spot shipments. The algorithm to create candidate schedules used in their approach is further addressed in a complementary study by Fagerholt and Christiansen (2000b). Kobayashi and Kubo (2010) address a scheduling problem occurring in oil distribution that is similar to the study by Fagerholt and Christiansen (2000a). They consider multiple time windows for the pickup and supply at ports, and introduce a decomposition approach. The shipping of bulk liquid chemicals is also studied by Jetlund and Karimi (2004), in which not all cargoes need to be served, but can be outsourced instead. Tanker schedules are determined to maximize profits, which depend on variable fuel and port costs as well as charter costs for vessels. Schedules are additionally restricted by tour duration constraints. The authors suggest a decomposition approach. Wang and Li (2018) study an in-port SRP for single tankers with up to 50 compartments for the transportation of chemicals. In this context, "in-port" refers to shipping operations to be performed within a single port. A central aspect of their study is the tank allocation problem, i.e., the allocation of chemicals to compartments with different properties (e.g., different coating materials). Moreover, the authors consider stability constraints and minimum volumes for the loading of compartments. As a solution approach, a two-phase heuristic is introduced in which candidate routes are generated using dynamic programming first, and a feasible tank allocation solution is searched for as a second step.

Multi-period problems. An inventory routing problem (IRP) for the transportation of liquid bulk products is discussed by Al-Khayyal and Hwang (2007). In this problem, some ports have a continuous demand for product types, whereas other ports produce those product types continuously. Moreover, for the complete planning horizon it needs to be ensured that the inventory levels at the ports neither fall below a minimum level nor exceed a maximum level. Alongside variable travel and port costs, loading and unloading costs also have to be taken into account. Siswanto, Essam, and Sarker (2011) study a similar problem, and introduce a multi-heuristic to simultaneously address the arising subproblems: routing, ship selection, loading and unloading activities. Christiansen et al. (2011) address an IRP in the cement industry with demand peaks that exceed available fleet capacities. They therefore additionally decide which demands should be fulfilled. The objective is to minimize costs for unfulfilled demands and transportation, while ensuring fea-

sible inventory levels. A GA is suggested to solve the problem. An IRP in oil distribution is considered by Agra, Christiansen, and Delgado (2013) in which inventory levels are only managed at demand ports and time windows are taken into account. The authors propose a model formulation as well as several valid inequalities. They use these inequalities to show that computing times can be reduced significantly and optimal solutions can be obtained for some real-life problems. Christiansen, Fagerholt, Rachaniotis, Tveit, and Øverdal (2015) study another in-port fuel supply problem with multiple trips. In this problem, large ships anchor at ports and are supplied with multiple types of fuel oils by smaller vessels. These operations have to be executed within certain time windows. Moreover, mandatory and optional demands are distinguished. Profit has to be maximized, while daily charter costs of vessels are considered. For the same problem, Christiansen, Fagerholt, Rachaniotis, and Stålhane (2017) show the superiority of a path-flow model. Santosa, Damayanti, and Sarkar (2016) solve an IRP for oil distribution with a GA. Their problem also takes cleaning operations within compartments into account, which enables the utilization of one compartment for several oil products if it is cleaned in between. Foss, Myklebust, Andersson, and Christiansen (2016) present an MIP and valid inequalities for another IRP with flexible product assignment. In their experiments they compare their model to approaches with fixed assignments and mixable products, and highlight the economic benefits of the flexible model. Siswanto, Eko Wiratno, Rusdiansyah, and Sarker (2019) study an IRP in oil distribution with multiple time windows. Similar to Siswanto et al. (2011), the authors propose a multi-heuristic approach to address the subproblems involved.

4.6. MCVRPs in other application contexts

4.6.1. Overview

Several publications on MCVRPs exist in other contexts. Each study refers to a specific field of application, namely bike-sharing systems, distribution of chemical products and passenger transportation. Table 6 summarizes the corresponding publications.

4.6.2. Related literature

Single-period problems. Li, Szeto, Long, and Shui (2016) introduce an MCVRP in the context of repositioning bikes in a bike sharing system. In this pickup and delivery problem, different types

Table 6
Characterization of MCVRP literature in other application contexts (in chronological order).

Publication	MCVRP-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Li et al. (2016)	x		x	x	x		x			x					
Cóccola et al. (2018)	x			x	x			x	x		x				x
Tellez et al. (2018)		x	x		x			x		x			x	x	x
Share in %	67	33	67	67	100	0	67	33	33	67	33	0	33	33	67

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

Table 7
Characterization of MCVRP literature for conceptual problems (in chronological order).

