Safety first portfolio choice based on financial and sustainability returns

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A B S T R A C T

The aim of this paper is to expand the methodological spectrum of socially responsible investing by introducing stochastic sustainability returns into safety first models for portfolio choice. We provide a foundation of the notion of sustainability in portfolio theory and establish a general model for generalized safety first portfolio management with probabilistic constraints and three specifications of it. Moreover, we prove theorems about conditions for unique optimal solutions and for the constraints of one model being more restrictive than those of another. In an empirical part, we calculate the costs of investing according to our approach in terms of less financial return.

1. Introduction

In recent years, investors’ behavior has fundamentally changed. Although the financial return is still important for the investment decision, environmental, social, and governance (ESG) issues of investment opportunities are beginning to receive more and more attention. The amount of invested capital in sustainable funds has been rapidly increasing over the last few years. In Europe, the market for socially responsible investments (SRI) has amounted to 4986 billion euros as of December 31, 2009 (see Eurosif, 2010). The growth from 2005 to 2009 in the euro amount of SRI is 338%.

Principally, we build on the ideas of SRI. However, we do not regard an investment’s sustainability as an objective property like the financial return. On the contrary, an investment’s sustainability depends on the individual preferences of every single investor. Every kind of investment can have non-financial impacts that are desirable for some investors. These impacts are summarized under the term sustainability.

Furthermore, the standard approach in SRI consists of screening methods which select the investable assets in a first step and optimize the portfolio conventionally in a second. This paper expands the methodological spectrum of SRI in four different ways. First, we provide a comprehensive foundation of sustainability in portfolio theory with axioms and definitions. Second, we introduce a general model for generalized safety first portfolio management with probabilistic constraints. Additionally we discuss three variants of this general model and establish conditions for unique optimal solutions. Our approach is not to represent investors’ preferences with an inclusion or exclusion criterion for every single asset as screening does, but rather with a scalable quantity capturing the quality of sustainability of an investment as an additional stochastic objective variable. Third, we treat the case of deterministic sustainability returns and show that under this assumption two of the three models suggested are equivalent. Fourth, we show the practical applicability of our models and calculate financial costs of investing sustainably.

The remainder of the paper is organized as follows. After a concise literature review directly following, Section 3 covers sustainability ratings and quantifies the notion of an investment’s sustainability return. We introduce three different models for downside risk portfolio choice in Section 4 and prove results on the solvability of the constructed models. Section 5 comprises an empirical application, while Section 6 concludes.

2. Literature review

Renneboog et al. (2008) may serve as a comprehensive review of the developments and methods in SRI. Whereas Guerard (1997) studies the performance differences of portfolios with various screening criteria, Bello (2005), Hamilton et al. (1993) compare the performance of sustainable and common funds. All of them show that there is no significant under- or overperformance of sustainable funds. Galema et al. (2008) consider the impact of SRI on stock returns and conclude that SRI has a significant impact on the stock returns. Benson and Humphrey (2008) find that SRI fund flow is less sensitive to returns than the fund flow of conventional funds and that SRI investors are less concerned about returns than conventional investors. Boilen (2007) suggests measuring the utility of a portfolio with multi-attributive utility functions, but he still shapes the SRI optimization problem in a binary manner, using an indicator function for the fulfillment of SRI attitudes.

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In contrast, Hallerbach et al. (2004) give a practical approach for portfolio selection utilizing multi-attributive preference functions. Ballestero et al. (2012) provide a financial-ethical bi-criteria model especially for SRI portfolio selection. Furthermore, studies like Abdelaziz et al. (2007), Steuer et al. (2007) argue that portfolio selection is a multi-objective problem. While all of the above references shape the sustainability quantity as deterministic, Dorfliter et al. (2011) introduce the idea of stochastic social returns and incorporate them into a Markowitz-like portfolio selection framework.

In recent years safety first approaches and variants of it as discussed in Haley and Whiteman (2008) or Huang (2008), all of which have been based on the pioneering work of Telser (1955), Roy (1952), have increasingly gained attention due to the growing practical relevance of downside risk.

Instead of a two-step portfolio selection with sustainability screening first and financial optimization second, which all of the papers mentioned in the first paragraph use, we establish models with financial and sustainable real-valued objective variables. This idea is also considered by all papers cited in the second paragraph above. However, of these approaches only Dorfliter et al. (2011) uses stochastic sustainability returns. We contribute to the literature of SRI by using this new concept in the context of safety first portfolio choice.

3. Modeling sustainability value and sustainability return

The basic idea is that an investment is characterized by different quantities, namely the initial wealth $V^0$ at the beginning, the final wealth $V'$ and a value of sustainability at the end of the investment period $[0, t]$.

3.1. Sustainability ratings

The growing demand for sustainable investments introduces some associated developments. International committees pass standards for sustainability reporting like the AccountAbility 1000 AccountAbility Principles. Sustainable rating agencies develop rankings for the sustainability of companies according to these reports and additional information. Most of these rankings are based on positive and negative indicators. Rating agencies score non-monetary values of these different positive indicators for each investment and condense indicators into factors. The scores of factors are aggregated to a number describing the grade of sustainability of a company as a random number. Therefore, it is appropriate to view this quantity as a ‘sustainability return’. Negative indicators represent the set of exclusion criteria used for negative screenings. Some of the agencies only provide ordinal rankings, while others compute real returns, representing a cardinal order. The sustainability ratings are based on historical data and upcoming projects (see for example Forrest et al., 2006). Taking the future actions into account, it is natural to consider the sustainability of a company as a random number.

3.2. Measuring sustainability value and return

In our approach we commence with determining the objective sustainability return of every single investment with respect to a set $F$ of factors, taken from an existing sustainability rating. In a second step an individual investor aggregates these objective values according to her preferences.

The objective sustainability return $\text{OSR}^{\delta,F}_t(F, \omega)$ of factor $F$ and state $\omega$ in investment period $[s, t]$ for investment $j$ is directly given by a sustainability rating. In an ex ante view the objective sustainability returns are clearly random variables. Knowing the invested initial wealth $V_j^0$ in investment $j$, the objective sustainability return can be transformed into an objective sustainability value.