Publication	MCVRP-specific Attributes										VRP-related Attributes ^a				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Repoussis et al. (2007)	x			x	x			x		x			x	x	x
Mendoza et al. (2010)	x		x		x			x		x		x			x
Mendoza et al. (2011)	x		x		x			x		x		x			x
Derigs et al. (2011)	x	x	x	x	x		x			x					
Wang et al. (2014)	x			x	x		x			x			x		
Abdulkader et al. (2015)	x		x		x			x		x					x
Archetti et al. (2015)		x		x	x		x			x					
Goodson (2015)	x		x		x			x		x		x			x
Huang (2015)	x		x		x			x		x		x			
Kabcome and Mouktonglang (2015)	x			x	x			x		x			x	x	
Archetti et al. (2016)		x		x	x		x	x	x	x					
Kaabi (2016)	x		x		x		x			x				x	x
Lahyani et al. (2017)	x			x	x		x			x				x	x
Mirzaei and Wöhlk (2017)	x		x		x		x	x		x					
Silvestrin and Ritt (2017)	x		x		x		x	x		x					x
Alinaghian and Shokouhi (2018)	x		x		x		x			x					x
Alemanly et al. (2018)	x			x	x		x			x			x		x
Reil et al. (2018)	x		x		x		x	x		x				x	x
Wang and Li (2018)	x		x		x		x			x					x
Share in %	89	16	63	42	100	0	63	58	5	100	0	21	21	26	63

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained.

of bikes, i.e., bikes with one or two seats, have to be repositioned between bike sharing stations such that known demands are satisfied. MCVs with spaces for specific bike types are available for transporting the bikes. They allow for the transportation of smaller bikes in a compartment dedicated to larger bikes subject to penalty costs. Similarly, penalties apply if demands for smaller bike types are fulfilled by offering larger bike types. Consequently, the objective function aims at minimizing the sum of variable travel costs and both types of penalties. The authors propose a GA. Another pickup and delivery problem with MCVs is studied by Tellez, Vercaene, Lehuédé, Péton, and Monteiro (2018). They consider the transportation of persons from pickup to delivery locations by a duration-constrained fleet of heterogeneous vehicles. Passengers have different space requirements, i.e., they require a normal seat or space for a wheelchair. The vehicle capacity can be adjusted to handle different demands. Moreover, each transportation request is specified by a time window, a service time and a maximal duration for the completion of a transportation request. In the objective function, the authors consider fixed vehicle costs, variable travel costs as well as duration-dependent driver wages. An LNS is proposed.

Multi-period-problems. An IRP with MCVs is studied by Cocco, Méndez, and Dondo (2018) in the context of the distribution of chemical products. Different chemical products have to be transported from suppliers with known production rates to customers who have storage tanks with given capacities and safety stocks. A service time is taken into account for each location, and each tour must comply with a duration limit. In addition to variable travel costs, the authors consider holding costs for storing product quantities at the suppliers' and customers' locations. The authors suggest a (heuristic) column generation procedure.

4.7. Conceptual MCVRPs without a specific application

4.7.1. Problem overview

Several papers study MCVRP-related problems without a specific or distinct application. Some similarities can nevertheless be observed for these problems. All of them consider shared compartments (SC), in most cases compartment sizes are fixed (FixS), and customer demands are unsplit (UD). A slightly larger number of studies deals with fixed compartment assignments (FixA) instead of flexible compartment assignments, whereas the consideration of single and multiple customer visits is evenly distributed. All conceptual studies are concerned with MCVRPs with a single period only (see Table 7). The discussion in the following section is thus structured according to the certainty of data.

4.7.2. Related literature

MCVRPs with deterministic demands. Derigs et al. (2011) present a general model for fuel and food distribution that takes different options of compartment flexibility into account. As a problem extension, they introduce incompatibilities for putting different product types in the same compartment as well as incompatibilities for the assignment of certain product types to certain compartments. The authors therefore explicitly allow for the mixing of some product types. They propose a solver suite of several algorithmic components for their general problem formulation, i.e., several construction and metaheuristic procedures. They use numerical experiments to identify combinations of these components that appear to be the most promising for the respective problem variants. Wang, Ji, and Chiu (2014), similar to Derigs et al. (2011), regard general compatibility constraints in order to define which