**Definition 1 (Objective sustainability value).** The objective sustainability value $\text{OSV}^{\delta,F}_t: \mathcal{F} \times \Omega \times \mathbb{R} \to \mathbb{R}$ of a factor $F \in \mathcal{F}$ is a real random number with sample space $\Omega$ representing the objective non-monetary value that is generated by factor $F$ of an investment $j$ at maturity $t$. The objective sustainability value depends on $V_j^0$ and is defined as

$$\text{OSV}^{\delta,F}_t(F, \omega, V_j^0) := V_j^0 \cdot \text{OSR}^{\delta,F}_t(F, \omega).$$

To shape the investor’s preferences, let $\delta \in \mathbb{R}$ be a real number and $F \in \mathcal{F}$ a factor of sustainable interest such as the environment. Then $\delta(F, \pi)$ denotes the strength of sustainable impact of factor $F$ on investor $\pi$. When factor $F$ has a positive impact on the investment decision of investor $\pi$, then $\delta(F, \pi) > 0$ holds. If an investor is indifferent with respect to factor $F$, we set $\delta(F, \pi) = 0$. An investor, who rejects the common interpretation of objective sustainability of factor $F$, has $\delta(F, \pi) < 0$. Using the notation of Definition 1 we can define the sustainability value of an investment $j$ for investor $\pi$.

**Definition 2 (Sustainability value).** The sustainability value $\text{SV}^{\delta,F}_t: \Omega \times \mathcal{F} \times \mathbb{R} \to \mathbb{R}$ of an investment $j$ is a real random number with sample space $\Omega$ representing the non-monetary value of an investment the investor $\pi$ receives at maturity $t$. Sustainability value

$$\text{SV}^{\delta,F}_t(\omega, \pi, V_j^0) := \sum_{F \in \mathcal{F}} \delta(F, \pi) \text{OSV}^{\delta,F}_t(F, \omega, V_j^0)$$

depends on the state $\omega$, the preference $\pi$ and initial wealth $V_j^0$.

Analogously to the objective sustainability return, a preference-dependent sustainability return exists.

**Definition 3 (Sustainability return).** The sustainability return $\text{SR}^{\delta,F}_t: \Omega \times \mathcal{F} \to \mathbb{R}$ of investment $j$ to investor $\pi$ in period $[s, t]$ with sample space $\Omega$ and preference space $\mathcal{F}$ is defined by

$$\text{SR}^{\delta,F}_t(\omega, \pi) := \frac{\text{SV}^{\delta,F}_t(\omega, \pi, V_j^0)}{V_j^0}.$$ 

An implication of the definitions from above is that the sustainability return can be expressed as a weighted sum over all factors $F$ of the objective sustainability returns with weights $\delta(F, \pi)$. Furthermore, sustainability returns evidently fulfill the property of portfolio additivity.

**Lemma 1 (Portfolio Additivity).** Let $w_1, \ldots, w_N$ the weights of $N$ assets with sustainability returns $\text{SR}^{\delta,F}_t$ ... $\text{SR}^{\delta,F}_t$. If the sustainability value of each factor is additive over different assets held then we have

$$\text{SR}^{\delta,F}_t = \sum_{j=1}^N w_j \text{SR}^{\delta,F}_t.$$ 

**Proof.** Lemma 1 follows straight from Definitions 2 and 3.

4. Model variants: some results and discussion

Next, we present and discuss three models for generalized safety first investors. All considerations below are based on one
single period; hence, we drop time and interval indices as well as parameters for the state and the investor’s preferences. Let $N \geq 2$ be the number of all assets the portfolio is supposed to comprise. Note that not all of these assets require a risky financial return. The weight of asset $i$ is $w_i$ and a well-defined portfolio satisfies $\sum_{i} w_i = 1$. In general, we permit short sales, which are characterized by negative $w_i$. There must be at least one risky asset with $w_i \neq 0$ to prevent computations from singularities.3

4.1. Approaches to safety first portfolio choice with stochastic sustainability returns

Even if it is not the standard view in SRI to model sustainability of an investment as a random number with a finite expected value and a variance, we regard this assumption as most realistic since even one can never predict to what extent the good intentions of an investor as a random number with a finite expected value and a variance, we regard this assumption as most realistic since every local optimal solution of a convex optimization problem is a global optimal solution. Below we present specifications of the general model representing three different approaches to deal with the tradeoff between $R$ and $SR$ in a safety first context. As the objective function we use the straightforward specification $\mathcal{P}(x,y) = (1 - \gamma)x + \gamma y$, where the scale parameter is $\gamma \in (0,1)$ with $\gamma = 0$ representing the case where only the financial return enters the objective function. We can derive the marginal rate of substitution between expected financial ($x$) and sustainability ($y$) return following

$$\partial \mathcal{P}(x,y)/\partial x = 1 - \gamma \quad \text{and} \quad \partial \mathcal{P}(x,y)/\partial y = \gamma$$

This quotient indicates how many units increase in the expected sustainability return one demands for a loss of one unit of the expected financial return while the objective function remains the same. Note that the sustainability return in general has one degree of freedom since the individual factors $\delta (\theta, F)$ could be multiplied by an arbitrary constant. However, when determining $\gamma$ according to the preferences, one has to take the general level of $SR$ into consideration and loses the degree of freedom again. Therefore $\gamma$ can be interpreted most easily if $R$ and $SR$ have approximately the same range. A value of $\gamma = 1/2$ would then imply that both returns are equally weighted in the objective function. If one then chooses to replace $SR$ by, for instance, $10 \cdot SR$, then $\gamma$ would have to change from $1/2$ to $1/11$ in order to express the same preferences as before.

4.1.1. General model

This subsection introduces the general structure of a portfolio problem of a generalized safety first investor with financial and sustainability interests. Let $A_i$ denote a random $I \times 2N$ matrix whose elements are multiples of $R$ and $SR$. Let be $t_2 = (1,1)^T \in \mathbb{R}^2$, then the Kronecker product $t_2 \otimes w$ denotes a column vector which is $w$ strung together twice. Let

$$K_i(w) := \mathcal{P} \left( \sum_{j=1}^{I} (A_i^{(j)}(t_2 \otimes w) \geq c_i^{(j)}) \right), \quad i = 1, \ldots , I$$

be a set of probabilities ($K(w) : \mathbb{R}^I \rightarrow [0,1]$) that depends on portfolio weights $w$, $R$ and $SR$ with vectors of thresholds $c_i \in \mathbb{R}^2$ and with $I \in \mathbb{N}$. Some appropriate specifications of $A_i^{(j)}$ and $c_i^{(j)}$ are listed in the following.

**Definition 4 (Generalized safety first investor).** An investor with financial and sustainable investment interests is called generalized safety first investor (GSFI), if she accepts a portfolio $w$ which fulfills every probability condition $K_i(w) \geq 1 - z_i$ for $i = 1, \ldots , I$ with a default probability $z_i \in \mathbb{R}$.

In general, a safety first investor maximizes a preference-dependent functional $\mathcal{P}(w \in \mathbb{R}^I, w \in \mathbb{R}^I)$ with $\partial \mathcal{P}/\partial w \mathbb{E}[R] \geq 0$, $\partial \mathcal{P}/\partial w \mathbb{E}[SR] \geq 0$ and $\mathcal{P} : \mathbb{R}^2 \rightarrow \mathbb{R}$ at the constraints given by Definition 4. Thus, the maximization problem is

$$\max_{w \in \mathbb{W}} \mathcal{P}(w \in \mathbb{E}[R], w \in \mathbb{E}[SR]) \quad \text{s. t.} \quad K_i(w) \geq 1 - z_i, \quad i = 1, \ldots , I \quad (1a)$$

with a convex set $\mathbb{W} \in \mathbb{R}^I$ with $\mathbb{W} := \{w \in \mathbb{R}^I | \sum_{i} w_i = 1 \}$, where $\mathbf{x}$ stands for $\mathbf{x}, \mathbf{y} \in \mathbb{R}, z \in \mathbb{R}$ and $b \in \mathbb{R}^n$. The set $\mathbb{W}$ contains all linear constraints on $w$ like lower and upper bounds $(l_u \in \mathbb{R}^I)$ for portfolio weights and other budget constraints, where $k \in \mathbb{N}$ is the number of all constraints. The risk preferences of every single investor are conveyed by the thresholds $c_i$ and the default probabilities $z_i$.

In general, problem (1) is a chance-constrained programming problem with random coefficient matrices $A_i$, which does not necessarily contain independent rows and is thus a non-convex programming problem. Kall and Mayer (2005, pp. 142–143) display the conditions under which problem (1) is an easy-to-solve convex programming problem. The intersection of two convex sets is convex. Thus, it is accurate to consider the probability constraints (1b) in this context only. It is also a well-known fact that every local optimal solution of a convex optimization problem is a global optimal solution. Below we present specifications of the general model representing three different approaches to deal with the tradeoff between $R$ and $SR$ in a safety first context. As the objective function we use the straightforward specification $\mathcal{P}(x,y) = (1 - \gamma)x + \gamma y$, where the scale parameter is $\gamma \in (0,1)$ with $\gamma = 0$ representing the case where only the financial return enters the objective function. We can derive the marginal rate of substitution between expected financial ($x$) and sustainability ($y$) return following

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4.1.2. Joint distribution model (JDM)

The first interpretation of a GSFI utilizes the joint probability distribution of financial and sustainability portfolio returns. A joint distribution type GSFI is defined as follows.

**Definition 5 (Joint distribution type GSFI).** An investor with preference functional $\mathcal{P}$ is called joint distribution type GSFI, if there are thresholds $c_F$ and $c_{SR}$ for financial and sustainability portfolio returns which she allows to underperform with joint probability $\gamma$. In this case the parameters of the general problem (1) are $I = 1, J = 2,

$$A = \begin{pmatrix} R_1 & \cdots & R_N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & SR_1 & \cdots & SR_N \end{pmatrix}, \quad C = \begin{pmatrix} c_F \\ c_{SR} \end{pmatrix}$$

and $z \in \mathbb{R}$. For a fixed portfolio $P$, we can determine the joint probability density function and the joint cumulative distribution function (cdf). The preference functional $\mathcal{P}$ is the convex combination of expected returns, where the relation between financial and sustainable quantities is reflected by $\gamma$. Hence, our first model for generalized portfolio choice is

$$\max_{w \in \mathbb{W}} \mathcal{P}[\{w | 1 - \gamma \mathbb{E}[R] + \gamma \mathbb{E}[SR] \} \cap \mathbb{W}], \quad \text{s. t.} \quad \mathcal{P}(w \in \mathbb{E}[R] + \gamma \mathbb{E}[SR]) \geq 1 - z \quad (2a)$$

with $c_F$ and $c_{SR}$ as thresholds and financial and sustainability portfolio returns $w \in \mathbb{E}[R] + \gamma \mathbb{E}[SR]$ are random numbers. The following theorem deals with the uniqueness of the solution if
The deterministic equivalent is not empty.

**Theorem 1.** The deterministic equivalent of chance-constrained programming problem (2) has a unique solution, if R and SR are multivariate normally distributed and the feasible set of the deterministic equivalent is not empty.