pair of product types can or cannot be assigned to the same compartment, and which product types can or cannot be assigned to a particular compartment. A reactive guided TS is proposed as a solution procedure. Archetti, Bianchessi, and Speranza (2015) study a problem with multiple product types to be delivered, but do not explicitly consider compartments as only the vehicles' total capacities are taken into account. They propose a branch-and-price-and-cut algorithm. Furthermore, Archetti, Campbell, and Speranza (2016) present an extensive study of four VRP variants, three of which can also be classified as MCVRPs. These variants differ with respect to the combination of split/unsplit customer demands and single/multiple customer visits. The authors use worst-case analyses to identify bounds on the maximal ratios of the objective function values between these different variants and show that total transportation costs decrease with increasing flexibility. In their analysis they also show that co-collection becomes more beneficial when, among other things, customers can be visited multiple times or the number of product types increases. Repoussis, Tarantilis, and Ioannou (2007) deal with an MCVRP with heterogeneous vehicles, time windows for the deliveries to customers, tour duration limitations and fixed vehicle costs. The proposed hybrid heuristic combines components from GRASP and VNS. Kabcome and Mouktonglang (2015) study a problem variant with soft time windows and a heterogeneous fleet. The evaluation of a solution is based on three cost types: distance- and quantity-dependent transportation costs as well as penalties for time window violations. Kaabi (2016) introduces a selective MCVRP with time windows and distance constraints, where "selective" refers to the characteristic that customers need not necessarily be visited. Instead, a profit is generated for each customer visited, and the total profit thus has to be maximized. The authors propose a GA that uses iterated local search for intensification. Abdulkader, Gajpal, and ElMekkawy (2015) deal with a basic MCVRP with tour duration constraints and propose a hybrid ACO where hybridization refers to the use of local search procedures to improve the solution of ants. Wang and Li (2018) develop a hybrid fruit fly optimization algorithm for a basic MCVRP with tour length limitations. Silvestrin and Ritt (2017) consider a basic MCVRP with tour duration constraints and develop an iterated TS that combines an iterated local search with TS to analyze the effect of split customer deliveries. Furthermore, Mirzaei and Wøhlk (2017) propose a branch-and-cut algorithm for a similar problem. An MCVRP with multiple depots, distance constraints and fixed vehicle costs is studied by Alinaghian and Shokouhi (2018). They introduce a VNS, an ALNS, and a hybrid ALNS where the hybridization refers to the incorporation of components from the VNS. Another MCVRP with heterogeneous fleet and multiple depots is examined by Alemany, Juan, Garcia, Garcia, and Ortega (2018). In addition to depots under the operator's ownership with given and possibly insufficient product supplies, external facilities can be visited for replenishment during or at the end of tours. Variable travel costs, fixed vehicle costs, and replenishment costs are considered. The latter are accounted for by incorporating hypothetical travel costs to external facilities. The problem is solved by a construction procedure combined with classical improvement operators. Reil, Bortfeldt, and Mönch (2018) investigate a VRP with backhauls and three-dimensional loading constraints. Their problem resembles an MCVRP as they distinguish customer requests between deliveries (linehaul requests) and collections (backhaul requests), which have to be transported in different compartments. Otherwise, linehaul requests might be blocked by backhaul requests during unloading. In addition to service times, tour duration limitations and time windows, several constraints specific to three-dimensional loading are considered. The two-stage objective function aims at minimizing the number of vehicles first, and the variable travel cost second. For their problem, the authors introduce

a hybrid metaheuristic containing elements of TS and evolutionary algorithms. Lahyani, Khemakhem, and Semet (2017) explore a traveling salesman problem (TSP) with a single MCV and revenues that are collected for serving customers. Moreover, each customer has a specific hard time window, and incompatibilities between product types and compartments are taken into account. In addition to revenues, variable travel costs and costs for waiting at the customer sites are regarded in the objective function. The corresponding TSP variant is solved using an MH approach.

MCVRPs with stochastic demands. Mendoza, Castanier, Guéret, Medaglia, and Velasco (2010) were the first to examine a stochastic version of the MCVRP with distance constraints. More precisely, they consider stochastic demands. Hence, the objective function aims at minimizing the variable transportation costs expected. They propose a memetic algorithm as a solution approach. Mendoza, Castanier, Guéret, Medaglia, and Velasco (2011) introduce three construction procedures as well as an improvement procedure for the same problem. Goodson (2015) consider tour duration limitations and the minimization of the expected total travel time. The problem is solved by a simulated annealing procedure. Huang (2015) studies a stochastic MCVRP with pickups and deliveries, which is further extended by a facility location component. Solutions to this problem are evaluated by expected variable transportation costs, fixed vehicle costs, depot opening costs, and expected penalties for violations of vehicle and depot capacities. The author introduces a decomposition approach in which subproblems are solved iteratively.