**Proof.** Solving problem (2) we issue a deterministic equivalent in the spirit of Watanabe and Ellis (1994) we consider a problem with multivariate normally distributed coefficients of matrix A. Therefore, as the deterministic equivalent of problem (2) we obtain

\[
\max_{w \in W} w' \left[ (1 - \gamma)E[R] + \gamma E[SR] \right] \quad \text{(3a)}
\]

s. t. \[\int_{Z_A} \int_{Z_B} \phi_2(z) dz_1 dz_2 \geq 1 - \alpha, \quad \text{(3b)}\]

where

\[
l(w) = \frac{c_S - \mu_{SR} w}{\sqrt{\text{var}(w)}} \quad \text{and} \quad l_{SR}(w) = \frac{c_{SR} - \mu_{SR} w}{\sqrt{\text{var}(w)}},
\]

\[
z_1(w) = \frac{R - \mu_R w}{\sqrt{\text{var}(w)}} \quad \text{and} \quad z_2(w) = \frac{SR - \mu_{SR} w}{\sqrt{\text{var}(w)}},
\]

\[
z = (z_1, z_2)^T, \quad \phi_2(w) = \left( \begin{array}{c} 1 \\ \rho(w_R, w_{SR}) \end{array} \right) \quad \text{and} \quad \phi_2(z) = \frac{1}{2\pi\sqrt{\text{det}(\phi(w))}} \exp \left\{ -\frac{1}{2} z' \phi(w)^{-1} z \right\}.
\]

The correlation coefficient of financial and sustainability portfolio returns is denoted by \(\rho(w_R, w_{SR})\) and the probability density function of the bivariate normal distribution with \(\mu = (0,0)\) and covariance matrix \(\phi(w)\) by \(\phi_2\). Constraint (3b) yields to a convex set because \(K(w)\) is quasi-concave due to the attitudes of the bivariate normal distribution. The objective function (3a) is constant, so it has a local optimum on the bounded feasible set, if this it not empty. Hence this solution is also a global optimal solution. □

4.1.3. Convolution model (CM)

The second approach is probably the most obvious one: We generate a convex combination of financial and sustainability portfolio returns.

**Definition 6** (Convolution type GSFI). An investor with preference functional \(\Psi\) is called convolution type GSFI, if there is a threshold \(c\) for the convex combination of financial and sustainability portfolio returns that he allows to underperform with probability \(\alpha\).

In this context, the problem is to maximize the combination of expected financial and sustainability portfolio returns under the constraint that the convex combination underperforms a threshold \(c\) with probability less than \(\alpha\). The parameters are \(l = 1, j = 1, A = ((1 - \gamma)\mathbf{R}, \ldots, (1 - \gamma)\mathbf{R}_0, \gamma \mathbf{S}_1, \ldots, \gamma \mathbf{S}_0), \quad c \in \mathbb{R} \quad \text{and} \quad \mathbf{z} \in \mathbb{R}.\) The formal notation is given by

\[
\max_{w \in W} w' \left[ (1 - \gamma)E[R] + \gamma E[SR] \right] \quad \text{(4a)}
\]

s. t. \[P(w_R \geq c_S) > 1 - \alpha, \quad \text{(4b)}\]

If \(\gamma = 0\) holds, an investor is not interested in sustainability; we obtain the standard safety first portfolio optimization problem in accordance with Telser (1955) without sustainable interests. On the contrary, an investor who is only interested in sustainability has \(\gamma = 1\). For each investor with \(0 < \gamma < 1\) and a fixed threshold \(c\), the constraint (4b) contains the tradeoff between financial and sustainable quantities. Let \(R_0 := (1 - \gamma)w_R + \gamma w_{SR}\) be the convex combination of financial and sustainability portfolio returns with \(\gamma \in (0,1)\).

**Theorem 2.** For multivariate normally distributed financial and sustainability portfolio returns the deterministic equivalent of problem (4) has a unique solution, if the feasible set is not empty.

**Proof.** A deterministic equivalent of problem (4) can be derived because of the normally distributed elements of \(\mathbf{z}\) the sum of 2N normally distributed random numbers \(R_i\) is normally distributed with parameters

\[
\mu_{\gamma} = w' \left[ (1 - \gamma)\mathbf{R}_0 + \gamma \mathbf{SR} \right],
\]

\[
\sigma_\gamma = \left( D(12 \otimes w) \right) \Sigma D(12 \otimes w) = \left( D(12 \otimes w) \right) \Sigma D(12 \otimes w) + \left( D(12 \otimes w) \right) \Sigma D(12 \otimes w) + \left( D(12 \otimes w) \right) \Sigma D(12 \otimes w),
\]

where \(\Sigma = \Sigma_{SR} S_{SR} + \Sigma_{SR} S_{SR} + \Sigma_{SR} S_{SR}\). The determinant of the convex combination of financial and sustainability portfolio returns is not constant, so it has a local optimum on the bounded feasible set, if this it not empty. Hence this solution is also a global optimal solution. □

4.1.4. Marginal distributions model (MDM)

The third model we describe in this paper differs from problems (2) and (4) in its constraints. Again, the objective function is the maximization of a convex combination of expected financial and sustainability portfolio returns. The risk constraints build on the marginal distributions of financial and sustainability portfolio returns.

**Definition 7** (Marginal type GSFI). An investor with preference functional \(\Psi\) is called marginal type GSFI, if there is a threshold \(c_R\) for the portfolio return which she allows to underperform with probability \(\alpha\) and a threshold \(c_{SR}\) for the portfolio sustainability return which she allows to underperform with probability \(\alpha_{SR}\).

The parameters are \(l = 2, j = 1, A_i = (R_i, \Gamma_i), \quad A = (0, 0, \ldots, 0), \quad A_j = (0, 0, \ldots, 0, \mathbf{SR}_1, \ldots, \mathbf{SR}_0), \quad (c_1, c_2) = (c_R, c_{SR})\) and \((\mathbf{z}_1, \mathbf{z}_2) = (\mathbf{z}_R, \mathbf{z}_{SR})\). The formal notation of the third model is given by

\[
\max_{w \in W} w' \left[ (1 - \gamma)E[R] + \gamma E[SR] \right] \quad \text{(6a)}
\]

s. t. \[P(w_R \geq c_R) > 1 - \alpha, \quad \text{(6b)}\]

\[
P(w_{SR} \geq c_{SR}) > 1 - \alpha_{SR}, \quad \text{(6c)}\]

Model (6) does not make use of any correlation between \(R\) and \(SR\). The lower bounds \(c_R\) and \(c_{SR}\) are fixed. Model (6) excludes all portfolios that underperform in one or both dimensions with a higher probability than \(\alpha_R\) and \(\alpha_{SR}\).

**Theorem 3.** If the marginal distributions of financial and sustainability portfolio returns are marginally normally distributed and the feasible set of the deterministic equivalent is not empty, a unique optimal solution exists.

**Proof.** The deterministic equivalent of problem (6) under normality assumptions is the convex programming problem (7)
max \ w'[(1 - \gamma)E[R] + \gamma E[SR]] \quad (7a)

s. t. \quad \Phi^{-1}(\alpha_0)\sqrt{w'\Sigma w} + w'\mu \geq c_R
\quad (7b)
\Phi^{-1}(\alpha_0)\sqrt{w'\Sigma w} + w'\mu \geq c_{SR}. \quad (7c)

with a local, thus global, optimal solution if the feasible set is not empty. ■

4.2. Joint vs. marginal distributions model

In this section we explore the connection between the feasible sets of the JDM and the MDM and proof a corresponding result. Both models differ from each other solely in terms of probability constraints. In contrast to the CM, where the probability constraint imitates the convex combination of the objective function and hence the feasible set depends on \( \gamma \), the feasible sets of the JDM and the MDM do not depend on that part of the investor’s preferences displayed by \( \gamma \). This implies a more flexible shaping. An investor can choose between financial and sustainable quantities in the objective function and the thresholds for both quantities in the probability constraints independently. By choosing \( \gamma = 0 \) both models are appropriate in shaping the preferences of an investor, who only maximizes the marginal distributions model’s optimal solution.

Theorem 4. Let the financial and sustainability portfolio returns of the marginal distributions model’s optimal solution (6) be distributed according to a bivariate distribution with positive quadrant dependence. Then the MDM becomes more restrictive than the JDM if

\[(1 - \alpha_0) \cdot (1 - \alpha_0) > 1 - \alpha.\]

Proof. Lehmann (1966) shows that every bivariate normal distribution with a positive correlation coefficient is positively quadrant dependent. In this case the proof follows straight from Theorem 4. ■

We display this observation in Appendix C in the supplementary material.

4.3. Safety first portfolio choice with deterministic sustainability returns

Since in state-of-the-art SRI practice a deterministic sustainability return is used, we now consider the GSFI approach under the assumption of deterministic sustainability returns. Let \( F_R(\cdot) \) be the appropriate cdf of the financial portfolio return \( w'R \). We consider a setup with a fixed investment period and a confidence level 1 – \( \alpha \) > 0.5. There are two different approaches derived from the general models in Section 4. The first one is the counterpart of the CM with a deterministic sustainability return, i.e.

max \ w'[(1 - \gamma)E[R] + \gamma SR] \quad (10a)

s. t. \quad 0 \geq c - \gamma w'SR - (1 - \gamma)F_R^{-1}(\alpha). \quad (10b)

We interpret this model as a sustainable shifted quantile model, which means that the \( \alpha \)-quantile of the financial portfolio return distribution is shifted by the sustainability return of the portfolio. The second model with deterministic sustainability return is deduced from the MDM and can be regarded as an improved screening portfolio selection. Hence, this model is given by

max \ w'[(1 - \gamma)E[R] + \gamma SR] \quad (11a)

s. t. \quad 0 \geq c_S - (1 - \gamma)F_R^{-1}(\alpha_S) \quad (11b)
0 \geq c_S - w'SR. \quad (11c)

Probability constraint (6c) from the general MDM is merged into a deterministic linear constraint (11c) and thereby unchanged in the deterministic equivalent. Therefore, a portfolio is feasible for this model if its sustainability return is higher than a given threshold and it satisfies the probability constraint for financial return. It is easy to show that the JDM is equivalent to (11) in this setting because without variability in the sustainability return the joint distribution essentially degenerates into the marginal distribution of the financial return with a deterministic value in the sustainability return which lies above the required level \( c_S \) with probability 0 or 1.

5. Empirical application and model comparison

To show the relevance of our theoretical models we next discuss several aspects of an empirical implementation. Here we also discuss the economic consequences of our approach.

5.1. Computational complexity

First, we analyze the computational effort of finding the optimal portfolio for 3, 50, 100 and 630 assets. The results are displayed in Table 1 in relation to the minimal computational time. Obviously, the JDM is the one with the highest computational effort. But the optimal solution of the portfolio problem with 630 risky assets from our sample lasts 34.5 minutes. In contrast, the CM and MDM have very low computational costs because there are no multivariate probabilities to deal with.

\footnote{This computation exercise was conducted under the assumption of normally distributed returns and with realistic values of all expected values, variances and covariances.}
5.2. Data and empirical methodology

To apply our methodology, the expected values, variances and covariances of the financial and sustainability returns need to be estimated using real world data. To compute the sustainability returns, we use annual ESG scores from the sustainability rating agency Inrate for the years dating from 2005 to 2009 for all companies in their rating universe. These ESG (Environment, Social, Governance) scores consist out of a high number of indicators, which score the efforts of each company in several fields of environmental protection, social issues and corporate governance issues. Corresponding to the shaping of objective and subjective sustainability returns, the scores of these indicators, that can be seen as the objective sustainability returns, are aggregated by special coefficients to a subjective sustainability return. In our case, we take the given coefficients from Inrate as the considered investors’ preference and use the provided ESG score as the subjective sustainability return for each asset. Basically, we suggest that not only ratings like the ones by Inrate can be used for the aggregation of subjective sustainability returns. Moreover, every investor gathers benefits from non-financial factors, which also can comprise being a home-biased investor or a sport supporter and several further issues. Every investor is required to score these factors to his own preference. Ratings provided by independent rating agencies are useful proxies for a wide set of factors but cannot cover all conceivable preferences. As we wish to be as objective as possible, we do not take into account further preferences except the ones incorporated into the Inrate ESG scores.

The range of the score is [0,100]. The company with the best score in 2009 had an ESG score of 90, and the one with the lowest had an ESG score of 38. As we are required to estimate returns and variance, we drop all companies that do not have annual ESG scores and obtain a sample with 735 stocks, most of which are also included in the MSCI World Index. We export monthly financial stock prices and market capitalization in USD from Thomson Reuters Datastream and estimate the monthly financial quantities using an expected weighted moving average model with decay factor 0.97 for a period dating from January 1, 1990 to December 1, 2009. We use annualized expected financial returns and variances in our portfolio choice.