5. Comparative review of MCVRP literature and future areas of research

This section provides a structured summary based on the identified attributes (Section 5.1) and solution approaches (Section 5.2). We identify similarities and differences in both sections, and discuss future areas of research.

5.1. Attributes of MCVRPs

We will first provide a general overview of the different attributes by area of application before discussing the MCVRP-specific and common VRP-related attributes in more detail.

5.1.1. Overview of attributes and areas of application

MCVRP literature is heterogeneous and similar characteristics can only be identified in some areas of application. Table 8 provides a summary across attributes and applications to highlight both similarities and dissimilarities. Comparing the respective dominant characteristics of each attribute, problems in waste collection and conceptual problems show a high degree of similarity, whereas problems in grocery distribution share few similarities with problems in fuel distribution, agricultural contexts or maritime transportation. In general, VRP-related extensions are considered most frequently in fuel delivery and maritime transportation, and to a lesser extent in agricultural problems, other contexts and conceptual publications. In waste collection and grocery distribution most publications focus on MCVRP-related attributes, while general VRP-related extensions are rarely considered. Finally, it is noteworthy that the consideration of stochastic demands is rare within the MCVRP literature. The following sections compare the literature with respect to MCVRP-specific and VRP-related attributes in greater detail.

Table 8
Share of problem attributes across all areas of application.

Application	MCVRP-specific Attributes										VRP-related Attributes ^a				
	(Share in %) ^b										(Share in %) ^b				
	FixS	FlexS	FixA	FlexA	SC	UC	SV	MV	SD	UD	MP	StoD	HF	TW	DC
Fuel	100	0	24	88	29	76	35	65	24	82	35	0	71	47	71
Waste	85	15	77	31	100	0	38	69	0	100	15	8	23	0	31
Agriculture	100	0	13	88	75	25	88	25	25	88	25	0	63	0	88
Grocery	33	78	89	11	100	0	33	67	11	89	11	0	0	56	11
Maritime	80	20	20	87	73	27	47	53	47	53	60	0	93	73	7
Others	67	33	67	67	100	0	67	33	33	67	33	0	33	33	67
Conceptual	89	16	63	42	100	0	63	58	5	100	0	21	21	26	63
Total share	83	19	48	60	79	23	50	57	19	85	25	6	46	36	46

^a MP: multi-period; StoD: stochastic demand; HF: heterogeneous fleet; TW: time windows; DC: duration/distance constrained

^b Sum of the shares of a specific attribute may exceed 100% since some publications study more than one characteristic.

5.1.2. MCVRP-specific attributes

Comparative literature review. With respect to MCVRP-specific attributes, three characteristics are found in more than two-thirds of all publications: fixed compartment sizes (83% of publications), shared compartments (79% of publications), and unsplit customer demands (85% of publications).

In fuel and agricultural contexts, MCVRPs with fixed compartment sizes are exclusively considered, while in waste collection, conceptual problems, and maritime transportation the related share is still at least 80%. Grocery distribution is the only area of application in which problems with flexible compartment sizes prevail as most publications deal with flexible or both fixed and flexible compartment sizes (78%). In general, flexible compartment sizes enable better utilization of the vehicle capacity. However, this requires special technology like moveable bulkheads or walls, which do not seem to be established equally well for different types of goods. Such technologies can be implemented more easily when solid product types have to be transported, as is the case in waste collection and grocery distribution, but applications in maritime transportation show that these technologies are also feasible for liquids.

Regarding compartment shareability, the utilization of shared compartments is always considered in waste collection, grocery distribution, other applications, and conceptual problems. Furthermore, more than 70% of publications in agricultural contexts and maritime transportation deal with shared compartments, whereas only 29% of the publications do so in the case of fuel delivery. Again, these findings can be attributed to the fact that in the former applications it is easier to differentiate between individual customer demands because of their solid consistency (e.g., by means of distinct roll cages for each customer demand in grocery distribution), or because differentiation is simply not relevant (e.g., collection of waste from different customers). Moreover, customers in fuel delivery typically receive full compartment loads since precisely splitting up liquids into multiple shares requires additional technical equipment (e.g., flow meters for trucks).

One of the attributes that can be observed most frequently across all applications is the assumption of unsplit customer demands (85%). With the exception of problems in maritime transportation, only scant attention has been paid to problems arising from split customer demands. From a customer perspective, this finding is reasonable. While split deliveries may accommodate cost advantages, the splitting of individual demands across multiple deliveries does not appear to be very customer friendly. In the case of waste collection, it is usually not possible to empty the contents of a single waste container only partially.