Following the sector-based screening analysis in the SRI literature (Guerard, 1997), we use ten market-capitalization weighted subportfolios, each consisting of ten randomly chosen companies belonging to the same industrial sector. These ten subportfolios are the investment instruments, which an investor uses to develop her portfolio. To achieve sustainability returns which are in a comparable range to the financial returns we linearly transform the ESG scores $S_j$ by $SR_j = \left( S_j - 50 \right) \cdot E[R]/[E(S_j) - 50]$ for all subportfolios $j \in \{1, \ldots, 10\}$. The ESG score of 50 is the minimum score a sustainably acceptable company should reach. Thus, values below 50 should correspond to a negative $SR$.

The covariance matrices of the sustainability returns are estimated using an expected weighted moving average model with decay factor 0.97 for the years between 2005 and 2009, while the expected sustainability returns are shown by the current scores. The expected financial and sustainability returns of each subportfolio are displayed in Table A.1, the estimated covariance matrices $\Sigma_F$, $\Sigma_R$ and $\Sigma_{SR}$ in Appendix A in the supplementary material. Both financial and sustainability returns are assumed to be multivariate normally distributed.\(^5\)

\(^5\) We check this by applying Royston’s test (Royston, 1982) to our financial and sustainability returns and find that our assumption of multivariate normally distributed returns cannot be rejected at any reasonable significance level.

The covariance matrices of the sustainability returns are estimated using real world data. To compute the sustainability returns, we use annual ESG scores from the sustainability rating agency Inrate for the years dating from 2005 to 2009 for all companies in their rating universe. These ESG (Environment, Social, Governance) scores consist out of a high number of indicators, which score the efforts of each company in several fields of environmental protection, social issues and corporate governance issues. Corresponding to the shaping of objective and subjective sustainability returns, the scores of these indicators, that can be seen as the objective sustainability returns, are aggregated by special coefficients to a subjective sustainability return. In our case, we take the given coefficients from Inrate as the considered investors’ preference and use the provided ESG score as the subjective sustainability return. Every investor is required to score these factors to his own preference. Ratings provided by independent rating agencies are useful proxies for a wide set of factors but cannot cover all conceivable preferences. As we wish to be as objective as possible, we do not take into account further preferences except the ones incorporated into the Inrate ESG scores.

5.3. Results

Now we calculate the optimal portfolios for the three models and different values of $\gamma$ and the thresholds.\(^6\) As a first model consistency check we compare the three models with respect to the optimal portfolios’ probabilities of exceeding the thresholds.\(^7\) We find that the CM may generate outcomes that violate the maximum probabilities very clearly. This is a phenomenon, which is implied by the effect that both return dimensions compensate for each other in this model.

Table 1

<table>
<thead>
<tr>
<th>Number of assets</th>
<th>3</th>
<th>50</th>
<th>100</th>
<th>630</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint distribution model</td>
<td>7.3</td>
<td>303.4</td>
<td>820.6</td>
<td>31417.0</td>
</tr>
<tr>
<td>Convolution model</td>
<td>1.0</td>
<td>25.4</td>
<td>69.0</td>
<td>11913.0</td>
</tr>
<tr>
<td>Marginal distributions model</td>
<td>1.0</td>
<td>27.7</td>
<td>73.1</td>
<td>9615.4</td>
</tr>
</tbody>
</table>

5.3.1. Portfolio weights variation

When considering the objective value, the expected financial and the expected sustainability return of the optimal portfolios for $\gamma = 0.9$ dependent on $c_{SR}$, the functions reveal segments with constant levels. This is due to the weight constraints. Table 2 displays how the portfolio weights change when increasing the sustainability threshold for $\gamma = 0.5$ in the JDM.\(^8\) Furthermore, we provide the expected financial and sustainability portfolio returns as well as the financial and sustainability portfolio variance of the optimal solutions in each case. Obviously, by increasing the threshold $c_{SR}$ as displayed here, riskier portfolios with weights on the upper and lower bounds are changed into more diversified portfolios. The result of this observation is a decrease in expected financial portfolio return, a decrease in financial portfolio standard deviation and an increase in expected sustainability portfolio return as well as a decrease in sustainability portfolio standard deviation, because a higher sustainable threshold $c_{SR}$ implies a higher sustainability risk aversion and simultaneously a higher demand for expected sustainability return.

5.3.2. Financial cost analysis

In the previous subsection one could see that the impact of a variation of $c_{SR}$ is higher for investors with small $\gamma$ since these investors mainly maximize the expected financial portfolio return. In Fig. 1, the line for an investor with $\gamma = 0$ shows how strongly the expected financial return decreases if the investor increases her sustainable threshold $c_{SR}$. Hence, the loss in the expected financial return can be viewed as the financial costs of a sustainable investment. We investigate this issue more precisely. As a benchmark, we model an investor with no sustainable interest, i.e. $\gamma = 0$ and $c_{SR} = \infty$ in the JDM and MDM. Basically, this kind of investor is only interested in financial risk and return.\(^9\) Next, we increase the

\(^6\) We restrict each portfolio weight to the interval $[0, 0.25]$. The values for $a$ are 0.13 in the JDM, 0.067 in the CM and $a = 0.00375$, $a = 0.04$ in the MDM.

\(^7\) Cf. Figure D.3 in the supplementary material.

\(^8\) We provide further tables with portfolio weights for several model and parameter combinations in Appendix E in the supplementary material.

\(^9\) With our GSFI models applied to $\gamma = 0$ (and $c_{SR} = \infty$ in the JDM and MDM) the optimal portfolios are even $\mu$-$\sigma$-efficient in the sense of Markowitz, at least for normally distributed financial returns (Arzac and Bawa, 1977), which is the assumption we apply.
Table 2
Portfolio weights of the optimal solutions of the JDM with \( \gamma = 0.5 \) in percent. It is due to rounding errors that the weights do not amount to 100 in all cases.

<table>
<thead>
<tr>
<th>( c_{SR} )</th>
<th>-0.020</th>
<th>-0.013</th>
<th>-0.006</th>
<th>0.001</th>
<th>0.008</th>
<th>0.015</th>
<th>0.022</th>
<th>0.026</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>24.8</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>11.5</td>
</tr>
<tr>
<td>( w_4 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( w_5 )</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>13.7</td>
</tr>
<tr>
<td>( w_6 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( w_7 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( w_8 )</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( w_9 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>2.0</td>
<td>11.1</td>
<td>22.5</td>
<td>25.0</td>
</tr>
<tr>
<td>( w_{10} )</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>23.0</td>
<td>14.0</td>
<td>6.2</td>
<td>0.0</td>
</tr>
<tr>
<td>( \mu_\delta )</td>
<td>0.144</td>
<td>0.144</td>
<td>0.144</td>
<td>0.144</td>
<td>0.143</td>
<td>0.136</td>
<td>0.126</td>
<td>0.114</td>
</tr>
<tr>
<td>( \sigma_\delta )</td>
<td>0.229</td>
<td>0.229</td>
<td>0.229</td>
<td>0.229</td>
<td>0.229</td>
<td>0.226</td>
<td>0.213</td>
<td>0.200</td>
</tr>
<tr>
<td>( \mu_{SR} )</td>
<td>0.115</td>
<td>0.115</td>
<td>0.115</td>
<td>0.115</td>
<td>0.115</td>
<td>0.114</td>
<td>0.116</td>
<td>0.120</td>
</tr>
<tr>
<td>( \sigma_{SR} )</td>
<td>0.029</td>
<td>0.029</td>
<td>0.029</td>
<td>0.029</td>
<td>0.029</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Table 3
Expected financial returns and costs of investing sustainably in the JDM for varying \( c_{SR}, c_{CR} \) and \( \gamma \). Line 2 displays the expected financial returns of the optimal portfolio without considering sustainability. Lines 3–6 display the differences between the optimal portfolios’ expected financial returns (according to the JDM) and line 2. The last column displays the average difference for \( c_{SR} \in [-0.2, -0.108] \).

<table>
<thead>
<tr>
<th>( c_{SR} )</th>
<th>-0.200</th>
<th>-0.177</th>
<th>-0.154</th>
<th>-0.131</th>
<th>-0.108</th>
<th>Average diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{CR} = -\infty, \gamma = 0 )</td>
<td>0.149</td>
<td>0.149</td>
<td>0.149</td>
<td>0.149</td>
<td>0.146</td>
<td>-0.005</td>
</tr>
<tr>
<td>( c_{CR} = 0, \gamma = 0 )</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.002</td>
<td>-0.009</td>
<td>-0.027</td>
<td>-0.005</td>
</tr>
<tr>
<td>( c_{CR} = 0, \gamma = 0.1 )</td>
<td>-0.005</td>
<td>-0.005</td>
<td>-0.005</td>
<td>-0.009</td>
<td>-0.027</td>
<td>-0.008</td>
</tr>
<tr>
<td>( c_{CR} = 0, \gamma = 0.5 )</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( c_{CR} = 0, \gamma = 0.9 )</td>
<td>-0.034</td>
<td>-0.035</td>
<td>-0.036</td>
<td>-0.043</td>
<td>-0.028</td>
<td>-0.036</td>
</tr>
</tbody>
</table>

sustainability threshold to \( c_{SR} = 0 \), which reflects a sustainability return equivalent to an ESG score of 50, and optimize portfolios for \( \gamma \in \{0, 0.1, 0.5, 0.9\} \). Note that for \( \gamma = 0 \) the investor is not interested in maximizing the sustainability return, but takes sustainability into account by the constraints in the JDM and the MDM.

In the JDM and the MDM, for fixed model specific default probabilities, a fixed sustainable threshold \( c_{SR} \) and a fixed \( \gamma \), the only free parameter with effect on the feasible set is the financial threshold \( c_{CR} \). An increase in \( c_{CR} \) means that the riskiness of the portfolio declines. Therefore, we also run the optimization for varying \( c_{CR} \). In line with Adler and Kritzman (2008) we regard the difference between the expected financial returns of a conventional and a sustainable investment as the cost of socially responsible investing. The expected return a sustainable investor achieves is compared to the optimal return at a threshold \( c_{SR} \), which is used as our term of risk in the following. We calculate the cost of sustainable interests by determining the difference between the optimal expected financial returns, i.e. without considering sustainability, and the expected financial return of the model and the parameter setting under consideration. Tables 3 and 4 display the results for the JDM. We receive similar results for the MDM, which are shown in Tables 5 and 6. Note that the feasible interval for \( c_{SR} \) is smaller for the MDM as compared to the JDM, an observation which is due to the fact that the MDM here is more restrictive than the JDM (cf. Theorem 4).

The methodology for the financial cost comparison for the CM is different to that of the JDM and the MDM. Because the common threshold depends on the financial and sustainability thresholds as well as on \( \gamma \), we would not obtain comparable results for different \( c_{SR} \) and different \( \gamma \). Therefore, we repeatedly use the combined threshold as the term of risk and calculate the optimal portfolio \( \bar{w} \) for a specific parameter set first. Subsequently, we take an inverse optimization step to calculate which threshold \( \bar{c} \) is suitable for receiving the calculated portfolio \( \bar{w} \) as the optimal portfolio of the financial investor optimization problem (\( \gamma = 0 \)). In a final step we optimize with the threshold \( \bar{c} \) and \( \gamma = 0 \). The optimal portfolio
Table 4
Expected sustainability returns and benefits of investing sustainably in the JDM for varying $c_p$, $c_{hp}$ and $\gamma$. Line 2 displays the expected sustainability returns of the optimal portfolio without considering sustainability. Lines 3–6 display the differences between the optimal portfolios’ expected sustainability returns (according to the JDM) and line 2. The last column displays the average difference for $c_k \in [-0.2, -0.108]$.