The proportions of problems with fixed (48% of all publications) or flexible (60%) assignments are balanced more evenly. However, the consideration of fixed and flexible assignments differs significantly across the various areas of application. The majority of publications (i.e., more than 75%) in waste collection and grocery distribution deal with a fixed assignment, while the opposite is true for fuel delivery, agricultural problems and maritime transportation. As for the latter applications, each compartment can usually be used for each product type (e.g., each type of fuel in fuel delivery, milk from different farms in milk collection, or chemical products transported in ships), as the items have identical transportation requirements. As a result, product types can be assigned flexibly to compartments. However, in grocery distribution and waste collection the assignment is more restricted, as different product types have varying transportation requirements (i.e., different temperatures in grocery distribution or different compression techniques in waste collection). Similarly, the proportions of single (50%) and multiple visits (57%) are balanced evenly, but differ between the distinct areas of application. This is particularly the case with agricultural problems, where the share of publications with single visits is 88%.

Future areas of research related to MCVRP-specific attributes. Flexible compartment sizes and split customer demands are underrepresented in the current literature, even though both allow for higher flexibility in distribution. However, the higher flexibility is accompanied by greater planning complexity. In general, more flexible distribution provides a larger degree of freedom concerning the routing decisions (e.g., via new options for joining the supply of different customers or product types), and consequently greater opportunities for cost savings. This has already been demonstrated in various areas of application (e.g., waste collection). Yet so far these opportunities have especially been neglected in agricultural contexts and fuel distribution. While the implementation of flexible compartment sizes may have been challenging in the past due to technical requirements, recent technological advances do support such approaches.

Sharing compartments across customers is underrepresented in fuel distribution, but technologically feasible. Analyzing how to make use of this flexibility is a promising research avenue. Moreover, only few publications deal with split customer demands. Splitting the demands of single customers increases the solution space of the corresponding routing problem, and may provide significant savings, as demonstrated in the case of other VRP variants (see e.g., Archetti & Speranza (2012)). Additionally, considering multiple customer visits in agricultural problems would also contribute to a more flexible supply of customers and further opportunities for cost savings.

Current models mainly deal with cost minimization. This can be explained by the fact that flexible compartments, flexible assignment and multiple visits offer distinctive cost advantages. However, profit impacts (e.g., service level agreements, delays or allowing split deliveries) and maintenance aspects (e.g., for flexible compartments) that are affected by the use of MCVs have not been the focus of research so far and should be considered in the future. Moreover, additional routing-relevant aspects like CO₂-emissions (MCV vs. SCV), energy savings resulting from split deliveries (e.g., for frozen products), consistency for deliveries (e.g., driver consistency for SCV vs. MCV, consistency across multiple periods, consistency across multiple product segments) as well as social and quality aspects (e.g., for long distance transports of certain products in shared compartments) offer new and relevant research perspectives. These aspects particularly motivate model enhancements with respect to multiple objectives.

The significant differences among applications show that MCVRP literature is driven by the specific problem setting and technological advances, such as enhanced compartment flexibility. Interdisciplinary research across engineering and operations research is a potential pathway to obtain inspiration from new technological developments while assessing the impact on routing. For instance, the use of fully electric and autonomous MCVs as well as MCVs with new compartment types should be assessed. This particularly includes a comprehensive cost and benefit assessment and the analysis of all decision-relevant aspects from loading to routing and unloading processes. Moreover, technological advances for MCVs could be used for new delivery options (e.g., within sharing economy concepts).

There are further research opportunities related to MCVRPs in other contexts. Problems in bike and ride sharing are becoming ever more relevant in practice, but just one publication is available in each of these fields so far. MCVs could be part of new city logistics concepts for home deliveries, and the corresponding MCVRPs need to be analyzed. Online grocery retailers for instance use multi-temperature trucks for home delivery (Wollenburg, Hübner, Trautrim, & Kuhn, 2018). Additionally, there may be entirely different areas of application unrelated to vehicle routing. In manufacturing, for instance, Guo, Geng, Takahashi, Wang, and Jin (2018) present the problem of assembling printed circuit boards, which is based on an MCVRP. A similar transfer could be possible to other related problems, such as machine scheduling problems.

5.1.3. General VRP-related attributes

Comparative literature review. We further compare the MCVRP literature with regard to prominent attributes commonly found in other VRP variants. These attributes are: tour length or tour duration limitations, fleet composition, consideration of time windows, number of planning periods and stochastic demand.

The most frequently studied extensions concern heterogeneous vehicle fleets and limitations of the tour lengths or the tour durations. Heterogeneous fleets are considered in 46% of all MCVRPs. This problem extension is frequently used in maritime transportation (93%), fuel delivery (71%), and agricultural problems (63%). Vehicle setups are rarely adjustable in these applications, and an alternative approach to gain flexibility in distribution is therefore to use different vehicle types. With respect to limitations of the tour lengths or durations, the consideration of such constraints is necessary to comply with legal regulations. Duration or distance constraints are imposed in 46% of all MCVRP publications. The largest share of problems with duration constraints can be found in agricultural contexts (88% of publications). Exceptions can be observed in the literature on grocery distribution and maritime transportation, in which there is only one publication each dealing with distance or duration constraints.

About one third (36%) of the problems studied take time window restrictions into account. Such constraints most frequently appear in fuel delivery (47%), grocery distribution (56%), and even more so in maritime transportation (73%). This finding seems reasonable since the respective applications are either strongly customer oriented or, in case of maritime transportation, deal with tightly planned handling capacities. The collection of waste on the other hand is more flexible with respect to the time of collection, and thus neglecting the consideration of time windows is appropriate.

Another attribute of VRP variants concerns the extension to multiple planning periods. This problem variant is studied in only 25% of all publications. Among those 25%, multiple planning periods are particularly often considered in maritime transportation problems due to their long-term nature. The focus on this aspect is also slightly greater in fuel delivery (35%), agricultural contexts (25%), and other applications (33%) compared to the remaining areas.

Lastly, little attention has been paid to stochastic demands so far. Only a few papers in waste collection and the area of conceptual problems consider stochastic demands, representing merely 6% of all publications. This may be justified by the fact that problems in other contexts usually deal with the delivery of products ordered in advance by customers. Demand quantities are therefore known, whereas this is not the case in waste collection.

Future areas of research related to general VRP-related attributes. Analyzing the MCVRP literature with regard to VRP related attributes, it becomes obvious that two out of five attributes considered have received little attention: multiple periods and stochastic demands.

Considering multiple periods is a reasonable extension whenever customers are flexible with respect to the period (e.g., day) of order fulfillment. This condition is commonly found in waste collection and some agricultural problems, in which only a certain frequency of visits needs to be ensured, but customers (e.g., households) may be indifferent to the actual date of service (e.g., collection of waste from containers). Moreover, if demands vary significantly across periods (due to weekly seasonality at petrol stations or in supermarkets, for example), multi-period models will better reflect the actual problem setting.

The second industry-relevant extension applies to the consideration of stochastic demands. These are neglected in basically all MCVRP applications and have only been studied for conceptual problems and a single waste collection problem so far. The integration of stochastic demands will contribute to better decision making particularly for applications where goods are collected (e.g., waste collection, collection of bikes, return of empties, etc.), and for tactical planning problems (e.g., definition of template routes for fuel or grocery distribution) where demands are not known beforehand. There are further aspects of MCVRPs that could be seen as stochastic such as travel times, service times, or the availability of customers themselves. Considering such stochastic or dynamic aspects and applying corresponding solution concepts could allow for more realistic investigations.

Other attributes have been considered more frequently, yet there are still various research possibilities. To begin with, the consideration of heterogeneous fleets offers a promising path for future research. Despite the fact that fleets of different truck types are often used in practice, only few publications in waste collection and none in grocery distribution deal with heterogeneous MCVs. Using trucks with trailers appears to be a reasonable delivery option especially in multi-product contexts, but only one publication has considered this extension and the resulting synchronization aspect in an MCVRP context. Companies use different truck sizes and types since the vehicles have been acquired at different points in time or in order to create more flexibility with respect to the

utilization of transportation capacity. Consequently, this attribute should also be part of corresponding routing problems in future literature.

While in other customer-oriented applications (e.g., fuel and grocery distribution) time windows are frequently taken into account, there is no corresponding consideration in an agricultural context. Furthermore, rest periods of truck drivers should be integrated in addition to time windows across all application areas in order to reach a higher degree of adaptation to practice, to enable more accurate planning and, above all, to meet existing legal regulations. In this context, there also appears to be a lack of publications in grocery distribution that deal with tour distance or duration constraints, although limitations of working hours apply to retailers in real-world applications.