<table>
<thead>
<tr>
<th>$c_k$</th>
<th>$0.200$</th>
<th>$0.177$</th>
<th>$0.154$</th>
<th>$0.131$</th>
<th>$0.108$</th>
<th>Average diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_k = -\infty, \gamma = 0$</td>
<td>0.106</td>
<td>0.106</td>
<td>0.106</td>
<td>0.106</td>
<td>0.104</td>
<td>–</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
<td>0.008</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.1$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
<td>0.008</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.5$</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.9$</td>
<td>0.024</td>
<td>0.024</td>
<td>0.024</td>
<td>0.022</td>
<td>0.007</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 5
Expected financial returns and costs of investing sustainably in the MDM for varying $c_p$, $c_{hp}$ and $\gamma$. Line 2 displays the expected financial returns of the optimal portfolio without considering sustainability. Lines 3–6 display the differences between the optimal portfolios’ expected financial returns (according to the MDM) and line 2. The last column displays the average difference for $c_k \in [-0.2, -0.116]$.

<table>
<thead>
<tr>
<th>$c_k$</th>
<th>$-0.200$</th>
<th>$-0.179$</th>
<th>$-0.158$</th>
<th>$-0.137$</th>
<th>$-0.116$</th>
<th>Average diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_k = -\infty, \gamma = 0$</td>
<td>0.149</td>
<td>0.149</td>
<td>0.148</td>
<td>0.140</td>
<td>0.110</td>
<td>–</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0$</td>
<td>0.000</td>
<td>0.000</td>
<td>–0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>–0.000</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.1$</td>
<td>–0.000</td>
<td>–0.000</td>
<td>–0.000</td>
<td>–0.000</td>
<td>–0.000</td>
<td>–0.000</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.5$</td>
<td>–0.005</td>
<td>–0.005</td>
<td>–0.003</td>
<td>–0.002</td>
<td>–0.001</td>
<td>–0.003</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.9$</td>
<td>–0.035</td>
<td>–0.035</td>
<td>–0.035</td>
<td>–0.032</td>
<td>–0.033</td>
<td>–0.027</td>
</tr>
</tbody>
</table>

Table 6
Expected sustainability returns and benefits of investing sustainably in the MDM for varying $c_p$, $c_{hp}$ and $\gamma$. Line 2 displays the expected sustainability returns of the optimal portfolio without considering sustainability. Lines 3–6 display the differences between the optimal portfolios’ expected sustainability returns (according to the MDM) and line 2. The last column displays the average difference for $c_k \in [-0.2, -0.116]$.

<table>
<thead>
<tr>
<th>$c_k$</th>
<th>$0.200$</th>
<th>$0.177$</th>
<th>$0.154$</th>
<th>$0.131$</th>
<th>$0.108$</th>
<th>Average diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_k = -\infty, \gamma = 0$</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
<td>–</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.003</td>
<td>0.008</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.1$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.003</td>
<td>0.008</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.5$</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.9$</td>
<td>0.024</td>
<td>0.024</td>
<td>0.024</td>
<td>0.022</td>
<td>0.007</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 7
Financial costs 0.000 0.000 0.000 0.000 0.000 0.000

Table 8
Expected sustainability returns and benefits of investing sustainably in the CM for varying $c_p$, $c_{hp}$ and $\gamma$. Line 2 displays the expected sustainability returns of the optimal portfolio without considering sustainability. Lines 4, 6 and 8 display the optimal portfolios’ expected sustainability returns. Lines 3, 5, 7 and 9 contain the differences between the optimal portfolios’ expected sustainability returns (according to the CM in the appropriate value of risk) and the corresponding line above. The last column displays the average difference for $c_k \in [-0.2, -0.148]$.

<table>
<thead>
<tr>
<th>$c_k$</th>
<th>$0.200$</th>
<th>$0.177$</th>
<th>$0.154$</th>
<th>$0.131$</th>
<th>$0.108$</th>
<th>Average diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_k = -\infty, \gamma = 0$</td>
<td>1.06</td>
<td>0.107</td>
<td>0.108</td>
<td>0.104</td>
<td>0.098</td>
<td>–</td>
</tr>
<tr>
<td>Sustainable benefits</td>
<td>0.00</td>
<td>–0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
<td>–0.000</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.1$</td>
<td>0.106</td>
<td>0.109</td>
<td>0.113</td>
<td>0.109</td>
<td>0.108</td>
<td>–</td>
</tr>
<tr>
<td>Sustainable benefits</td>
<td>0.00</td>
<td>0.002</td>
<td>0.007</td>
<td>0.009</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.5$</td>
<td>0.106</td>
<td>0.109</td>
<td>0.113</td>
<td>0.109</td>
<td>0.108</td>
<td>–</td>
</tr>
<tr>
<td>Sustainable benefits</td>
<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$c_k = 0, \gamma = 0.9$</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td>–</td>
</tr>
<tr>
<td>Sustainable benefits</td>
<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
possesses the same risk in terms of the CM and only maximizes financial quantities. Hence, we use this portfolio as the financial benchmark model and calculate the differences to the sustainable investor’s portfolio. The results are shown in Tables 7 and 8.

The results show that JDM, CM and MDM investors have to give up expected financial return if they account for sustainability. For instance, a sustainable investor with $\gamma = 0.5$ (and $c_{SR} = 0$) loses, on average, 0.6% in the JDM and 0.3% in the MDM, both with regard to the expected financial return. In reverse, she increases her expected sustainability return by an absolute value, on average, by 0.8% for the JDM and by 0.8% for the MDM. In this setting, the CM choses all weights equal to the lower or upper bound and thus does not reveal any variation. Finally, Fig. 2 visualizes the variation of the expected financial return in all three models for varying parameters $c_{SR}$, $c_{SR} \gamma$ and $\gamma$.

6. Conclusion

We present a mathematical framework for modeling the sustainability return as a computable quantity. Based on sustainability ratings, one can derive objective sustainability returns of every sustainability dimension of an investment asset. These objective sustainability returns are then aggregated linearly according to the investor’s preferences. Based on these considerations we present a general model for safety first portfolio selection with stochastic financial and sustainability returns and introduce the notion of sustainability return, the conditions determining the feasible set may vary. We specify this general model in three different forms, namely the convolution type, the marginal distributions type and the joint distribution type generalized safety first investor, and show that each model has an optimal solution under the assumptions. The models presented are suggestions for the real world investors. Our results only concern theoretical properties and implementational details of these models. It is up to the investors to find those variants best describing their views and needs.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2012.02.034.

References