Simply considering volume or weight constraints with respect to a vehicle’s capacity may not be feasible in contexts in which products or product batches are relatively large in relation to the vehicle capacity (e.g., when pallets or roll cages are used). Thus, taking two- or three-dimensional loading constraints into account more extensively might enable valuable analyses. This will give valuable insights from a conceptual perspective when compartment sizes are flexible, as this would result in a two-stage packing problem in which small items (products) have to be packed in large items (compartments), and the large items themselves have to be packed in even larger ones (vehicles).

Expanding the MCVRP to pickup and delivery problems is a relevant topic, but has not yet been investigated extensively. In grocery distribution it is necessary to pick up empty roll cages at retail outlets and transport them back to the depot, for example. In other areas of application it may be necessary to empty and refill the truck during a tour at certain locations and consolidation points (e.g., for waste disposal). Similarly, the MCVs may be used for backhauling, such as for picking up material from suppliers after customer deliveries.

Traditional routing problems are part of various higher-level planning problems – of strategic network planning, for instance. The use of MCVs and their impact within these planning problems should be assessed. MCVs may influence service levels (e.g., lead time of different product types) and total logistics costs. Purchasing decisions relating to transportation capacity, such as fleet mix or in-/outsourcing, are also affected by the application of MCVs. Tactical planning problems, such as driver capacity management, need to be aligned when an MCVRP is being considered. Integration into end-to-end transportation chains (e.g., from waste pickup to disposal or from warehouses to retail shelves) and an investigation into dependent and decision-relevant costs are required. The integration of preparing and picking of different product types that need to be combined for joint transport in an MCV is relevant if different product types require specific processes and lead times. Finally, addressing the question of how multi-compartment transportation can be applied within multi-modal transportation networks or cross-docking applications may be another relevant path for future research.

5.2. Solution approaches for MCVRPs

We further analyze the MCVRP literature with respect to solution procedures. Overall, it can be observed that most publications (76%) propose heuristics, while fewer studies (17%) introduce exact solution procedures.

Exact solution approaches. MCVRPs extend the CVRP that is known to be NP-hard (Toth & Vigo (2014)). Thus, MCVRPs are NP-hard problems themselves, and the application of heuristic solution procedures appears to be generally reasonable. Exact solution procedures may nevertheless be applied if (in practice) computa-

tion time is less important or when (in numerical experiments) they provide benchmarks for gaining insights into specific problem properties. A lack of exact solution procedures can be observed with respect to MCVRP literature. For problems in both waste collection and agricultural contexts in particular, only one exact approach has been introduced so far, whereas this number is slightly higher for problems in grocery distribution (2), maritime transportation (3), conceptual problems (3), or fuel delivery (4) (see Table 9). Five of the methods proposed are branch-and-cut approaches, six are branch-and-price / column generation methods, and the remaining four are other exact methods such as branch-and-bound or Lagrangean approaches. (As some publications introduce no or multiple procedures, the sum of ratios does not necessarily add up to 100%.)

Heuristics approaches. Table 10 provides an overview of the heuristics approaches that have been applied. Metaheuristics that have been proposed at least three times are explicitly listed, i.e., TS, LNS, VNS, GA, ACO, MH, whereas all other heuristics are summarized within “other metaheuristics” and “non-metaheuristics”.

The majority of publications introduce metaheuristics, and LNS (11%), GA (13%), VNS (11%), and TS (7%) are the most frequently selected methods. All major types of metaheuristics have been proposed for conceptual problems. In contrast, there seems to be potential for the application of further approaches for problems in grocery distribution or maritime transportation, as LNS approaches have mostly been considered for the former, and GAs for the latter. For problems in fuel distribution and waste collection, the variety of methods appears to be slightly higher.

Future areas of research related to solution approaches. It goes without saying that opportunities exist for research into new, improved or alternative solution approaches, particularly because most algorithms proposed so far have been designed for very specific problems. As has been shown, only a minority of publications suggest exact approaches. There appears to be a lack of innovative concepts to solve MCVRPs to optimality or to combine exact and heuristic approaches to improve existing results. Furthermore, with increasing computational power, exact approaches may be able to solve instances of practice-relevant sizes, providing benchmarks and assessing the solution quality of heuristics.

With respect to heuristic approaches, the variety of methods that have been proposed is quite different among areas of application. Heuristic approaches that perform very well for one application could be evaluated for other problem variants as well. In particular, further population-based metaheuristics could be tested for problems in fuel distribution, further local search-based metaheuristics for problems in waste collection, agricultural contexts and maritime transportation, and both types of metaheuristics for problems in grocery distribution. Multi-period problems, multi-objective formulations or the further integration of relevant con-

Table 9
Overview of exact solution approaches.

	Exact total	Branch-and-cut	Branch-and-price/ column generation	Others
Fuel	4 (24)	1 (6)	2 (12)	2 (12)
Waste	1 (8)	1 (8)	0 (0)	0 (0)
Agriculture	1 (13)	1 (13)	0 (0)	0 (0)
Grocery	2 (22)	1 (11)	0 (0)	1 (11)
Maritime	3 (20)	0 (0)	3 (20)	0 (0)
Other	0 (0)	0 (0)	0 (0)	0 (0)
Conceptual	3 (16)	1 (5)	1 (5)	1 (5)
Total	14 (17)	5 (6)	6 (7)	4 (5)

Absolute number of papers introducing a certain type of approach; percentage share in brackets.

Table 10
Overview of heuristic solution approaches.

	Heuristics total	TS	LNS	VNS	GA	ACO	MH	Other MetaH	Non-MetaH
Fuel	14 (82)	1 (6)	1 (6)	4 (24)	1 (6)	0 (0)	3 (18)	0 (0)	6 (35)
Waste	11 (85)	0 (0)	0 (0)	2 (15)	3 (23)	2 (15)	1 (8)	3 (23)	2 (15)
Agriculture	6 (75)	2 (25)	0 (0)	1 (13)	1 (13)	0 (0)	1 (13)	1 (13)	1 (13)
Grocery	8 (89)	0 (0)	5 (56)	0 (0)	0 (0)	0 (0)	0 (0)	3 (33)	0 (0)
Maritime	7 (47)	0 (0)	0 (0)	0 (0)	3 (20)	0 (0)	0 (0)	0 (0)	4 (27)
Other	3 (100)	0 (0)	1 (33)	0 (0)	1 (33)	0 (0)	0 (0)	0 (0)	1 (33)
Conceptual	15 (79)	3 (16)	2 (11)	2 (11)	2 (11)	1 (5)	1 (5)	6 (32)	3 (16)
Total	64 (76)	6 (7)	9 (11)	9 (11)	11 (13)	3 (4)	6 (7)	13 (16)	17 (20)

Absolute number of papers introducing a certain type of approach; percentage share in brackets; TS: Tabu search; LNS: Large neighborhood search; VNS: Variable neighborhood search; GA: Genetic algorithm; ACO: Ant colony optimization; MH: Matheuristic; MetaH: Metaheuristics.

straints for time windows, weight, distance or duration will require alternative heuristics (e.g., ALNS). A solver suite that is adjustable to the different attributes would be a valuable contribution to MCVRP literature. This should include an in-depth comparison of heuristic performance related to the different attributes.

Innovative concepts are required in order to provide better forecasts of stochastic demands and stochastic travel and processing times. Such forecasts may be based, for instance, on machine learning concepts and approaches from prescriptive analytics that integrate data estimation and optimization. Furthermore, in order to identify parameters of heuristics that strongly influence the quality of obtained solutions and the computational performance of the algorithm, extensive numerical experiments are necessary. Self-governing methods for the algorithmic configuration are therefore becoming important. This includes the general solution approach as well as the parameter selection during the optimization procedure. Further attempts could be based on simulation optimization approaches.

Finally, the availability of test instances is currently restricted to individual publications. A comprehensive database that contains relevant data sets structured by area of application and attributes derived in this paper would be a good starting point for further research and benchmarking approaches.

6. Conclusion

The available literature on MCVRPs has steadily grown over the past decades and especially during recent years, totaling 84 publications so far. Most of these papers have been published during the last 10 years. This underlines the increasing importance of MCVs in research and industry, which comprises not only classical areas of application (fuel deliveries, for instance), but also newly emerging ones (such as bike sharing). Yet so far neither a uniform typology for MCVRPs nor a comprehensive survey on different MCVRP variants have been available in order to provide a concise overview of the literature, which appears to be rather heterogeneous. This literature review on MCVRPs is intended to fill this gap. We have introduced a conceptual model formulation that provides a basis for further model developments and covers existing MCVRP variants and extensions. Moreover, we have identified and defined attributes that allow the distinction of varying MCVRP variants and consolidated the distinct characteristics in a comprehensive typology for MCVRPs. We have analyzed the existing MCVRP literature using the typology and grouped MCVRPs within different areas of application and solution approaches. Finally, we have identified existing gaps in research and indicated future research opportunities. In short, especially a higher degree of flexibility using flexible compartment sizes, split customer demands or multiple periods, and variants adapted to real-world re-

quirements, such as dynamic or stochastic components, appear to be valuable directions for future research.

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